The Intrinsic Shape Distribution of a Sample of Elliptical Galaxies

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ABSTRACT

We apply the dynamical modeling approach of Statler (1994b) to 13 elliptical galaxies from the Davies and Birkinshaw (1988) sample of radio galaxies to derive constraints on their intrinsic shapes and orientations. We develop an iterative Bayesian algorithm to combine these results to estimate the parent shape distribution from which the sample was drawn, under the assumption that this parent distribution has no preferred orientation. In the process we obtain improved estimates for the shapes of individual objects. The parent shape distribution shows a tendency toward bimodality, with peaks at the oblate and prolate limits. Under minimal assumptions about the galaxies’ internal dynamics, 35% of the objects would be strongly triaxial ($0.2 < T < 0.8$).

However, the parent distribution is sensitive to the assumed orbit populations in the galaxies. Dynamical configurations in which all galaxies rotate purely about either their long or short axes can be ruled out because they would require the sample to have a strong orientation bias. Configurations in which the mean motion about the short or long axis is either “disklike”—dropping off away from the symmetry planes—or “spheroidlike”—remaining roughly constant at a given radius—are equally viable. Spheroidlike rotation in the long-axis or short-axis tube orbits significantly lowers the abundance of prolate or oblate galaxies, respectively. If rotation in ellipticals is generally disklike, then triaxiality is rare; if spheroidlike, triaxiality is common.

Subject headings: galaxies: elliptical and lenticular, cD—galaxies: kinematics and dynamics—galaxies: structure
1. Introduction

Over the years a number of attempts have been made to derive the intrinsic shape distribution of elliptical galaxies from observations (Hubble 1926, Sandage et al. 1970, Noerdlinger 1979, Marchant & Olson 1979, Richstone 1979, Binggeli 1980, Binney & de Vaucouleurs 1981; for a review see Statler 1996). Generally the results of these efforts have been ambiguous, and interest in the problem waned somewhat in the 1980s. But more recent developments have sparked renewed attempts to crack this classic chestnut. Among these developments are the recognition that halo shapes may serve as a diagnostic of galaxy formation physics (Dubinski & Carlberg 1991, Weil & Hernquist 1996), and indications that Hamiltonian chaos, dissipation, or both may either force triaxial equilibrium configurations to evolve slowly toward axisymmetry or render them altogether impossible (Dubinski 1994, Merritt & Fridman 1996, Merritt & Quinlan 1998).

Studies of central surface brightness profiles using HST suggest that the fundamental properties of elliptical galaxies may be bimodally distributed. There appears to be a dichotomy between high-luminosity, slowly rotating systems with shallow central cusps and boxy isophotes, and lower luminosity, rotationally supported systems with steeper cusps and a tendency for diskiness (Lauer et al. 1995; see also Kormendy & Bender 1996). Since rapid rotation and triaxiality are generally regarded as being incompatible, one might anticipate a bimodal distribution of triaxialities. Tremblay & Merritt (1996) find that low- and high-luminosity elliptical galaxies have different distributions of apparent ellipticity, which would imply different distributions of true shapes. Merritt & Tremblay’s work joins that of Fasano & Vio (1991), Ryden (1992, 1996), and Fasano (1996) as successors to the classical photometric approaches pioneered by Hubble, Sandage, and others.

However, photometric methods, while effective in constraining the distribution of
overall flattenings, reveal little about the frequency of axisymmetry vs. triaxiality in the population. Uncovering this information requires the use of kinematic data and dynamical models to connect the kinematics to the shape of the gravitational potential. In the rare cases where well-defined, equilibrium gas disks are present, emission-line kinematics can yield excellent constraints on the shape of the potential if one assumes that the gas is on closed orbits (Bertola et al. 1991). But for the majority of ellipticals, methods relying primarily on stellar kinematics are essential. Approaches of this type were originated by Binney (1985) and enlarged upon by Franx et al. (1991) and Tenjes et al. (1993). Statler (1994a, 1994b) introduced major refinements, including improved dynamical models and a Bayesian approach to model fitting. This method has great potential to place quite narrow constraints on the triaxialities of individual galaxies for which very high quality stellar kinematic data are available. Unfortunately, the number of such galaxies is still very small, and is likely to increase at only a modest pace in the short term.

Our goal in this paper is to see what can be learned from the larger sample of galaxies with stellar kinematic data of less-than-ideal quality already in the literature. We focus on the Davies & Birkinshaw (1988, hereafter DB) sample of radio ellipticals, all of which have kinematic data on multiple position angles and are photometrically well studied. In the process, we extend the statistical methods of Statler (1994b) and show how to estimate the parent shape distribution from a sample of galaxies for which the data may be very inhomogeneous. Our method will thus continue to be generally applicable as new data are obtained.

In the next section of the paper we describe the general statistical approach for determining the parent distribution of a set of intrinsic quantities from measurements of related, but different, observable quantities, and show the particular application of this approach to the shape problem. In § 3, we discuss our treatment of the data and define a
subsample of the DB galaxies which we are able to model reliably. Section 4 presents the results for the parent distribution, and examines systematic effects relating to unknown aspects of the stellar dynamics. Section 5 compares our results to those of previous studies, and § 6 sums up.

2. Estimating the Parent Distribution

2.1. Basic Idea

In previous papers (Statler 1994b, 1994c, Statler et al. 1999) we describe a Bayesian approach to inferring the intrinsic shapes of individual elliptical galaxies, based on dynamical models that predict the mean radial velocity field (VF) by solving the equation of continuity for the stellar “fluid.” For triaxial systems with negligible figure rotation, the streamlines of the mean motion in the main families of circulating orbits are dictated by the triaxiality of the mass distribution; thus for a given shape, orientation, and set of boundary condition parameters describing the internal orbit populations, the line-of-sight VF can be calculated. The calculation of the VF is very fast, so the multidimensional parameter space can be adequately explored.

For each set of parameters, we calculate the probability that the observed VF and surface brightness distribution would, with the known observational errors, be obtained from the corresponding model. This yields a multidimensional likelihood function $L(T, c, \Omega, \mathbf{d})$, where $T$ is the triaxiality of the total mass distribution, $c$ is the short-to-long axis ratio of the luminosity distribution, $\Omega = (\theta, \phi)$ is the orientation of the galaxy, and the vector $\mathbf{d}$ represents the remaining dynamical parameters (see § 2.4). The likelihood is integrated over an assumed prior distribution in the dynamical parameters $\mathbf{d}$ and an isotropic prior
distribution in $\Omega$ to give a two-dimensional likelihood $L(T,c)$.\(^3\) To obtain the Bayesian estimate of the galaxy’s shape, $L(T,c)$ is multiplied by a model for the parent shape distribution and normalized. In most of our previous work, this model has been a flat distribution, $F(T,c) = \text{const}$, meaning that we have estimated the shape of each galaxy in isolation. The likelihood $L_i(T,c)$ for each galaxy $i$ is basically a data point with an error ellipse, i.e., a measurement of $T$ and $c$. Of course, in this case the errors are strongly non-Gaussian since we work with the actual probability distribution. The goal now is to combine these measurements for a sample of galaxies into an estimate of the parent distribution from which the sample was drawn.

Our algorithm is conceptually simple. We start with a flat model for the parent distribution, $F(T,c) = \text{const}$. The Bayesian posterior probability density $P_i(T,c)$ for each galaxy is the normalized product of $L_i$ with the model parent $F$. (At this stage, $P_i = L_i$.) We stack the $P_i$’s on top of each other, add them up, smooth the sum with a nonparametric smoothing spline, and normalize the result. This gives us an improved model for the parent distribution $F$, which we multiply by the $L_i$’s and feed iteratively into the same procedure. Note that, after the first iteration, the statistical estimate of the shape of each galaxy in the context of the whole sample, $P_i$, is different from the estimate of its shape in isolation, $L_i$. This difference arises from the requirement that the sample be drawn from an isotropic distribution of orientations. Note also that since all operations are performed on distributions that are already integrated over $\Omega$, the isotropy of the parent distribution is guaranteed.

A one dimensional toy problem can demonstrate that this algorithm works, even when the $L_i$ distributions are strongly non-Gaussian. Consider a set of objects, each with a value of some intrinsic property $X$ between 0 and 1. $X$ is not measurable, but a related quantity

\(^3\)See Statler et al. (1999) for a more rigorous formulation of this statement.
$x$ is. Suppose that for any object an observer has a uniform probability of measuring any value of $x$ between 0 and $X$. It is easy to show that a single measurement $x_i$ implies a likelihood $L_i(X)$ that is zero for $X < x_i$ and proportional to $X^{-1}$ for $X > x_i$. Figure 1 shows the algorithm in action. The likelihoods $L_i(X)$ for 9 measured $x$ values are shown in Fig. 1a. These functions are multiplied by the initially flat parent distribution (b), summed (c), and smoothed to produce the new parent (d). The result after 20 iterations (e, solid line) is a decent representation of the true parent distribution (dotted line). The functions $P_i(X)$ giving the estimates of the $X$ values for the individual objects (f) differ from the original $L_i$’s but are consistent with the parent distribution.

### 2.2. Statistical Rationale

The stack-smooth-iterate algorithm is a general technique that is closely related to Lucy’s Method (Lucy 1974) and penalized likelihood (Wahba & Wendelberger 1980, Silverman 1986, Green & Silverman 1994).\(^4\) Readers uninterested in the statistical details are welcome to skip directly to § 2.5.

Imagine a population of objects, each of which possesses some value of an intrinsic property (or set of properties) $X$, distributed according to the parent distribution $F_p(X)$. Let there be an observable quantity (or set of quantities) $x$ which is related to $X$ by a conditional probability distribution $P(x|X)$. The distributions are normalized so that $\int dX F_p(X) = \int dx P(x|X) = 1$. From models, we calculate the likelihood $L(X|x)$ that a given measurement of $x$ was obtained from an intrinsic value $X$. We assume that the

\(^4\)Our method is similar, but not identical, to a well-established approach developed by Wahba & Wendelberger (1980). We are grateful to the referee for directing us to this important paper.
likelihoods are explicitly normalized so that $\int dX\, L(X|x) = 1$. If the models take into account all physical effects and measurement errors exactly, then $L(X|x)$ and $P(x|X)$ are mathematically the same function. For measurements $x$, the estimate of $X$ is given by the posterior density,

$$P(X) = \frac{F(X)L(X|x)}{\int dX\, F(X)L(X|x)},$$  \hspace{1cm} (1)

where $F(X)$ is the current estimate of the parent distribution. This is the standard Bayesian approach for individual objects.

The goal is to find the parent distribution $F(X)$ that maximizes the joint probability of obtaining the measurements $x_i$ for the set of $n$ objects $i = 1, \ldots, n$. The logarithm of this probability is given by

$$\ln P = \ln \prod_i \int dX\, F(X)P(x_i|X) = \sum_i \ln \int dX\, F(X)L(X|x_i).$$ \hspace{1cm} (2)

If we write the measurements in terms of a distribution of observables, $W(x) = \sum_i \delta(x - x_i)$, then we can interpret the integral

$$W_m(x) \equiv \int dX\, F(X)L(X|x)$$ \hspace{1cm} (3)

as giving the distribution of observables that would be predicted if the model parent distribution $F(X)$ were correct. With these definitions, equation (2) becomes

$$\ln P = \int dx\, W(x) \ln W_m(x).$$ \hspace{1cm} (4)

If we subtract the constant $\int dx\, W(x) \ln W(x)$ from the quantity in equation (4), we get the Lucy $H$-function,

$$H_L = \int dx\, W(x) \ln \frac{W_m(x)}{W(x)}.$$ \hspace{1cm} (5)

In the statistics literature, $-H_L$ is known as the “Kullback-Leibler information distance” between model and data (Silverman 1986).
Lucy’s method works to increase $H_L$ (decrease the information distance) by iteratively applying the rule

$$F_{\text{new}}(X) = F(X) \int dx \frac{W(x)}{W_m(x)} L(X|x).$$

(6)

Using equations (1) and (3), this can be written as

$$F_{\text{new}}(X) = \int dx W(x) P(X) = \sum_i P_i(X).$$

(7)

Thus one iteration of Lucy’s method is identical to our scheme of “stacking” the posterior densities. In the absence of smoothing, our approach will seek a parent distribution that maximizes the likelihood of the observed sample.

For a finite sample, however, the maximum-likelihood parent distribution will be a set of spikes at the maximum of each $L(X_i|x_i)$, so it is ill-advised to iterate without a penalty function that enforces smoothness. Moreover, for a realistically small sample, there is not a unique $F(X)$ that maximizes $\ln P$ unless such a penalty function is present to lift the degeneracy. To implement this penalty at each iteration, we regard $F_{\text{new}}(X)$, computed according to equation (7), as a noisy realization of an underlying smooth function $F^{*}_{\text{new}}(X)$. This function is estimated using a smoothing spline, and $F_{\text{new}}(X)$ is replaced by its smoothed counterpart.

2.3. Smoothing Splines and Cross-Validation

Smoothing splines may be defined in any number of dimensions; here we summarize the two-dimensional case. This discussion is adapted from Green & Silverman (1994) and Silverman (1986).

The estimate of the parent distribution, $F_{\text{new}}(X)$, is defined on a discrete grid of $n$ values of $X$. (We drop the subscript “new” in what follows for brevity.) In our case, $X = (T, c)$, and we have $F(T_1, c_1), ..., F(T_n, c_n)$, from which we want to determine the
underlying smooth function $F^*(T, c)$. For a trial function $g(T, c)$, a penalized sum of squares of the residuals is given by

\[ S(g) = \sum_{i=1}^{n} [F(T_i, c_i) - g(T_i, c_i)]^2 + \alpha J(g), \]  

(8)

where $\alpha > 0$ is a “rate of exchange” between the usual goodness of fit measure and the penalizing function $J(g)$, given by

\[ J(g) = \int \int dT dc \left\{ \left( \frac{\partial^2 g}{\partial T^2} \right)^2 + 2 \left( \frac{\partial^2 g}{\partial T \partial c} \right)^2 + \left( \frac{\partial^2 g}{\partial c^2} \right)^2 \right\}. \]  

(9)

The penalizing function measures the rapid variation and departure from local linearity in $g$. The functions which minimize $S(g)$ are known as thin plate splines, which are analogous to natural cubic splines in one dimension. Algorithms for calculating thin plate splines are implemented in the routines DTPSS and DPRED in the GCVPACK package (Bates et al. 1987), which is available from Netlib.\(^5\)

The problem of finding $F^*(X)$ is now reduced to determining a single parameter $\alpha$. Likelihood cross validation provides a method for doing this automatically. The premise is that the best estimate of $\alpha$ should produce the distribution which best predicts all future data points. Since one is generally not gifted with prescience, one proceeds by removing one measurement from the sample and calculating the likelihood that that measurement would be obtained in the parent distribution found from the other measurements. Repeating this procedure for each measurement in turn and then averaging the likelihoods yields the cross validation (CV) score.

In our case, the measurements are the individual normalized likelihoods $L(X_i|x_i)$. We remove the $i$th measurement from our data set and create a new distribution $F_{-i}(X)$ using the methods above. For a given value of $\alpha$, the likelihood that a single $L(X_i|x_i)$ is drawn

\(^5\)http://netlib.org/gcv
from the smoothed model parent distribution $F^s_i(X;\alpha)$ is given by

$$\mathcal{L}_{-i} = \ln \left( \int dX F^s_i(X;\alpha) L(X|x_i) \right).$$

(10)

Averaging the $\mathcal{L}_{-i}$’s gives the likelihood cross validation score,

$$CV(\alpha) = n^{-1} \sum_{i=1}^{n} \mathcal{L}_{-i},$$

(11)

and maximizing $CV(\alpha)$ provides the best estimate for $\alpha$. To ensure uniqueness of the final result, we compute the maximum of $CV(\alpha)$ only once, on the first iteration, and fix the smoothing parameter for all subsequent iterations to its initial value. In some cases there is not a unique maximum in $CV(\alpha)$; instead, $CV(\alpha)$ is nearly flat up to some $\alpha_0$, beyond which it turns over. We set $\alpha$ to its turnover value, and we see no indication that this choice biases the results.

2.4. Implementation

Our numerical implementation follows from that described in Statler (1994b). The treatment of individual galaxies is essentially the same, except for details noted in § 3 below. The grid of dynamical parameters used in the models is also the same as in the earlier work; to aid the reader in § 4 we give a brief overview here.

The models assume that (1) rotation of the figure (i.e., tumbling) is negligible; (2) short-axis tube and long-axis tube mean motions can be represented by confocal streamlines (Anderson & Statler 1998); (3) the luminosity density $\rho_L$ is stratified on similar ellipsoids, $\rho_L(r,\theta,\phi) = \hat{\rho}_L(r)\rho^*_L(\theta,\phi)$; and (4) the velocity field obeys a “similar flow” ansatz outside the tangent point for a given line of sight, $\mathbf{v}(r,\theta,\phi) = \tilde{v}(r)\mathbf{v}^*(\theta,\phi)$. The last two assumptions are needed for projecting the models. The results are insensitive to the accuracy of these assumptions as long as $\hat{\rho}_L(r)$ and $\hat{\rho}_L(r)\tilde{v}(r)$ decrease faster than $r^{-2}$. This requirement
limits the validity of the models to regions where the rotation curve is not steeply rising. As a further simplification we adopt power laws for the luminosity density and the velocity scaling law: $\bar{\rho} \sim r^{-k}$ and $\bar{v}(r) \sim r^{-l}$. The index $k$ is determined from surface photometry, and we nominally adopt $l = (0, \pm \frac{1}{2})$, omitting the $l = -\frac{1}{2}$ case when $k \leq 2.5$. It turns out that the results are not very sensitive to either of these parameters.

Remaining properties of the phase space distribution function are described by a scalar constant $C$ and a function of one variable $v^*(t)$. These parameters describe the mean velocity across the $xz$ plane on one fiducial shell, which in turn determines the velocity field over the whole shell once the triaxiality $T$ and the luminosity density are specified. The “contrast” $C$ is defined as the ratio of the $y$ component of the mean velocity on the $x$ axis to that on the $z$ axis, on the fiducial shell. The function $v^*(t)$ gives the angular dependence of the mean velocity across the $xz$ plane on the fiducial shell. The variable $t$ is a rescaled polar angle, given, for spherical shells, by

$$t = \begin{cases} 
2 - \frac{\sin^2 \theta}{T}, & \theta < \sin^{-1} \sqrt{T}, \\
\cos^2 \theta, & \theta > \sin^{-1} \sqrt{T},
\end{cases}$$

where $\theta$ is the usual polar angle. The relation for ellipsoidal shells is given in § Section 3.1 of Statler (1994b). By definition, $v^*(0) = C$ and $v^*(2) = 1$.

The model grid comprises 8 different assumptions for the variation of $C$ with intrinsic shape. In four of these $C$ is constant: $C = 0$ (long-axis tube dominated), 0.5, 1, and infinite (short-axis tube dominated). Four more functional forms for $C(T, c)$ are introduced to mimic certain self-consistent models, and are given in equations (11) – (14) of Statler (1994b). The function $v^*(t)$ is taken to be either piecewise-constant or piecewise-linear in each of the intervals $[0, 1)$ and $(1, 2]$ (in the linear cases dropping to zero at $t = 1$). This function describes how the mean rotation speed in each of the tube orbit families declines away from the symmetry plane that contains its parent orbits. For example, the mean rotation in the Galaxy drops with height above the disk plane as one moves into
the more pressure-supported halo. At the other extreme, a maximally rotating isothermal sphere has constant rotation speed at all latitudes. Accordingly, we refer to linear $v^*(t)$ as “disklike” rotation, and constant $v^*(t)$ as “spheroidlike” rotation. A model can be disklike or spheroidlike in either short-axis or long-axis tubes. One should avoid the impression that disklike rotation necessarily implies a two-component structure; in an oblate disklike model, the mean rotation speed $45^\circ$ up from the equatorial plane is half of the in-plane value, a much gentler transition than in a genuine disk-halo system.

The likelihoods $L_i(T, c)$ for each galaxy are computed on a $20 \times 20$ rectangular grid on the intervals $0 \leq T \leq 1$ and $0.4 \leq c \leq 1$. Smoothing a function over a finite domain creates problems near the edges unless suitable boundary conditions are imposed to minimize this effect. The thin plate spline does not impose any strict boundary conditions but rather sees the area outside of the boundaries as lacking information. The penalizing function $J(L_i(T, c))$ is therefore the only part to contribute to the penalized sum of squares of the residuals, forcing the function $L_i(T, c)$ to be flat outside the boundaries (see section 2.3). In practice this has the effect of biasing $L_i(T, c)$ towards closed contours and reduced variability near the edges (Green & Silverman 1994). The effect however is limited and our results show little if no evidence of it.

We find that, in practice, convergence of the parent distribution can be rather slow, as peaks grow at the expense of valleys that sink toward zero. With our sample of only 13 objects, iterating until a stringent convergence criterion is satisfied may be dangerous. In order to be conservative in our conclusions regarding the frequency of triaxiality, we stop iterating when the maximum fractional change in $F(T, c)$ per iteration falls below 10%. Typically this occurs after about 7 iterations.
3. Data

3.1. Kinematics

All of the galaxies modeled are taken from the sample of radio ellipticals for which DB obtained multiple position angle rotation curve measurements. The sample contains more E3–E4 and fewer E0 galaxies than the general population and, as DB point out, includes an overabundance of “unusual” objects. Where appropriate we have supplemented the DB data with data from Franx et al. (1989), Binney et al. (1990), Bender et al. (1994) and Fried & Illingworth (1994).

The published rotation curves are first oriented to match our convention that radii west of north are positive. Since the models assume that the rotation curves are antisymmetric, we fold the profiles about the center of the galaxy to reduce the formal errors in the average rotation velocity. For each individual galaxy we approximate by eye the radius at which the rotation curve flattens and use the data outside of this in the models. At large radii the kinematic data become unreliable for reasons which vary from galaxy to galaxy (see section 3.3). We therefore set an outer radius beyond which we discard the data. We average the data points which are left between the inner and outer radii on each PA, weighted by the inverse square of the published errors. The uncertainty associated with the average is taken to be the $(1/\sigma^2)$-weighted standard deviation following Statler (1994c). The inner and outer radii and the adopted mean velocities are given in columns 4, 5 and 9, respectively, in Table 1.

3.2. Photometry

With the exception of the data for NGC 4839, all of the photometry is drawn from Peletier et al. (1990), who tabulate the ellipticity, major axis PA and surface brightness as
functions of radius. Similar photometry for NGC 4839 is drawn from Joergensen et al. (1992). For each galaxy the adopted major axis position angle is the average between the inner and outer limiting radii. The ellipticities are determined by taking the unweighted mean in the same interval, with the standard deviation serving as the uncertainty. The adopted mean ellipticities and major axis PAs are given in columns 2 and 3 of Table 1.

The slope of the surface brightness profile is calculated by differentiating numerically. The surface brightness slope is then deprojected into a volume brightness slope ($k$) by adding 1. Although this is strictly valid only for pure power-law profiles, it is fine for our level of approximation. For most galaxies the logarithmic slope of the surface brightness profile is not constant in the relevant intervals, and so two values are used that span the ranges of $k$. We compute all of the models using both values. The spanning values of $k$ for all of the galaxies are in columns 6 and 7 of Table 1.

### 3.3. Notes on Individual Galaxies

Of the 14 galaxies in the DB sample, four, NGC 1600, NGC 4374, NGC 4636 and NGC 4839, do not show any significant rotation at DB’s level of accuracy. We therefore model them using only their photometric data. A fifth object, NGC 4278, does show significant rotation but is not used. It shows a 20° isophotal twist between 20″ and 60″ and a drop in rotation velocity to zero outside of 20″ that conflicts with our assumption that the rotation curve is flat at large radii. It could be modeled but a more sophisticated method involving fitting at multiple radii would be required.

Details of how we have handled the data for the remaining sample of 13 galaxies are as follows:

**NGC 1600, NGC 4374, NGC 4636 and NGC 4839.** Because of the lack of
any significant rotation in these galaxies, photometric data alone is used to estimate their shape likelihood distributions. The triaxialities of these four galaxies are therefore poorly constrained. They are included on the grounds that omitting them could bias our results away from strongly triaxial systems, most of which are probably slowly rotating. In each case, the data are averaged from the center out to the largest radius for which kinematic data is available (see Table 1). The ellipticity of NGC 1600, NGC 4374 and NGC 4636 varies by about 0.10 in this interval, but the ellipticity of NGC 4839 rises steadily from 0.20 near the center to 0.50 at 32".

**NGC 315.** The turnover radius of the rotation curve is easily located at 5" by inspection. At 27" on PA 40 the rotation velocity is more than 3σ from the mean. Since we do not know if this is a real effect, we eliminate the total of six data points outside of 20".

**NGC 741.** The relatively low surface brightness of this galaxy results in large uncertainties in the kinematic data. Nonetheless, its six position angle measurements of the rotation curve make it very attractive to model. Outside of 15" some rotation appears on PA 10 and PA 40. Although it is not at all clear that the turnover radius of the rotation curve has been found, all of the data from 15" to the outermost data point at 30" are used.

**NGC 1052.** The data for this galaxy is the best for any in the sample. DB present kinematic data on four PAs, Binney et al. (1990) on the major and minor axis and Fried & Illingworth (1994) on the major axis. The turnover radius of the rotation curve is easily seen to be at 15" for this galaxy. A velocity difference of almost 50 km/sec between the southern and northern parts of the galaxy outside of 37" on PA 117 and PA 164 in both the DB and Binney et al. (1990) data sets imply that NGC 1052 may not be antisymmetric outside of this radius as is assumed in our models. The total of 11 questionable points outside of 35" are therefore eliminated. Fried & Illingworth (1994) measure the rotation curve to be flat out to 40" on PA 117.
**NGC 3379.** The turnover radius of the rotation curve appears at about 15″ so only data from this radius to the outermost data point at 34″ is used.

**NGC 3665.** The rotation curve is flat from inside of 5″ to close to 30″, but there is a discontinuity at 10″ where the ellipticity suddenly drops from 0.35 to almost zero. The ellipticity then slowly rises to approximately 0.2 at 15″ outside of which it is constant. There is also a 10° isophotal twist in the same range. Only data outside of 15″ is used.

**NGC 4261.** DB provide kinematic data on four position angles out to 55″. The major and minor axis data are supplemented with data from Bender et al. (1994), who place their slits 4° from those of DB. The average of the slit positions of the two papers is therefore used when combining the two datasets. Although this does introduce some error into the data it is very minor compared to the uncertainty in the velocity measurements. This galaxy is clearly a minor axis rotator with a turnover radius of the rotation curve at 20″.

**NGC 4472.** The turnover radius of the rotation curve on all the PAs is at approximately 25″. Between 3″ and 30″ the ellipticity of NGC 4472 increases from 0.06 to 0.17 but is constant outside this range, so only data points beyond 30″ are used. Outside of 60″ two data points on PA 160 are more than 4σ from the average so we discard the points outside of this radius on all position angles.

**NGC 4486.** This galaxy’s slow rotation makes the errors relatively large in the velocity measurements, but outside of 20″ the data is statistically consistent with a flattening of the rotation curve out to the last datapoint at 60″. The data between these radii are therefore used.

**NGC 7626.** This is another slow rotator. The only significant rotation is on the minor axis. Outside of 20″ the rotation curve seems to reverse, but the errors are so large that the reversal is not statistically significant. We model the data only inside 20″. A turnover
radius in the rotation curve on the major axis at approximately 5″ sets the inner radius.

4. Results

4.1. The “Maximal Ignorance” Shape Distribution

As in previous papers, we take the result from an unweighted combination of all models to represent the case of “maximal ignorance,” i.e., minimal assumptions as to the character of the internal dynamics. The parent shape distribution after seven iterations is shown in Figure 2a. The distribution is plotted in terms of $T$ and $c$ such that oblate spheroids, prolate spheroids, and spheres lie, respectively, along the right, left, and top margins. This distribution is bimodal, dominated by one group of nearly oblate, moderately flattened systems and a second group of rounder, nearly prolate systems. The valley between the two peaks represents a dearth of very triaxial galaxies. The bimodality is almost entirely a consequence of the kinematic data; to illustrate, we show in Figure 2b the result obtained from photometry alone, ignoring the kinematics. As discussed in the Introduction, photometry is effective in constraining the overall flattening distribution but reveals little about triaxiality.

A more succinct description of the frequency of triaxiality in this distribution comes from the one-dimensional distribution $F(T)$, obtained by integrating Figure 2a over $c$. The result is shown in Figure 3a. We somewhat arbitrarily set boundaries at $T = 0.2$ and $T = 0.8$ to delineate “nearly oblate,” “triaxial,” and “nearly prolate” regions. By this definition, the maximal ignorance distribution is 47% nearly oblate, 18% nearly prolate, and 35% triaxial. If we continue iterating beyond our nominal stopping criterion, the triaxial fraction decreases further, so we can take this as a conservative estimate of the rarity of triaxial systems implied by our subsample of the DB galaxies. The result is influenced
somewhat by the four galaxies without kinematic data; if these objects are omitted, the fractions change to 55% oblate, 25% prolate, and 21% triaxial.

We can obtain some measure of whether the DB subsample is representative of the elliptical galaxy population at large by calculating the expected ellipticity distribution for a randomly-oriented population drawn from the inferred parent. We plot this as the smooth curve in Figure 3b, compared with the observed ellipticity distribution from Ryden (1992). The two distributions are similar, though our predicted distribution contains a slight excess of very round galaxies. A Kolmogorov-Smirnov (KS) test implies a 14% probability that the observed sample was drawn from our distribution. However, the KS probability can be affected by details of how the mean ellipticities are defined. The ellipticities tabulated by Ryden are weighted by luminosity, whereas we exclude data from the brightest parts of the galaxies. Applying a systematic shift as small as \( \Delta \epsilon = 0.019 \) to our expected distribution would increase the KS probability to 99%. We conclude that our maximal ignorance parent distribution is consistent with the ellipticities of the general population of elliptical galaxies.

The final posterior densities describing the shapes of the individual galaxies in the sample with rotation data are shown in Figure 4. Some well-known objects are found to have well constrained triaxialities; NGC 1052, NGC 3379, and NGC 4472 are probably oblate or nearly so. The famous minor-axis rotator NGC 4261, not surprisingly, turns out to be most likely prolate, though there are oblate models not excluded at the 2\( \sigma \) level. The shapes of other objects are not as well constrained, and bimodal posterior densities are less a consequence of the kinematic data for the individual galaxies than a reflection of the parent distribution.
4.2. Dynamical Configurations That Can Be Ruled Out

Just as the marginal posterior densities describing the shape of each galaxy (Fig. 4) can be computed for a given parent distribution, we can compute marginal densities describing the orientation of each galaxy according to

\[ P_i(\Omega) = \int dT \int dc \frac{1}{4\pi} F(T, c) L_i(T, c, \Omega). \]  \hspace{1cm} (13)

The 4-dimensional likelihoods \( L_i(T, c, \Omega) \) are obtained by integrating the original likelihood function \( L_i(T, c, \Omega, d) \) over the dynamical parameters. The factor \( 1/4\pi \) reflects the assumed isotropy of the parent distribution; in other words we have assumed that the 4-dimensional parent has the form \( F(T, c, \Omega) = F(T, c)/4\pi \). We could, in fact, have worked our whole procedure in 4 dimensions instead of 2. Had we done so, isotropy of the parent would have been imposed at each iteration by explicitly smoothing away all of the \( \Omega \) dependence from the stacked \( P_i(T, c, \Omega) \) functions. One would expect, for a plausible set of models leading to a plausible parent distribution, that the stacked \( P_i \)'s should have an \( \Omega \) dependence not too far from isotropic, before it is smoothed away. This gives us an important consistency check: the sum, \( \Sigma_i P_i(\Omega) \), of the final posterior densities from equation (13) ought to be reasonably flat. Even though the parent distribution is, by construction, isotropic, there is no guarantee that the sample is isotropic. If we find a strong orientation bias in the sample despite assuming an isotropic parent, this constitutes a contradiction and signals a false assumption.

Two applications of this test are shown in the bottom four panels of Figure 2. Figure 2c shows the parent distribution derived under the assumption that all galaxies rotate about their intrinsic long axes \( (C = 0) \). Most objects are close to oblate and quite flat, with a small but significant fraction of rounder, triaxial systems. This is clearly different from the maximal-ignorance distribution in Figure 2a. However, this case can be ruled out by the orientation distribution of the sample, shown in Figure 2e. For the galaxies all to be
long-axis rotators, we must be seeing them in nearly the same orientation; the line of sight lies inside one of two 45 deg-wide cones for about 40% of the sample.

A better quantitative measure of the orientation bias is the rms deviation of $\Sigma_i P_i(\Omega)$ from perfect isotropy, normalized to unit mean; we refer to this as the *sample anisotropy*, $A_s$. For the case in Figure 2e, $A_s = 1.17$. Table 2 gives $A_s$ values for the distributions calculated using various subsets of the dynamical models. Unfortunately, it is not straightforward to link a value of $A_s$ with a confidence limit. The expected $A_s$ distribution for an ensemble of random isotropic samples depends on the forms of the individual $P_i$’s, which depends on both data and models. We can make a very rough correspondence to an easier statistical problem if we imagine that each $P_i(\Omega)$ simply marks a fraction $f$ of the sphere as allowed and a fraction $1-f$ as excluded. The $P_i$’s are then $n$ patches thrown down at random onto the sphere. At a random point on the sphere, the number $m$ of overlapping patches is given by a binomial distribution. In Table 2, we find that, except for the $C = 0$ and $C = \infty$ cases, all of the models using the kinematic data hover around $A_s \approx 0.2$. This would follow from the binomial distribution for $f = 0.63$, which is a not-unreasonable characterization of the $P_i$’s. We calculate that, if 0.2 is the expected $A_s$ for a random sample of 13 objects and if $A_s$ is distributed as in the patch problem, we can reject cases with $A_s > 0.40$ at 99% confidence and cases with $A_s > 0.48$ at 99.9% confidence. A more realistic simulation with 9 patches that exclude 50% of the sphere and 4 that exclude only 10% gives very similar results. Thus the hypothesis that elliptical galaxies rotate about their long axis is firmly ruled out. Of course, this is neither a particularly surprising nor new result; Binney (1985) reached the same conclusion from essentially the same data.

Figure 2d shows the parent distribution under the assumption that all objects rotate around their intrinsic short axes ($C = \infty$, also known as “zero intrinsic misalignment”). Here, most objects are triaxial, again very different from the maximal ignorance result.
Figure 2f shows the orientation distribution for the sample, which has $A_s = 0.43$. The assumption of zero intrinsic misalignment for all systems is excluded at approximately the 99.6% confidence level. This result differs from that of Franx et al. (1991), who were able to reproduce the observed distribution of ellipticities and kinematic misalignment angles\textsuperscript{6} with a family of triaxial models rotating about their short axes. We have not explored the source of this disagreement in depth. While it may be due simply to our smaller sample, we suspect that the models with which Franx et al. can fit galaxies with large kinematic misalignments fail on more detailed comparison with multi-position-angle data.

### 4.3. Dynamical Configurations That Cannot Be Ruled Out

Of the parent distributions we have derived from various subsets of the dynamical models, we find no other cases that can be ruled out on the basis of the $A_s$ values. Some of the unexcludable cases nonetheless differ significantly from the maximal ignorance distribution. Of particular interest are the cases in the last four rows of Table 2, for which the derived parent distributions are shown in Figure 5. These distributions differ only in whether the mean rotation is assumed to be disklike or spheroidlike (see § 2.4). The parent distribution is more sensitive to this assumption than to any of the other dynamical parameters, save for the cases already ruled out above.

Figure 6 shows the triaxiality distributions for these four cases, indicating the fraction of nearly oblate, nearly prolate, and triaxial systems as defined in § 4.1. The prevalence of axisymmetric systems over triaxial ones is significantly affected by the rotation characteristics, in the sense that axisymmetry becomes less common if rotation is more

\textsuperscript{6}For galaxies with only major and minor axis kinematics, the misalignment angle is $\tan^{-1}(v_{\text{minor}}/v_{\text{major}})$. 

spheroidlike. Moreover, the fractions of nearly prolate and nearly oblate objects are largely determined, respectively, by the character of the rotation in the long-axis and short-axis tubes. The peak at the prolate limit seen in the maximal ignorance result disappears entirely if the rotation in the long-axis tubes is spheroidlike. The dominant peak at the oblate limit is lowered by nearly a factor of two if the short-axis tube rotation is spheroidlike rather than disklike.

We consider this the most important result in this paper: *if rotation in ellipticals is generally disklike, then triaxiality is rare; if spheroidlike, triaxiality is common.* It follows that understanding the shapes of elliptical galaxies is closely linked with understanding whether weak disks are common structural components. It also follows that a physical understanding of what conditions during formation are likely to impose disklike or spheroidlike rotation on a hot stellar system would be extremely valuable.

5. Discussion

5.1. Previous Results on the Shape Distribution

A number of attempts have been made in the past to determine the parent intrinsic shape distribution of elliptical galaxies, mostly using photometry alone. It is interesting to see how our maximal-ignorance result compares to some of these.

Ryden (1992) fits a parent distribution to a sample of 171 measured ellipticities by letting the distribution assume the form of a circular Gaussian in axis ratio space. She finds a best-fit center to the distribution at $b = 0.98, c = 0.69$, implying that the most common shape is nearly oblate. The distribution is wide, however, with 61% of galaxies having a triaxiality between 0.2 and 0.8, compared to 35% for our sample. This difference may be attributable to Ryden’s assumption of a single peak; our method shows that the
distribution may be bimodal. When recast in terms of \((T, c)\), Ryden’s distribution has a short-to-long axis ratio expectation value \(\langle c \rangle = 0.68\), similar to our value of 0.71.

Lambas et al. (1992) take a similar approach using 2135 measurements of ellipticities from the APM Bright Galaxy Survey. Using a Monte Carlo technique they find the elliptical Gaussian in axis ratio space which best reproduces their observations. Their results are remarkably different both from ours and from Ryden’s. They find the center of their distribution at a flattening \(c = 0.55\) with a width of 0.2 in that dimension, implying that 30% of ellipticals have \(c < 0.4\). The main reason for this difference is an excess of flat galaxies in their sample. Only 2% of the galaxies in Ryden’s (1992) sample have an apparent ellipticity \(\epsilon > 0.6\), but the APM sample has 30% to 40% in that range. Lambas et al. do not offer an explanation for this apparent inconsistency with previous photometric studies of elliptical galaxies. Conceivably a large S0 contamination could be the cause.

A nonparametric, maximum-entropy shape distribution for the Ryden (1992) ellipticity sample is derived using a modified Lucy’s method by Statler (1994a). He finds a rather broad distribution in triaxiality, with 47% of the galaxies having \(T < 0.5\) compared with 70% in our distribution.

Using the same data as Statler (1994a), Tremblay and Merritt (1995) use a nonparametric maximum penalized likelihood estimator to derive the maximum-entropy shape distribution. They find that it is weakly bimodal and weighted towards oblate figures. Our distribution is significantly more bimodal and predicts fewer triaxial galaxies.

Using the same technique, Tremblay and Merritt (1996) estimate the parent distribution from a sample of 220 ellipticities. They assume that all galaxies have the same triaxiality and then proceed to calculate the distribution of intrinsic flattenings \(c\). They find that a pure oblate or prolate distribution is inconsistent with the available data and that a division of intrinsic flattenings exists between bright and faint galaxies with peaks at \(c = 0.75\) and
$c = 0.65$ respectively. All our galaxies are bright and therefore our expectation value of $c = 0.71$ agrees well with theirs.

Although the above studies, with the exception of Lambas et al. (1992), give similar results for the axis ratio $c/a$, none is able to put any real constraints on triaxiality, even when large samples are used. This demonstrates the need to include kinematic data in the models. Franx, Illingworth and de Zeeuw (1991) attempt to address this need by including the misalignment between the photometric and kinematic axes in their models. Studying a sample of 38 ellipticals, they conclude that a wide variety of distributions are consistent with the data, including ones similar to ours with both an oblate and a prolate peak.

5.2. Previous Results for Individual Galaxies

Some of the individual galaxies in our sample have been modeled previously. Statler (1994c) treats NGC 3379 using essentially the same data and methods applied here, except that the galaxy is fit in isolation, using a flat parent distribution. The result is that flattened nearly oblate shapes or rounder triaxial configurations are allowed by the data. Compared with this earlier result, the posterior density shown in Figure 4 is more constrained toward small $T$ due to the preference for near axisymmetry in the parent distribution.

Some objects in our sample have available additional kinematic or morphological constraints which are not included in our models. The best-studied example is NGC 1052, which has been modeled by Binney et al. (1990), Tenjes et al. (1993), and Plana and Boulesteix (1996). This galaxy has the best constrained shape in our sample; Figure 4 shows only a small permitted region around the oblate spheroid with $c = 0.63$. The small triaxiality supports the use of axisymmetric models by Binney et al. (1990) to constrain the phase space distribution function. Applying the Jeans equation to the observed surface
photometry and comparing the predicted velocity dispersion and azimuthal streaming to the observed kinematics, they find that NGC 1052 is consistent with a two integral distribution function. Tenjes et al. (1993), using the method of Franx, Illingworth and de Zeeuw (1991) and the presence of a gas disk to constrain the viewing angles, find that, depending on the specific kinematic model used, $c$ lies between 0.4 and 0.6 and the triaxiality is well constrained between 0.56 and 0.61. These values imply a very highly triaxial galaxy, and lie well outside of our 95% highest posterior density region. Using similar methods Plana & Boulesteix (1996) calculate the triaxiality to be 0.48 with a flattening of 0.5. This is much flatter and more triaxial than our result. It is possible that including orientation constraints from the gas would alter our derived shape. However, the results of Tenjes et al. (1993) and Plana and Boulesteix (1996) are very sensitive to the orientation of the disk, and consequently to assumptions about its intrinsic flatness and circularity; even a small error here could change their results dramatically.

In a study similar to that of Binney et al. (1990), Van der Marel et al. (1990) model the distribution functions of NGC 3379, NGC 4261, and NGC 4472. Their use of oblate axisymmetric models for NGC 3379 and NGC 4472 is supported by our results for these galaxies. For NGC 4261 they fit the observations to a prolate model with $c = 0.59$, which is consistent with our triaxiality estimate and is within our 95% highest posterior density region.

At the risk of disappointing the reader, we have avoided discussing the orientations of individual galaxies in the sample. This is, admittedly, counter to the original motivation of DB, which was to determine if there is any relationship between the orientations of the galaxies and their radio jets. Although we have calculated orientation constraints for each of the galaxies, a full discussion of this topic would of necessity be lengthy, and is outside the scope of this paper. We will deal with this issue in a future publication.
6. Summary and Conclusions

By combining photometric and kinematic data with dynamical models using the method of Statler (1994b), we have derived constraints on the intrinsic shapes and orientations of 13 ellipticals from the Davies and Birkinshaw (1988) sample of radio galaxies. Using an iterative Bayesian approach we have then combined those results to estimate the parent shape distribution from which they were drawn, under the assumption that this parent distribution has no preferred orientation. In the process we have obtained improved constraints on the shapes of the individual objects.

We have found that the parent shape distribution shows a tendency toward bimodality, with peaks at the oblate and prolate limits. In the distribution derived under minimal assumptions about the galaxies’ internal dynamics, only about a one-third of the objects would be strongly triaxial ($0.2 < T < 0.8$). However, the parent distribution does depend on dynamical assumptions. Some of these assumptions can be ruled out because they would require the sample to have a strong orientation bias; configurations in which all galaxies rotate purely about either their long axes or their short axes can be excluded on these grounds. On the other hand, configurations in which the mean motions in the short-axis and long-axis tube orbits are either disklike—dropping off away from the symmetry planes—or spheroidlike—staying approximately constant at a given radius—cannot be distinguished at this point. Whether the rotation is disklike or spheroidlike has a strong effect on the inferred shape distribution. Spheroidlike rotation in the long-axis or short-axis tubes, respectively, significantly reduces the fraction of nearly prolate or nearly oblate galaxies; bimodality is completely eliminated if the long-axis tubes are spheroidlike and the short-axis tubes disklike. In a nutshell, if rotation in ellipticals is generally disklike, then triaxiality is rare; if spheroidlike, triaxiality is common.

This inferential link between diskiness and axisymmetry complements the intuitive
physical notion that the two ought to go hand in hand. There is evidence from the width of the Tully-Fisher relation that the disks of spiral galaxies are very nearly circular (Franx & de Zeeuw 1992), and indications from numerical experiments that growing even a weak disk in a triaxial halo can render the latter axisymmetric (Dubinski 1994). Whether weak disks in elliptical galaxies are detectable is another long-standing issue receiving renewed attention (Magorrian 1999). High-accuracy, multi-position-angle kinematic mapping may be able to reveal hidden disks, but the expected signatures are subtle. Some support is lent to the possibility that weak disks may be common by the kinematic similarities that the “standard elliptical” NGC 3379 shares with the S0 galaxy NGC 3115 (Statler & Smecker-Hane 1999). Theoretically, however, the origin of these particular kinematic features is not understood. As we have stressed, a physical understanding of the processes that may establish disklike or spheroidlike rotation in a hot stellar system is sorely needed.

We are indebted to Barbara Ryden and the referee, David Merritt, for numerous constructive comments. This work was supported by NASA Astrophysical Theory Program Grant NAG5-3050 and NSF CAREER grant AST-9703036.
Table 1. Observational data used in models

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<td>129</td>
<td>−9±7</td>
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<td></td>
<td></td>
<td>159</td>
<td>−4±8</td>
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<td>NGC 4636</td>
<td>0.12±0.03</td>
<td>152</td>
<td>15</td>
<td>48</td>
<td>…</td>
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<tr>
<td>NGC 4839</td>
<td>0.29±0.07</td>
<td>62</td>
<td>4</td>
<td>29</td>
<td>…</td>
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<td>…</td>
<td>…</td>
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</tr>
<tr>
<td>NGC 7626</td>
<td>0.11±0.01</td>
<td>7</td>
<td>5</td>
<td>20</td>
<td>9/4</td>
<td>10/4</td>
<td>28</td>
<td>−25±26</td>
<td>1</td>
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<td></td>
<td>87</td>
<td>−8±14</td>
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<td>118</td>
<td>14±14</td>
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<td></td>
<td></td>
<td>149</td>
<td>0±9</td>
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<td></td>
<td></td>
<td></td>
<td>178</td>
<td>16±9</td>
</tr>
</tbody>
</table>

$^a$Degrees from minor axis

$^b$Only one value of $k$ needed since the logarithmic slope is almost constant between $R_{\text{min}}$ and $R_{\text{max}}$

Table 2. Sample Anisotropies from Various Dynamical Assumptions

<table>
<thead>
<tr>
<th>Case</th>
<th>Restriction</th>
<th>$A_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NK</td>
<td>No kinematics, photometry only</td>
<td>0.10</td>
</tr>
<tr>
<td>0</td>
<td>Maximal Ignorance (all models)</td>
<td>0.18</td>
</tr>
<tr>
<td>1</td>
<td>$C(T, c) = \text{“Prescription 1”}^a$</td>
<td>0.19</td>
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<tr>
<td>2</td>
<td>$C(T, c) = \text{“Prescription 2”}^a$</td>
<td>0.16</td>
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<tr>
<td>3</td>
<td>$C(T, c) = \text{“Prescription 3”}^a$</td>
<td>0.20</td>
</tr>
<tr>
<td>4</td>
<td>$C(T, c) = \text{“Prescription 4”}^a$</td>
<td>0.17</td>
</tr>
<tr>
<td>5</td>
<td>$C = \infty$ (around short axis)</td>
<td>0.43</td>
</tr>
<tr>
<td>6</td>
<td>$C = 1$</td>
<td>0.17</td>
</tr>
<tr>
<td>7</td>
<td>$C = 0.5$</td>
<td>0.26</td>
</tr>
<tr>
<td>8</td>
<td>$C = 0$ (around long axis)</td>
<td>1.17</td>
</tr>
<tr>
<td>X1</td>
<td>spheroidlike/spheroidlike$^b$</td>
<td>0.32</td>
</tr>
<tr>
<td>X2</td>
<td>disklike/disklike$^b$</td>
<td>0.20</td>
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<tr>
<td>X3</td>
<td>spheroidlike/disklike$^b$</td>
<td>0.21</td>
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<tr>
<td>X4</td>
<td>disklike/spheroidlike$^b$</td>
<td>0.13</td>
</tr>
</tbody>
</table>

$^a$Prescriptions 1 – 4 given by equations (11) – (14) of Statler (1994b).

$^b$Form of $v^*(t)$ for long axis/short axis tubes.
REFERENCES


Fasano, G. 1996, in Fresh Views of Elliptical Galaxies, A. Buzzoni, A. Renzini, & A. Serrano, eds. (San Francisco: Astronomical Society of the Pacific), 37


Statler, T. S. 1996, in Fresh Views of Elliptical Galaxies, A. Buzzoni, A. Renzini, & A. Serrano, eds. (San Francisco: Astronomical Society of the Pacific), 27

This manuscript was prepared with the AAS \LaTeX{} macros v4.0.
Fig. 1.— Stack-smooth-iterate algorithm in 1 dimension, finding the distribution of the intrinsic quantity $X$ from measurements of an observable $x$. Likelihoods $L_i(X)$ for 9 measured $x$ values (a) are multiplied by the initial model parent distribution (b), summed (c), and smoothed to produce an improved parent (d). Result after 20 iterations (e, solid line) is a decent match to the true parent distribution (dotted line). Estimates of $X$ values for individual objects (f) differ from the original likelihoods.

Fig. 2.— (a–d) Parent intrinsic shape distributions, in the space of triaxiality $T$ of the mass distribution and flattening $c_L$ of the light distribution, obtained under different assumptions for the internal dynamics of the galaxies. In each panel, round prolate galaxies are at upper left, flattened oblate galaxies at lower right; objects in between are triaxial. Contours enclose 68% and 95% of the total probability. (a) “Maximal ignorance” distribution, an unweighted combination of all models; (b) result from photometry only, omitting all kinematic data; (c) rotation solely around the short axis; (d) rotation solely around the long axis. (e, f) Orientation distribution of the sample, for the shape distributions in (c, d) respectively. Centers of the circles correspond to views down the short axis, top and bottom edges to views down the long axis, and extreme right-hand edge to views down the intermediate axis. These cases can be ruled out because of the strong orientation bias implied for the sample.

Fig. 3.— (a) Triaxiality distribution in the maximal ignorance case, obtained by integrating Figure 2a over flattening $c$. Dashed lines demarcate nearly prolate (left), triaxial (center), and nearly oblate (right) regions. Percentages indicate fractions of total probability in each region. (b) Expected distribution of apparent ellipticities in the maximal ignorance case (smooth curve). Histogram shows ellipticity distribution for a sample of 165 galaxies from Ryden (1992).

Fig. 4.— Posterior probability densities in the intrinsic shape plane for each of the 9 galaxies which show significant rotation, using the maximal ignorance parent distribution. Contours
indicate the 68% and 95% highest posterior density regions.

Fig. 5.— Parent intrinsic shape distributions obtained under different assumptions for the “disklike” or “spheroidlike” character of the rotation in long-axis and short-axis tubes, as indicated.

Fig. 6.— Triaxiality distributions for the same four cases in fig. 5, obtained by integrating the parent distributions over $c$. Spheroidlike rotation in long-axis or short-axis tubes suppresses the fraction of prolate or oblate objects, respectively.
Fig. 1
Fig. 3
Fig. 4
Fig. 5
long-axis tubes
Disklike  Spheroidlike

Fig. 6