We study the Autler-Townes spectrum of a V-type atom coupled to a single-mode, frequency-tunable cavity field at finite temperature, with a pre-selected polarization in the bad cavity limit, and show that, when the mean number of thermal photons $N \gg 1$ and the excited sublevel splitting is very large (the same order as the cavity linewidth), the probe gain may occur at either sideband of the doublet, depending on the cavity frequency, due to the cavity-induced interference.

Within recent years, there has been a resurgence of interest in the phenomenon of quantum interference [1]. The principal reason is that it lies at the heart of many new effects and applications of quantum optics, such as lasing without population inversion [2], electromagnetically-induced transparency [3], enhancement of the index of refraction without absorption [4], fluorescence quenching [5–8] and spectral line narrowing [6].

The basic system consists of a singlet state connected to a closely-spaced excited doublet by a single-mode laser. Cardimona et al. [5,6] studied the effect of quantum interference on the resonance fluorescence of such a system, and found that it can be driven into a dark state in which quantum interference prevents any fluorescence from the excited sublevels, regardless of the intensity of the exciting laser. We have recently shown that quantum interference can also lead to narrow resonances, transparency and gain without population inversion in the probe absorption spectrum of such an atomic system [9].

Harris and co-workers [2] generalized the V-type atom to systems where the excited doublets decay to an additional continuum or to a single auxiliary level, in addition to the ground state. They found that at a certain frequency, the absorption rate goes to zero due to destructive interference whereas the emission rate remains finite. It is possible to amplify a laser field at this frequency without population inversion being present. In the case of a single auxiliary level, quantum interference can lead to the elimination of the spectral line at the driving laser frequency in the spontaneous emission spectrum [7] and transparency in the absorption spectrum [10].

It is important for these effects that the dipole moments of the transitions involved are parallel, so that the cross-decay terms are maximal. From the experimental point view, however, it is difficult to find isolated atomic systems which have parallel moments [2,5,11,12].

Various alternative proposals [11,13] have been made for generating quantum interference effects. For example, if the two upper levels of a V-type atom are coupled by a microwave field or an applied laser, the excited doublet becomes a superposition, so that as the atom decays from one of the excited sublevels it drives the other. For such systems, the cross-decay terms are evident in the atomic dressed picture [13]. A four-level atom with two closely-spaced intermediate states coupled to a two-mode cavity can also show the effect of quantum interference [11]. In fact, the experimental observation of the interference-induced suppression of spontaneous emission was carried out in sodium dimers where the excited sublevels are superpositions of singlet and triplet states that are mixed by a spin-orbit interaction [8,12].

We have recently also proposed a scheme for engineering of quantum interference (parallel or anti-parallel dipole moments) in a V-type atom coupled to a frequency tunable, single-mode cavity field with a pre-selected polarization at zero temperature [14]. We have found that the effects of the cavity-induced interference are pronounced only when the cavity detuning $\delta$ and the excited doublet splitting $\omega_{21}$ are much less than the cavity linewidth $2\kappa$. Here we shall extend the study to a cavity damped by a thermal reservoir at finite temperature, so that the mean number of thermal photons, $N$, in the cavity mode is nonzero. We show that, even in the case of $\delta$ and $\omega_{21}$ being the same order of the cavity linewidth $2\kappa$, the cavity-induced interference is still significant when $N \gg 1$, and that interference-assisted gain may occur in one component of the Autler-Townes doublet for certain cavity resonant frequency.

Our model consists of a V-type atom with the ground state $|0\rangle$ coupled by the single-mode cavity field to the excited doublet $|1\rangle, |2\rangle$. Direct transitions between the excited sublevels $|1\rangle$ and $|2\rangle$ are dipole forbidden. The master equation for the total density matrix operator $\rho_T$ in the frame rotating with the average atomic transition frequency
\[ \omega_0 = (\omega_{10} + \omega_{20})/2 \] takes the form
\[ \dot{\rho}_T = -i [H_A + H_C + H_I, \rho_T] + \mathcal{L}\rho_T, \]
where
\[ H_C = \delta a \dagger a, \]
\[ H_A = \frac{1}{2}\omega_{21} (A_{22} - A_{11}), \]
\[ H_I = i (g_1 A_{01} + g_2 A_{02}) a \dagger - h.c., \]
\[ \mathcal{L}\rho_T = \kappa (N + 1) (2a \rho_T a \dagger - a \dagger a \rho_T - \rho_T a a \dagger) + \kappa N (2a \dagger \rho_T a - a a \dagger \rho_T - \rho_T a a \dagger), \]
with
\[ \delta = \omega_C - \omega_0, \quad \omega_{21} = E_2 - E_1, \quad g_i = e_\lambda \cdot d_{0i} \sqrt{\hbar \omega_C / 2\epsilon_0 V}, \quad (i = 1, 2). \]

Here \( H_C, H_A \) and \( H_I \) are the unperturbed cavity, the unperturbed atom and the cavity-atom interaction Hamiltonians respectively, while \( \mathcal{L}\rho_T \) describes damping of the cavity field by the continuum electromagnetic modes at finite temperature, characterized by the decay constant \( \kappa \) and the mean number of thermal photons \( N \); \( a \) and \( a \dagger \) are the photon annihilation and creation operators of the cavity mode, and \( A_{ij} = |i\rangle \langle j| \) is the atomic population (the dipole transition) operator for \( i = j \) (\( i \neq j \)); \( \delta \) is the cavity detuning from the average atomic transition frequency, \( \omega_{21} \) is the splitting of the excited doublet of the atom, and \( g_i \) is the atom-cavity coupling constant, expressed in terms of \( d_{ij} \), the dipole moment of the atomic transition from \( |j\rangle \) to \( |i\rangle \), \( e_\lambda \), the polarization of the cavity mode, and \( V \), the volume of the system. In the remainder of this work we assume that the polarization of the cavity field is pre-selected, i.e., the polarization index \( \lambda \) is fixed to one of two possible directions.

In this paper we are interested in the bad cavity limit: \( \kappa \gg g_i \), that is the atom-cavity coupling is weak, and the cavity has a low \( Q \) so that the cavity field decay dominates. The cavity field response to the continuum modes is much faster than that produced by its interaction with the atom, so that the atom always experiences the cavity mode in the state induced by the thermal reservoir. Thus one can adiabatically eliminate the cavity-mode variables, giving rise to a master equation for the atomic variables only [15], which takes the form,
\[ \dot{\rho} = -i [H_A, \rho] + \{ F(\omega_{21})(N + 1) [g_1]^2 (A_{01} \rho A_{10} - A_{11} \rho) + g_1 g_2^* (A_{01} \rho A_{20} - A_{21} \rho)] + F(-\omega_{21})(N + 1) [g_2]^2 (A_{02} \rho A_{20} - A_{22} \rho) + g_1^* g_2 (A_{02} \rho A_{10} - A_{12} \rho)] + F(\omega_{21})N [g_1]^2 (A_{10} \rho A_{01} - \rho A_{01}) + g_1^* g_2 A_{20} \rho A_{01}] + F(-\omega_{21})N [g_2]^2 (A_{20} \rho A_{02} - \rho A_{02}) + g_1^* g_2 A_{10} \rho A_{02}] + h.c. \]
where \( F(\pm \omega_{21}) = [\kappa + i(\delta \pm \omega_{21}/2)]^{-1} \).

Obviously, the equation (4) describes the cavity-induced atomic decay into the cavity mode. The real part of \( F(\pm \omega_{21})[g_j]^2 \) represents the cavity-induced decay rate of the atomic excited level \( j (=1, 2) \), while the imaginary part is associated with the frequency shift of the atomic level resulting from the interaction with the vacuum field in the detuned cavity. The other terms, \( F(\pm \omega_{21})g_i g_j^* \) (\( i \neq j \), however, represent the cavity-induced correlated transitions of the atom, i.e., an emission followed by an absorption of the same photon on a different transition, \( |1\rangle \rightarrow |0\rangle \rightarrow |2\rangle \) or \( |2\rangle \rightarrow |0\rangle \rightarrow |1\rangle \), which give rise to the effect of quantum interference.

The effect of quantum interference is very sensitive to the orientations of the atomic dipoles and the polarization of the cavity mode. For instance, if the cavity-field polarization is not pre-selected, as in free space, one must replace \( g_i g_j^* \) by the sum over the two possible polarization directions, giving \( \Sigma_\lambda g_i g_j^* \propto d_{0i} \cdot d_{0j}^\lambda \) [11]. Therefore, only non-orthogonal dipole transitions lead to nonzero contributions, and the maximal interference effect occurs with the two dipoles parallel. As pointed out in Refs. [2,5,11,12] however, it is questionable whether there is an isolated atomic system with parallel dipoles. Otherwise, when the polarization of the cavity mode is fixed, say \( e_\lambda = e_x \), the polarization direction along the \( x \)-quantization axis, then \( g_i g_j^* \propto (d_{0i})_x (d_{0j}^\lambda)_x \), which is nonvanishing, regardless of the orientation of the atomic dipole matrix elements.

It is apparent that if \( \kappa \gg \delta, \omega_{21} \), the frequency shifts are negligibly small [14], and this equation (4) reduces to that of a V-atom with two parallel transition matrix elements in free space [5,6,9]. In the following we shall discuss
the effect of quantum interference in the situation of $\omega_{21} \geq \kappa$ and $N \gg 1$, by examining the steady-state absorption spectrum of such a system, which is defined as

$$A(\omega) = \Re \int_0^\infty \lim_{t \to \infty} \langle [P(t + \tau), P^\dagger(t)] \rangle e^{i\omega \tau} d\tau,$$

where $\omega = \omega_p - \omega_0$, and $\omega_p$ is the frequency of the probe field and $P(t) = d_1 A_0 + d_2 A_0$ is the component of the atomic polarization operator in the direction of the probe field polarization vector $e_p$, with $d_i = e_p \cdot \mathbf{d}_{0i}$. With the help of the quantum regression theorem, one can calculate the spectrum from the Bloch equations,

$$\langle \dot{A}_{11} \rangle = -[F(\omega_{21}) + F^*(\omega_{21})] |g_1|^2 [(N + 1)\langle A_{11} \rangle - N \langle A_{00} \rangle]$$

$$- F(-\omega_{21})g_1^* g_2 (N + 1) \langle A_{12} \rangle - F^*(-\omega_{21}) g_1 g_2^* (N + 1) \langle A_{21} \rangle,$$

$$\langle \dot{A}_{22} \rangle = -[F(-\omega_{21}) + F^*(\omega_{21})] |g_2|^2 [(N + 1)\langle A_{22} \rangle - N \langle A_{00} \rangle]$$

$$- F^*(\omega_{21}) g_1^* g_2 (N + 1) \langle A_{12} \rangle - F(\omega_{21}) g_1 g_2^* (N + 1) \langle A_{21} \rangle,$$

$$\langle \dot{A}_{12} \rangle = -F(\omega_{21}) g_1 g_2^* (N + 1) \langle A_{11} \rangle - F^*(-\omega_{21}) g_1 g_2^* (N + 1) \langle A_{21} \rangle + [F(\omega_{21}) + F^*(-\omega_{21})] g_1 g_2^* N \langle A_{00} \rangle$$

$$- [F^*(\omega_{21}) |g_1|^2 (N + 1) + F(-\omega_{21}) |g_2|^2 (N + 1) + i\omega_{21}] \langle A_{12} \rangle,$$

$$\langle \dot{A}_{01} \rangle = -\left[ F(\omega_{21}) |g_1|^2 (2N + 1) + F(-\omega_{21}) |g_2|^2 N - i\frac{\omega_{21}}{2} \right] \langle A_{01} \rangle - F(-\omega_{21}) g_1^* g_2 (N + 1) \langle A_{02} \rangle,$$

$$\langle \dot{A}_{02} \rangle = -\left[ F(\omega_{21}) |g_1|^2 N + F(-\omega_{21}) |g_2|^2 (2N + 1) + i\frac{\omega_{21}}{2} \right] \langle A_{02} \rangle - F(\omega_{21}) g_1^* g_2^* (N + 1) \langle A_{01} \rangle.$$ (6)

To monitor quantum interference, we insert a factor $\eta = 0, 1$ in the cross transition terms $g_i g_j^\dagger$. When $\eta = 0$, the cross transitions are switched off, so no quantum interference is present. Otherwise, the effect of quantum interference is maximal.

Figure 1 shows the Autler-Townes spectra for $g_1 = g_2 = 10$, $\kappa = \omega_{21} = 100$, $N = 10$, and different cavity detunings. In the absence of the interference ($\eta = 0$), two transition paths, $|0\rangle \leftrightarrow |1\rangle$ and $|0\rangle \leftrightarrow |2\rangle$, are independent, which lead to the lower and higher frequency sidebands of the absorption doublet, respectively. It is not difficult to see that the spectral heights and linewidths are mainly determined by the cavity-induced decay constants $\gamma_i$ ($i = 1, 2$) of the excited states, which have the forms

$$\gamma_1 = \frac{\kappa |g_1|^2}{\kappa^2 + (\delta + \omega_{21}/2)^2}, \quad \gamma_2 = \frac{\kappa |g_2|^2}{\kappa^2 + (\delta - \omega_{21}/2)^2}.$$ (7)

which vary with the cavity frequency. It is evident that $\gamma_1 < \gamma_2$ when $\delta > 0$, and both $\gamma_1$ and $\gamma_2$ decrease as $\delta$ increases. Noting that the lower and higher frequency peaks have respective linewidths $\Gamma_1 = \gamma_1 (2N + 1) + \gamma_2 N$ and $\Gamma_h = \gamma_1 N + \gamma_2 (2N + 1)$, and are proportional to $\Gamma_{1h}$, the lower frequency sideband is slightly higher than the higher frequency one in the case of $\delta > 0$ and both the sidebands can be narrowed by increasing the cavity detuning, see for example, the dashed lines in the following three figures.

Whereas, the spectral features are dramatically modified in the presence of the cavity induced interference ($\eta = 1$). When the cavity is resonant with the average frequency of the atomic transitions, $\delta = 0$, the doublet is symmetric, and its sidebands are higher and wider than that for $\eta = 0$, as shown in the frames 1(a), 2(a) and 3(a). Otherwise, it is asymmetric. Either sideband of the doublet can be suppressed, depending upon the cavity frequency, e.g., the higher frequency sideband is suppressed for $\delta = 10$, 50 and 100, see in Figs. 1(b)–1(d), while the sideband is enhanced for $\delta = 200$, shown in Fig. 1(e). When the cavity frequency is far off resonant with the atomic transition frequencies, say $\delta = 500$ in Fig. 1(f), the absorption spectra for $\eta = 0$ and 1 are virtually same, that is the effect of the cavity induced interference is negligible small.

Rather surprisingly, the frame 1(c) shows probe gain in the higher frequency sideband, without the help of any coherent pumping. Moreover, increasing the mean number of thermal photons $N$ may enhance the probe gain, see for instance, in Fig. 2 for $N = 20$, in which the higher-frequency probe gain even occurs for a relative small cavity detuning, say $\delta = 10$ in the frame 2(b). Contrastively, when the detuning is very large, the probe beam can be amplified at the lower-frequency sideband, rather than at the higher-frequency one, as shown in the frame 2(c) for $\delta = 200$ for example. Fig. 2 also exhibits that the linewidths are broadened for large number of thermal photons.

We present the Autler-Townes spectrum for a large excited level-splitting, $\omega_{21} = 200$, and a large number of thermal photons, $N = 20$, in Fig. 3, in which the more pronounced gain, comparing with that for $\omega_{21} = 100$, is displayed at either the lower-frequency sideband for $\delta = 10$, 50 and 100, or the higher-frequency sideband for $\delta = 200$. One can also find that for the large level-splitting, the effect of the cavity-induced interference is still significant when $\delta = 500$, as shown in Fig. 3(f), where the lower frequency peak is almost suppressed while the other is greatly enhanced.
However, when $\delta \gg \omega_{21}$, say $\delta = 1000$ for instance, the effect of the interference disappears (we have exhibited no figure here).

In what follows, we shall see that the probe gain is a direct consequence of the cavity-induced quantum interference between the two transition paths, $|0\rangle \leftrightarrow |1\rangle$ and $|0\rangle \leftrightarrow |2\rangle$. The gain at different sidebands has different origin. To show this, we first plot the steady-state population differences between the excited sublevels and the ground level, $\langle A_{11} \rangle - \langle A_{00} \rangle$ and $\langle A_{22} \rangle - \langle A_{00} \rangle$, and the coherence between the excited sublevels, $\langle A_{12} \rangle$, against the cavity detuning $\delta$ in Fig. 4 for $g_1 = g_2 = 10$, $\kappa = 100$, $\omega_{21} = 200$ and $N = 20$. It is clearly that the steady-state populations and coherence are highly dependent on the cavity frequency. The coherence is symmetric with the cavity detuning and reaches the maximum value at $\delta = 0$, while the population differences are asymmetric. Furthermore, the population inversion may be achieved for certain cavity frequency, for example, if $143.8 < \delta < 650$, then $\langle A_{11} \rangle - \langle A_{00} \rangle > 0$, while $\langle A_{22} \rangle > \langle A_{00} \rangle$ in the region of $-650 < \delta < -143.8$. Therefore, the gain in the region of $-143.8 < \delta < 143.8$ must stem from the cavity-induced steady-state coherence between the two dipole-forbidden excited sublevels, rather than from the population inversion between the two dipole transition levels. Whereas, the population inversions may result in the probe gain when the cavity detuning is in the regions of $-650 < \delta < -143.8$ and $143.8 < \delta < 650$. We thus conclude that, in the case of $\delta > 0$, as shown in Figs. 1-3, the gain at the lower-frequency sideband comes from the contribution of the steady-state atomic coherence $\langle A_{12} \rangle$, while the gain at the other sideband is attributed to the steady-state population inversion ($\langle A_{11} \rangle > \langle A_{00} \rangle$).

Noting that, in the absence of the interference ($\eta = 0$), $\langle A_{11} \rangle = \langle A_{22} \rangle = N/(3N+1)$, $\langle A_{00} \rangle = (N+1)/(3N+1)$, and $\langle A_{12} \rangle = 0$ are independent of the cavity detuning, the cavity frequency dependence of the steady-state populations and coherence manifests the cavity-induced quantum interference.

To further explore the origin of the probe gain, we separate the Autler-Townes spectrum into two parts, in which one corresponds to the contribution of the populations, while the other results from the coherence, in Fig. 5 for $g_1 = g_2 = 10$, $\kappa = 100$, $\omega_{21} = 200$, $N = 20$, and various cavity frequencies. It is obvious that when $\delta = 0$, 50 and 100, the contributions of the coherence to the spectrum are negative (i.e., probe gain), whereas the populations make positive contributions, see, for example, in frames 6(a)–6(c). One can also see that the spectral component resulting from the populations is symmetric only when $\delta = 0$, otherwise, it has different values at the lower and higher frequency sidebands, which are proportional to $\langle (A_{00} - A_{11}) \rangle$ and $\langle (A_{00} - A_{22}) \rangle$, respectively. As shown in Fig. 4, if the cavity detuning is zero, then $\langle (A_{00} - A_{11}) \rangle = \langle (A_{00} - A_{22}) \rangle$, whereas, $\langle (A_{00} - A_{11}) \rangle > \langle (A_{00} - A_{22}) \rangle$ for $\delta = 50$ and 100. As a result, the lower frequency peak is higher than that of the other one in the cases of $\delta = 50$ and 100. Therefore, the total spectrum may exhibit the probe gain at the higher frequency sideband at these cavity frequencies, see, for example, in Figs. 3(c) and 3(d). The gain is purely attributed to the cavity-induced steady-state atomic coherence. However, when $\delta = 200$, the situation is reverse: the coherence gives rise to the probe absorption, while the populations lead to the gain at the lower frequency sideband, due to the population inversion between the levels $|0\rangle$ and $|1\rangle$, as illustrated in Fig. 4.

In summary, we have shown that maximal quantum interference can be achieved in a V-type atom coupled to a single-mode, frequency-tunable cavity field at finite temperature, with a pre-selected polarization in the bad cavity limit. There are no special restrictions on the atomic dipole moments, as long as the polarization of the cavity field is pre-selected. We have investigated the cavity modification of the Autler-Townes spectrum of such a system, and predicted the probe gain at either sideband of the doublet, depending upon the cavity resonant frequency, when the excited sublevel splitting is very large (the same order as the cavity linewidth) and the mean number of thermal photons $N \gg 1$. The gain occurring at different sidebands has the various origin: in the case of $\delta > 0$, the lower frequency gain is due to the nonzero steady-state coherence, while the higher frequency one is attributed to the steady-state population inversion. Both the nonzero coherence and population inversion originate from the cavity-induced quantum interference.

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FIG. 1. Absorption spectrum for $g_1 = g_2 = 10$, $\kappa = 100$, $\omega_{21} = 100$, $N = 10$, and $\delta = 0, 10, 50, 100, 200, 500$ in (a)–(f), respectively. In Figs. 1–3 the solid curves represent the spectrum in the presence of the maximal interference ($\eta = 1$), whilst the dashed curves are the spectrum in the absence of the interference ($\eta = 0$).

FIG. 2. Same as FIG. 1, but with $N = 20$.

FIG. 3. Same as FIG. 1, but with $\omega = 200$ and $N = 20$.

FIG. 4. The steady-state population differences and coherence vs the cavity detuning, for $g_1 = g_2 = 10$, $\omega = 200$, $N = 20$ and $\eta = 1$. The solid, dashed and dot-dashed lines respectively represent $(\langle A_{11} \rangle - \langle A_{00} \rangle)$, $(\langle A_{22} \rangle - \langle A_{00} \rangle)$ and $\text{Re}(\langle A_{12} \rangle)$.

FIG. 5. Different contributions to the absorption spectrum, for $g_1 = g_2 = 10$, $\kappa = 100$, $\omega_{21} = 200$, $N = 20$, $\eta = 1$, and $\delta = 0, 50, 100, 200$ in (a)–(d), respectively. The solid curves represent the contributions of the population differences, whilst the dashed curves are the ones of the coherences.