Evaporation of near-extremal Reissner-Nordström black holes

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The formation of near-extremal Reissner-Nordström black holes in the S-wave approximation can be described, near the event horizon, by an effective solvable model. The corresponding one-loop quantum theory remains solvable and allows to follow analytically the evaporation process which is shown to require an infinite amount of time.

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Black holes are the most fascinating objects in General Relativity. Since Hawking discovered that they emit thermal radiation [1], it has been a long standing puzzle to explain their thermodynamical properties, in terms of some microscopic structure, and to understand the dynamical evolution beyond Hawking initial scheme where the gravitational field was treated as a fixed background. Extremal and near-extremal charged black holes have recently played a fundamental role in String Theory, where in some special cases it has been possible to give a statistical explanation of the Bekenstein-Hawking area law for their entropy [2]. Moreover, the scattering of low-energy particles off extremal black holes also provides a convenient setting to study the evaporation process including back-reaction effects. By throwing a long-wavelength particle into an extremal hole, a non-extremal configuration is created and quantum-mechanically one expects it to decay via Hawking emission back to extremality. To render this problem tractable one can boil it down considering a low $q$ and incoming neutral matter with zero angular momentum. In this context, dilatonic black holes [3] were extensively considered since the scattering particle-hole and the ensuing information loss problem can be analyzed in an analytical framework. This is so because the problem can be reduced to study a two-dimensional effective theory [4] which turns out to be solvable even when back-reaction effects are included.

Let us consider the solutions of Einstein-Maxwell gravity with null infalling matter

$$ds^2 = -\left(\frac{r - r_+}{r^2}\right) dv^2 + 2dr dv + r^2 d\Omega^2, \quad F = e^{\phi} \epsilon_2,$$

where $\epsilon_2$ is the volume element of the unit $S^2$. The only nonzero component of the stress tensor is given by

$$T_{vv} = \frac{\partial_v \left( r_+(v) + r_-(v) \right)}{8\pi r^2},$$

where $f^2 = G$ is Newton’s gravitational constant. One can describe the formation of a non-extremal black hole by sending a low-energy shock wave

$$T_{vv} = \frac{\Delta m}{4\pi r^2} \delta (v - v_0),$$

in the extremal geometry ($v < v_0$). This model can be described by an effective two-dimensional theory given by

$$I = \int d^2 x \sqrt{-g} \left[ R_{\phi} + L_{\phi} V(\phi) - \frac{1}{2} \right] \nabla f^2,$$

where the field $f$ represents the null matter with $(\partial_v f)^2 \equiv T_{vv} = 4\pi r^2 T_{vv}$, and

$$\phi = \frac{r^2}{4f^2}, \quad V(\phi) = (4\phi)^{-2} - q^2 (4\phi)^{-2}. $$

The two-dimensional metric is related to the $r - v$ projection of (1) by the conformal rescaling

$$ds^2 = \sqrt{\phi} d\bar{s}^2.$$

The extremal black hole is recovered for the zero of the potential $V(\phi_0) = 0$, which corresponds to $\phi_0 = q^2/4$. This fact...
suggests us to consider the effective near-horizon and near-extremal theory defined by the expansion of (5) around \( \phi_0 \)
\[
\phi = \phi_0 + \tilde{\phi} , \quad m = l q + \Delta m .
\] (8)

We obtain
\[
I = \int d^2 x \sqrt{-g} \left[ (R + 4 \frac{4}{l q^2}) \tilde{\phi} - \frac{1}{2} \nabla J \right] + O(\tilde{\phi}^2) ,
\] (9)

where the leading order term is just the Jackiw-Teitelboim (JT) model, which now arises as the effective theory governing the dynamics near extremality and close to the horizon. A d = 1 realization of the AdS\(_{d+1}\)/CFT\(_d\) correspondence [13] in the JT model exactly accounts for the deviation from extremality of the Bekenstein-Hawking entropy of RN black holes [14]. Therefore one can also expect to obtain an exact picture of the evaporation process near the horizon. The formation of a near-extremal black hole due to a shock wave (4) can be pushed down to the JT metrics of constant negative curvature. For \( v < v_0 \) we have the extremal RN configuration and its near-horizon geometry is given by the Robinson-Bertotti anti-de Sitter geometry [15]
\[
ds^2 = -\frac{2}{l q^3} \tilde{x}^2 dudv ,
\] (10)

where \( r_0 \) is the extremal radius. After the rescaling (7) the two-dimensional metric in null conformal coordinates becomes
\[
ds^2 = -\frac{2}{l q^3} \tilde{x}^2 du dv ,
\] (11)

with \( u = v + \frac{l^2 q^2}{2} \). Proceeding in a similar way for the near-extremal configuration \( v > v_0 \), we obtain
\[
ds^2 = -\frac{2}{l q^3} \tilde{x}^2 - l \Delta m) d\tilde{u} d\tilde{v} ,
\] (12)

with
\[
\tilde{u} = v + \sqrt{2q^3 \Delta m} \arctanh \sqrt{\frac{2}{2q^3 \Delta m}} \tilde{x} .
\] (13)

The coordinates \((v, u)\) and \((v, \tilde{u})\) are the radial null coordinates corresponding to those of RN \((l + r^* , l - r^*)\) before and after the shock wave. Imposing the continuity of (11) and (12) along \( v = v_0 \) one obtains
\[
u = v_0 + a \cosh \frac{\tilde{u} - v_0}{a} ,
\] (14)

where \(a = \sqrt{2 q^3 / \Delta m}\). From this relation we can work out immediately the outgoing energy flux of Hawking radiation in terms of the Schwarzian derivative between the coordinates \(u\) and \(\tilde{u}\)
\[
\langle T^f_{\Delta u} \rangle = -\frac{1}{2 \Delta u} \langle u, \tilde{u} \rangle = \frac{1}{12 \pi a^2} = \frac{\pi l^2}{12} T_H .
\] (15)

We observe that this flux is constant and coincides with the thermal value of Hawking flux for near-extremal RN black holes, where \(T_H\) is Hawking’s temperature. This fact can be understood easily since the AdS\(_5\)/S\(_3\) geometries associated to (11) and (12) represent indeed the near-horizon limit of the RN geometries (1) due to the shock wave (4). So the constant thermal flux for every value of \(\tilde{u}\) corresponds to the flux measured by an inertial observer at future null infinity approaching the event horizon of the RN black hole. In the light of this remark it is interesting to point out that the JT theory also describes the dynamics of extremal and near-extremal RN black holes close to the horizon in the presence of a spherically symmetric Klein-Gordon field. This is so because, as it is well-known, the scalar field propagates freely near the horizon.

Our purpose now is to analyze the back-reaction effects in the evaporation process. The one-loop effective theory is obtained by adding the non-local Liouville-Polyakov term to the classical action. We then get (\(\lambda^2 = l^{-1} q^{-2}\))
\[
I = \int d^2 x \sqrt{-g} \left[ R \tilde{\phi} + 4 \lambda^2 \phi - \frac{1}{2} \sum_{i=1}^N \nabla J_i^2 \right] - \frac{N h}{96 \pi} \int d^2 x \sqrt{-g} R \Box^{-1} R + \xi N \frac{h}{12 \pi} \int d^2 x \sqrt{-g} \lambda^2 ,
\] (16)

where we have considered the presence of \(N\) scalar fields. The parameter \(N\) allows us to consider the theory in the large \(N\) limit, keeping \(N h\) fixed. Moreover we have also added a local counterterm (in the form of a 2d cosmological constant which corresponds to the freedom of adding a constant to the 2d conformal anomaly), mimicking the analysis of dilatonic gravity theory [5], to ensure that the extremal geometry remains an exact solution of the one-loop theory at \(\xi = 1\). Our results are the same irrespective of the value of \(\xi\). We should mention now that for the region we are interested in, the conformal factor \(\sqrt{\sigma}\) of (7) is almost constant and therefore the semiclassical quantization in terms of the Einstein-Maxwell action and the JT action are equivalent. Far from the horizon this is no more true. The equations of motion derived from (16) in conformal gauge \(ds^2 = -e^{2\phi} dx^+ dx^-\) are (from now on we take \(\xi = 1\))
\[
2 \partial_{\phi} \partial_{-\rho} + \lambda^2 \phi^2 \phi = 0 ,
\] (17)
\[
\partial_\phi \partial_{-\rho} \phi' + \lambda^2 \phi' e^{2\phi} = 0 ,
\] (18)
\[
\partial_{\phi} \partial_{-\rho} f_i = 0 ,
\] (19)
\[
-2 \partial_{\phi}^2 \phi + 4 \partial_{\rho} \rho \partial_{-\rho} \phi = T^f_{\Delta u} - \frac{N h}{12 \pi} t_{\Delta u} - \frac{N h}{12 \pi} (\partial_{\rho} \rho)^2 - \partial_{-\rho} \rho .
\] (20)

The functions \(t_{\Delta u} (x^\pm)\) are related with the boundary conditions of the theory and depend on the quantum state of the system. The Liouville equation (17) has the general solution
\[
ds^2 = -\frac{\partial_{\phi} A_{\phi} \partial_{-\rho} A_{-\rho} - (A_{\phi} A_{-\rho})^2}{(1 + 2 A_{\phi} A_{-\rho})^2} dx^+ dx^- .
\] (21)
Before the shock wave these coordinates $x^\pm$ correspond to the RN coordinates $(v, u)$ and after $(v > v_0)$ the relation is
\begin{equation}
 v = x_0^+ + a \arctanh \left( \frac{x^+ - x_0^+}{x^- - x_0^+} \right),
\end{equation}
which can be readily obtained using the classical relation (23), is given by
\begin{equation}
 \tilde{\phi} = lq^3 \frac{1 - \Theta(x^+ - x_0^+)}{a^2 (x^+ - x_0^+)(x^- - x_0^+)}.
\end{equation}

The expression (30), and also the function $F(x^+)$, admits a series expansion in powers of $\hbar$, where the classical term, obtained using the classical relation (23), is given by
\begin{equation}
 x^- = x^+ - \frac{2F}{F'},
\end{equation}
represents the location of the extremal radius and $\partial_{x^+} \tilde{\phi} = 0$ defines the inner and outer apparent horizons in the spacetime of the evaporating black hole
\begin{equation}
 x^- = x^+ - \frac{F'}{F'} \pm \frac{\sqrt{F'^2 - 2F F''}}{F''}.
\end{equation}

The intersection of these three curves takes place when
\begin{equation}
 F'^2 - 2F F'' = 2\nu F m(x^+) = 0,
\end{equation}
where $\nu = lq^3$ is a characteristic time of the evaporating black hole. From the numerical graph of the function $F(x^+)$ it is clear that this happens at a finite...
value of $x^+$ for which $F(x^+_{\text{int}}) = 0$ and moreover from eq. (34) it also follows that $F'(x^+_{\text{int}}) = 0$. One can also show that $F''(x^+_{\text{int}})$ is nonzero while all the other derivatives vanish, so that locally close to the intersection point $F(x^+)$ behaves as a parabola with exponentially suppressed corrections. This is enough to prove, see (32) and (33), that the intersection point belongs to the AdS boundary, i.e. $r^+ = x^+_{\text{int}}$. The graph of all these curves is shown in Fig. 2, where the saddle point in the outer apparent horizon curve $r^+$ signals the transition from the strong to the weak back-reaction regimes as described in [11]. At the end-point the curves $\phi = 0$ and $\partial_+ \phi = 0$ are null and the dilaton function is well represented asymptotically by the extremal solution

$$\phi = \frac{\partial_+^2 F(x^+_{\text{int}})}{2} \frac{(x^+ - x^+_{\text{int}})(x^- - x^-_{\text{int}})}{x^- - x^+} + \ldots$$

(35)

where the dots are nothing but exponentially small corrections. So, as time passes the solution becomes 'more and more extremal' but without actually coming back to the extremal state due to the infinite evaporation time. Due to the fact that $T_H = 0$ for the extremal black hole our exact results are in agreement with the third law of thermodynamics applied to black holes (see for instance [17]), despite the fact that the weak energy condition is violated close to the horizon, as well as with those obtained using the adiabatic approximation [10].

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