In-Plane Elliptic Flow of Resonance Particles in Relativistic Heavy-Ion Collisions

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We analyze the second Fourier coefficient $v_2$ of the pion azimuthal distribution in non-central heavy-ion collisions in a relativistic hydrodynamic model. The exact treatment of the decay kinematics of resonances leads to almost vanishing azimuthal anisotropy of pions near the midrapidity, while the matter elliptic flow is in-plane at freeze-out. In addition, we reproduce the rapidity dependence of $v_2$ for pions measured in non-central Pb + Pb collisions at 158.4 GeV. This suggests that resonance particles as well as stable particles constitute the in-plane flow and are important ingredients for the understanding of the observed pion flow.

The main goals of relativistic nuclear collisions are to determine the nuclear equation of state (EOS) under extreme conditions and to understand a new phase of deconfined nuclear matter, the quark-gluon plasma (QGP) [1]. Since it is the pressure gradient perpendicular to the collision axis that causes various transverse collective flows, such as radial flow, directed flow, and elliptic flow, in relativistic nuclear collisions, these flows observed in the final state are expected to carry the information about the EOS [2]. If the QGP phase is created in nuclear collisions, the quark matter expands, cools down, and goes through the "softest point" [3] where the ratio of the pressure to the energy density takes its minimum as a function of the energy density. Therefore, it is expected that the suppression of the collective flows is not only a signal for the existence of the QGP [3–6] but also a useful tool to determine the EOS near the phase transition region.

Some experimental groups reported that the inverse slope parameters of the transverse mass spectra for the non-multistrange hadrons $\pi$, $K$, $p$, and $d$ in central Pb + Pb collisions at 158.4 GeV at the CERN Super Proton Synchrotron (SPS) are parametrized by two common values, the freeze-out temperature and the transverse flow velocity [7]. This implies that these particles constitute the radial flow and the local thermalization at least among those particles is achieved in central collisions. It has been, however, an open question whether equilibration is achieved even in non-central collisions. In this Letter, we study the elliptic flow that is made of stable particles and resonances with a hydrodynamic model, and show that the final pion distribution is well described in this approach, if decay kinematics is appropriately taken into account.

The rapidity dependence of azimuthal anisotropy for particle $i$ is characterized by the coefficients $v_n^i (n = 1, 2, 3...)$ in the Fourier expansion of the azimuthal distribution of the particle [8]:

$$\frac{dN^i}{d\phi dY} = \frac{1}{2\pi} \frac{dN^i}{dY} \left[ 1 + 2v_1(Y) \cos \phi + 2v_2(Y) \cos 2\phi + \cdots \right],$$  \hspace{1cm} (1)

where $\phi$ is the azimuthal angle measured from the reaction plane and $Y$ is the rapidity. Non-vanishing $v_1$ and $v_2$ imply the formation of directed and elliptic flows, respectively. Experimentally, $v_2(Y)$ is of the order of four percent around the midrapidity ($Y = 2.92$) for low transverse momentum ($50 < p_T < 350$ MeV/$c$) charged pions in non-central Pb + Pb collisions that correspond to the impact parameter range $6.5 < b < 8.0$ fm [9]. The value of the observed $v_2$ has been claimed to be smaller than that predicted for direct pions in hydrodynamic models.

In the following, we consider not only direct pions but also indirect pions that are from resonance decays. Thus, we need to strictly distinguish the matter flow before freeze-out and the observed particle flow. In this Letter we use a term “in-plane” when the hydrodynamic flow before freeze-out is directed preferentially to the positive and the negative $x$-axes on the transverse plane and a term “positive elliptic flow” when $v_2$ of observed particles is positive. Here the $x$-axis is defined as the direction of impact parameter in non-central collisions. When one only considers particles directly emitted from freeze-out hyper-surface, “positive elliptic flow” means that the hydrodynamic flow is “in-plane”. However, once feeding from resonance decays is included, the above two are no longer equivalent.

First, let us suppose that a resonance particle with mass $m_R$ is emitted from freeze-out hyper-surface and decays into two identical daughter particles with mass $m$ in the vacuum. In the rest frame of the resonance particle the decay is isotropic and the daughter particles are uniformly distributed with respect to the azimuthal angle when one averages the spin of the resonance particles. This is, however, not the case anymore if the resonance is moving in a reference frame. To see this more clearly, let us assume that the resonance particle moves with velocity $(V_{Rz}, V_{Ry}, V_{Rx}) = (V, 0, 0)$
in a reference frame. If \( V \) is larger than the critical value \( V^* = p^*/E^* \), where \( p^* = \sqrt{m_R^2/4-m_T^2} \) and \( E^* = m_R/2 \), the probability that the daughter particle is emitted with an angle \( \phi \) from the \( x \)-axis peaks at \( \phi \pm = \pm \sin^{-1}\left[ (1-V^2)^{1/2}/(MV) \right] \) and \( \phi \) is limited in a range, \( \phi_- \leq \phi \leq \phi_+ \) [10]. Two peaks are due to the Jacobian singularity in the Lorentz transformation from the resonance rest frame to the reference frame. As a result, the opening angle between the daughter particles tends to remain finite even if the resonance particle moves at a large velocity. This is the reason why the equivalence between “in-plane flow” and “positive elliptic flow” breaks down when one includes the decay of resonances in the final state.

The multiplicity of pions through two body decays of resonances is given by

\[
N_{R \to \pi X} = \int \frac{d^3p_R}{4\pi p^*} b_{R \to \pi X} d\phi \int ds W_R(s) \frac{g_R}{(2\pi)^3} \frac{p_R^\mu \sigma^\mu_R}{(p_R u_\mu - \mu)/T_f + 1},
\]

where \( u^\mu \), \( \mu \), \( T_f \), and \( \sigma_R \) are, respectively, the four-dimensional fluid velocity, the chemical potential, the freeze-out temperature, and the freeze-out hyper-surface. \( p_L \) is the longitudinal momentum of pion. \( p_R^\mu \) and \( g_R \) are, respectively, the resonance four momentum in the reference frame and the degeneracy, and \( -(+ \pm) \) is for boson (fermion) resonances. \( b_{R \to \pi X} \) is the branching ratio of the decay process and \( W_R \) is the Breit-Wigner type function, which takes account of the finiteness of the resonance width. For \( W_R \), we adopt the form used in Ref. [11]. The Jacobian of the Lorentz transformation from the resonance rest frame to an arbitrary reference frame \( J(p_L, \phi; V_R) \) is defined by

\[
dp^*_L d\phi^* = J(p_L, \phi; V_R) dp_L d\phi, \quad \text{where the quantities with (without) } * \text{ are the ones in the resonance rest (reference) frame.}
\]

Two typical shapes of the Jacobian as functions of the azimuthal angle in \( \rho \to \pi \) are shown in Fig. 1 (a). If \( V^*_R = (V^*, 0, 0) \) and \( V^*_R < V^*(\sim 0.93c \text{ in this process}) \), \( J(p_L = 0, \phi; V_R) \) has a broad peak around \( \phi = 0 \) at the same azimuthal angle of the resonance particle as expected. However, if \( V^*_R > V^* \), \( J(p_L = 0, \phi; V_R) \) has a finite value only in the range \( \phi_- < \phi < \phi_+ \) and two sharp peaks appear at \( \phi_+ \) in addition to the original broad one as explained above.

In order to get an idea about the effect of thermal smearing, we estimate the azimuthal distribution of pions through the above process, taking the following simple model: There are only two fluid elements with local fluid velocities \((v_x, v_y, v_z) = (\pm 0.5c, 0, 0)\). Both elements are assumed to be thermalized at the temperature \( T_f = 140 \text{ MeV} \). The pion distribution from \( \rho \)-meson decays in each fluid element is shown in Fig. 1 (b). The sharp peaks at \( \phi \sim \pm 1.21 \pm 1.93 \) in the Jacobian are smeared by thermal motion of \( \rho \)-mesons, but they are still visible in the azimuthal distribution of pions from \( \rho \)-mesons in the fluid element with \( v_x = 0.5c (-0.5c) \), while the peaks at \( \phi = 0 (\pi) \) are completely washed out. The superposition of the two distributions leads to an azimuthal distribution with broad peaks at \( \pm \pi/2 \), as shown in Fig. 1 (b). In this simple model, pions from \( \rho \)-meson decay have negative \( v_2 \), i.e., the elliptic flow in the final state is negative, while the motion of the two fluid elements is in-plane.

We next carry out realistic hydrodynamic simulations for the space-time evolution of \( \text{Pb + Pb} \) collisions at 158A GeV to see how large the effect of the Jacobian singularity is for the final state pion distribution. We first assume that the hot and dense nuclear matter produced in heavy-ion collisions is in local thermal equilibrium after \( t_0 = 1.55 \text{ fm/c} \) since the two nuclei touched [12]. Then, we describe the space-time evolution of nuclear matter after this time by using a relativistic hydrodynamic model without assuming the Bjorken’s scaling solution [13] or the cylindrical symmetry along the collision axis [14–17]. Thus, it is possible to discuss the rapidity dependence of elliptic flow \( v_2(Y) \) for charged pions through resonance decays as well as for charged pions directly emitted from the freeze-out hyper-surface in this model. Our numerical algorithm for the hydrodynamic simulation is categorized into the piecewise parabolic method (PPM) [18], which is known as a very robust scheme for the non-relativistic gas with shock waves. We have extended the PPM to the relativistic hydrodynamic equation [17]. We use a model EOS with a first order phase transition between the QGP phase and the hadron phase. The QGP phase is assumed to be free gas composed of quarks with \( N_f = 3 \) and gluons. For the hadron phase we adopt a resonance gas model, which includes all baryons and mesons up to the mass of 2 GeV [19], together with an excluded volume correction [20]. We use the critical temperature at zero baryon density, \( T_c(n_B = 0) = 160 \text{ MeV} \). The two model EOS’s are matched by imposing the Gibbs’ condition for phase equilibrium on the phase boundary. The numerical results of the hydrodynamic simulation give us the momentum distribution of hadrons through the Cooper-Frye formula [21] with a freeze-out temperature \( T_f = 140 \text{ MeV} \). We have used this formula for the \text{direct} emission of \( \pi^- \), \( K^- \), and \( p \). In addition, we have taken into account negative pions from the decays of \( \rho \), \( \omega \), \( K^* \), and \( \Delta \) in the final state.

In the numerical simulation we have fixed the impact parameter at 7.0 fm. We have chosen the initial parameters in the hydrodynamic simulation to reproduce the preliminary data of rapidity distribution of negative hadrons and net protons for the impact parameter range 6.0 < \( b < 8.0 \text{ fm} \) by the NA49 collaboration [22]. The corresponding central energy density \( E_0 \) and baryon number density \( n_0 \) are, respectively, 10.0 GeV/fm\(^3\) and 0.76 fm\(^{-3}\). The energy density and the baryon number density on the transverse plane are assumed to be in proportion to the standard Woods-Saxon function with a diffuseness parameter \( \delta_r = 0.5 \text{ fm} \) at the initial time. We have assumed the initial transverse flow to vanish.

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We first discuss the effect of the Jacobian singularity for pions through $\rho$-meson decays in the realistic hydrodynamic calculation. The momentum distribution of $\rho$-mesons is free from the effects of the Jacobi function and is given by the second integral in Eq. (2), i.e., the integral with respect to $s$ and $\sigma_{ij}$. In Fig. 2, we show $v_2$ for the $\rho$-mesons directly emitted from freeze-out hyper-surface and for pions through $\rho \rightarrow \pi \pi$. The elliptic flow of $\rho$-mesons is positive and in-plane. Nevertheless, the $v_2$ for pions through $\rho \rightarrow \pi \pi$ almost vanishes near the midrapidity due to the decay kinematics. This implies that the effect of the Jacobian singularity, which we discussed above with a simple model, survives even in this realistic calculation. The behavior of the $v_2$'s of pions from the other resonances included in our calculation is similar to this.

Finally, we discuss the rapidity dependence of the observed pion elliptic flow obtained from our fully three-dimensional hydrodynamic calculation. In Fig. 3 we compare our results with the experimental data by the NA49 Collaboration [9]. The solid line represents $v_2$ for the total charged pions. For comparison, $v_2$ for pions directly emitted from freeze-out hyper-surface (dashed line) and that for pions through resonance decays (dotted line) are shown separately. Our results were obtained by summing up pion distribution over a $p_T$ range, $50 < p_T < 350$ MeV/c. The experimental data corresponds to an impact parameter range $6 < b < 8.0$ fm. The solid line, which has a maximum value at midrapidity [17], is in good agreement with the experimental data [23]. The $v_2$ for indirect pions from resonance decays vanishes near midrapidity as discussed above. This reduces $v_2$ for the total pions by about 40 % near midrapidity. This figure tells us that hydrodynamic description, which assumes the local thermal equilibrium, works well also for the expansion stage of non-central collisions at the SPS energy [24].

A few remarks on other calculations are in order here. At midrapidity, our result for the total pions is consistent with the previous result obtained by a two-dimensional hydrodynamic model [24]. The authors of Ref. [24] assumed that the longitudinal expansion can be described by the Bjorken’s scaling solution [13] and numerically simulated the evolution of nuclear matter only in the transverse directions. Hence they could not obtain the rapidity dependence of elliptic flow. They also took into account resonance particles up to the mass of the (1232), but concluded that resonance decays reduce the momentum anisotropy for pions by only 10-15 %. In addition, they did not discuss the details of the reduction mechanism. Liu et al. [25] compared their results from a transport model, the Relativistic Quantum Molecular Dynamics (RQMD), with the experimental data and concluded that the model calculations are in reasonable agreement with experimental data. The experimental data of elliptic flow was, however, later updated [9], and the agreement is not as good as before anymore. Soff et al. [26] also obtained $v_2(Y)$ for pions from a microscopic transport model, the Ultrarelativistic Quantum Molecular Dynamics (UrQMD), but the situation is similar to the RQMD model. Recently, it was argued that the suppression of the elliptic flow is due to partial thermalization [2,27,28]. However, as we have shown, this is not needed to explain the data. According to our result, the suppression rather implies the full thermalization of the system, including resonances.

In summary, we have investigated the elliptic flow of pions in Pb + Pb non-central collisions at 158A GeV in a relativistic hydrodynamic model. As sources of pions, we have considered not only direct pion emission from the freeze-out hyper-surface but also decays of resonance particles. Pions from resonance decays suppress the azimuthal anisotropy in the midrapidity region as much as 40 %. Taking this effect into account, we were able to reproduce the experimental data of the rapidity dependence of $v_2$. These results lead to our conclusion that the pions and the resonance particles constitute thermalized in-plane elliptic flow and that the hydrodynamic picture is applicable to the expansion stage in the non-central collisions at the SPS energy. The Jacobian singularity effect should exist also in cascade calculations such as RQMD and UrQMD if the decay kinematics is appropriately taken into account. It will be interesting to see how important this mechanism is for the suppression in those calculations.

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[12] $t_0$ is the passage time when the Lorentz contracted incident nucleus finishes penetrating the target nucleus in the center of mass system.
[23] Recently Dinh et al. found that the Hanbury-Brown Twiss effect creates apparent azimuthal anisotropies. According to their results, we can compare our result with the corrected data by this effect: P. M. Dinh, N. Borghini, and J.-Y. Ollitrault, Phys. Lett. B 477, 51 (2000).
FIG. 1. (a) Jacobian as a function of azimuthal angle in the Lorentz transformation between two reference frames (see text). The solid and dashed lines correspond to $V = 0.94c$ and $0.9c$, respectively. (b) Azimuthal distribution (in arbitrary units) of pions at midrapidity through decays of $\rho$-mesons from two fluid elements which are moving in opposite directions. The velocities of fluid elements are assumed to be $(v_x, v_y, v_z) = (\pm 0.5c, 0, 0)$. 

$J(p_L=0, \phi)$

$\frac{dN}{d\phi}$, arb. units
FIG. 2. Azimuthal anisotropy $v_2$ for $\rho$-mesons directly emitted from freeze-out hyper-surface (solid line) and for pions through $\rho \rightarrow \pi \pi$ (dashed line) in non-central Pb + Pb collisions at 158 $A$ GeV.

FIG. 3. Azimuthal anisotropy $v_2$ for total charged pions (solid line), direct pions (dashed line), and pions from resonance decays (dotted line) as functions of rapidity. The experimental data was measured by NA49 in Pb + Pb collisions at 158 $A$ GeV. See text for details.