Brane Worlds, the Cosmological Constant and String Theory

S. P. de Alwis, A. T. Flournoy and N. Irges

Department of Physics, Box 390, University of Colorado, Boulder, CO 80309.

Abstract

We argue that traditional methods of compactification of string theory make it very difficult to understand how the cosmological constant becomes zero. String inspired models can give zero cosmological constant after fine tuning but since string theory has no free parameters it is not clear that this is allowed. Brane world scenarios on the other hand while they do not answer the question as to why the cosmological constant is zero do actually allow a choice of integration constants that permit flat four space solutions. In this paper we discuss gauged supergravity realizations of such a world. To the extent that this starting point can be considered a low energy effective action of string theory (and there is some recent evidence supporting this) our model may be considered a string theory realization of this scenario.

---

1 e-mail: dealwis@pizero.colorado.edu, flournoy@pizero.colorado.edu, irges@pizero.colorado.edu
1 Introduction

The traditional method of compactification of superstring theory involves writing the ten dimensional manifold as a direct product of a non-compact Minkowskian four manifold and a compact Euclidean six manifold $M_{10} = M_4 \otimes M_6$, with the size of the compact space being set by the string scale. The four manifold is identified with the space we observe around us and is therefore taken to be flat. The compact manifold is taken to be a Ricci flat Kähler manifold (i.e. a Calabi-Yau space or an orbifold) so that we obtain $\mathcal{N} = 1$ supersymmetry (SUSY) in the four manifold [1].

The latter is required for at least two reasons. The first is the so-called hierarchy problem involving the stabilization of the weak scale to quantum corrections. With (softly broken) $\mathcal{N} = 1$ SUSY one can protect this hierarchy. The other issue is that of the cosmological constant (CC). Again supersymmetry offers the hope of stabilizing this against quantum corrections i.e. if the ultra violet theory (say at the string scale) has zero cosmological constant then as long as SUSY is preserved it will remain zero. In this case case however once SUSY is broken it is difficult to see how one can prevent a CC, at least of order the mass splitting between fermions and bosons, from being generated.

Actually getting a realistic model where this is achieved would be progress, since there would also be standard model phase transitions which are also of the same order and could in principle cancel the SUSY breaking effects. Nevertheless at this point it is, we believe, fair to say that even this has not yet been achieved in a natural way, since typical cosmological constants that are generated upon SUSY breaking are at an intermediate scale of $O(10^{11} \text{GeV})$ (which is the actual scale of SUSY breaking in the so-called hidden sector).

To get a zero (or small) CC requires fine tuning of some parameter in the scalar potential of $\mathcal{N} = 1$ supergravity. However string theory has no tunable parameters. In fact it is not very clear how to even generate the potential for the moduli (the dilaton, the size and shape of the compact manifold) but it is generally agreed that, either stringy or low energy field theoretic, non-perturbative effects will give such a potential and stabilize these moduli \(^1\).

Let us first think about the field theoretic stabilization mechanism. One possibility is to have several gauge groups in the hidden sector whose gauginos condense. This is the so-called race track model. The effective superpotential needs to be a sum of three exponentials of the moduli fields, two are needed to get a weak gauge coupling minimum and an intermediate scale of SUSY breaking and a third is needed in order to fine tune the cosmological constant to zero. This model has various problems associated with the steepness of the potential (see [2] for a recent discussion) but even apart from that it seems unlikely that the sort of fine tuning that is required is allowed by string theory. An alternative would be to have the moduli stabilized by string scale physics. In this case however one needs to find field theoretic models which solve the practical cosmological constant problem i.e. obtain a model with CC only at the weak scale as advocated in [2]. Even this seems a non-trivial task; in particular it seems that one needs a constant in the superpotential to do so and to get a zero (or small) cosmological constant would seem to require a fine tuning that would really be at variance with what one expects from string theory. Similar remarks apply to all models proposed so far for getting standard model physics out of string theory (including many brane world type scenarios based on type I string theory) once one tries to get flat four dimensional space after SUSY breaking.

\(^1\)For a recent discussion highlighting the problems involved with references to earlier literature see[2]
The main point of this paper is to propose a string theoretic brane world scenario for getting a zero cosmological constant based on arguments sketched in [3]. This would imply a radical departure from what happens in the standard string compactification models discussed in the above paragraphs. Instead of compactifying to four flat dimensions with $\mathcal{N} = 1$ supersymmetry, we compactify the ten dimensional theory (here we will concentrate on type IIB) on a five sphere (or squashed sphere) to five dimensions [4]. In contrast to the standard compactifications we will now have a potential for at least the T-moduli (i.e. the size and shape of the squashed sphere) and of course a bulk cosmological constant. What we have is five dimensional gauged supergravity but the four dimensional world, instead of being a further compactification of this, is viewed as a brane sitting in this five dimensional bulk. As discussed in [3], there are various possibilities for getting flat four dimensional space on the brane without fine tuning\(^2\) none of which explain why the cosmological constant is zero. But unlike in the standard compactifications, where it is difficult to understand even how the CC becomes tuned to zero, in our case we believe there is a viable scenario whereby with a choice of integration constants and a compactification parameter (which is not determined by the ten dimensional theory) flat space four dimensional solutions can be obtained.

Although our discussion is confined to an analysis of a five dimensional effective action coming from compactifying a ten dimensional action for type IIB supergravity on a (squashed) sphere, we believe that is quite likely that there is a string construction in such a Ramond-Ramond (RR) background. In fact, recently there has been considerable progress in constructing such theories [7]. Thus we think that the model that we propose here could be promoted to the complete string theory.

Another issue that we should discuss here is that of singular brane configurations versus smooth configurations. We take the point of view that our branes are D-branes and/or orientifold planes so that as far as the low energy effective action is concerned their thickness is infinitesimal.\(^3\) Their transverse size in other words is set by the string scale and to the extent that we are working in the low energy effective action we can ignore their thickness. Thus we will be looking for solutions to the bulk equations in the presence of these branes, which will act as singular source terms as for instance in section 3 of [8]. We believe that the considerations of [9] where the authors look for brane scenarios that try to mimic the Randall - Sundrum (RS) scenarios [10] and/or [11] without putting in sources (and finding a negative result) are not relevant to our construction. On either side of our branes we will have smooth solutions to the supergravity equations which will then be matched at the position of the branes. It should also be pointed out here that a model for the real world would in fact require a brane which carried gauged degrees of freedom and so we really would want to think in terms of D-branes rather than smooth brane like configurations. The picture that we have (i.e. basically that in [10],[11]) actually implies the physical existence of a brane (on which the standard model should live) placed in the bulk supergravity space.

Our model has the explicit D-brane/orientifold planes at the fixed points of the $\frac{\mathbb{Z}_2}{\mathbb{Z}_2}$ orbifold in the $(x^4)$ fifth direction. We assume that the ten dimensional dilaton is frozen by string scale dynamics.

\(^2\)For a discussion of a case which apparently does not involve a choice of integration constant but involves a bulk singularity see the first paper of [5]. Apart from the difficulty of interpreting this naked singularity (which is common to all the models in both papers of [5] this model has the problem that the brane tension chosen is not renormalization group (RG) invariant. For comments on these models along these lines see [3], [6].

\(^3\)We stress that although flat space considerations imply that gravity cannot be confined to a D-brane (as one wants for a one brane RS type scenario) in a warped background this is not necessarily the case.
When the squashing modes are constant but at a certain critical point with non-zero values, the bulk is a $\mathcal{N} = 2$ background. Turning on the breathing mode $\varphi$ of the sphere should not affect the supersymmetry. In the limit where the radius of the circle goes to infinity the solution with $\varphi = constant$ being frozen by string scale dynamics will correspond to AdS solutions with $\mathcal{N} = 2$. The supersymmetry of the D/orientifold 3-plane would be $\mathcal{N} = 1$. This would give by adjustment of an integration constant (in this case it is actually a compactification curvature scalar), a solution of the RS type without fine tuning and with $\mathcal{N} = 1$ supersymmetry. For finite values of the radius we have two brane solutions with non-constant solutions for the breathing mode of the type discussed in [12]. In this case, in addition to the constant coming from compactification, there are also two integration constants from the metric factor and breathing mode and also the radius of the circle - four constants in all that will be determined by the two pairs of matching conditions at each brane. Thus all the constants are completely fixed. In either case the real world is taken to be on a 3-brane and upon breaking of $\mathcal{N} = 1$ supersymmetry on the brane the effective cosmological constant would get adjusted to zero by the mechanism explained in [3].

As stressed in this last reference this is of course not a solution to the cosmological constant problem since there is no explanation of why the integration constants chosen to give the values that they need to have in order to get flat branes. Nevertheless we believe it is progress in that at least there is an explanation as to how one might get a zero CC in a string theoretic scenario after SUSY breaking.\(^4\)

It should also be pointed out that the brane world scenario appears to be the only context, within string theory, that the idea of using the free parameter (from the point of view of the ten dimensional theory) in the potential of gauged supergravity, to adjust the cosmological constant to zero, can be made use of. This is because this mechanism can only be used in type IIA or IIB theories and the standard model/real world in such theories necessarily lives on a D-brane.

## 2 The String Theory Setup

We wish to consider type IIB string theory compactified on a five sphere or squashed sphere. Typically this background is obtained by turning on the five form RR flux. We assume that string theory on such a background exists perhaps along the lines discussed by Berkovits et al [7]. If this is indeed justified, it should follow that the entire D-brane machinery can be taken over.

Thus we look at $S^1/Z_2$ orientifold of the five dimensional theory resulting from the (squashed) sphere compactification with $S^1$ being a circle of radius $R$ and the $Z_2$ action being $x^4 \rightarrow -x^4$. The theory is a five dimensional gauged supergravity which contains in addition to the dilaton-axion, forty other scalars some of which can be interpreted as squashing modes of the five sphere. These fields are massless in the limit when the gauge coupling is zero. In addition there is a massive (compactification scale) breathing mode and of course the gauge fields which along with the two - two form fields associated with the F and D strings will be set to zero in the following. Also we assume that the ten dimensional dilaton is frozen by string scale dynamics.

Let us first consider the sphere compactification. The ten dimensional metric is written as

$$ds_{10}^2 = e^{2\alpha \varphi} ds_5^2 + e^{2\beta \varphi} ds^2(S^5)$$

\(^4\)While this paper was being prepared for publication a paper which discusses an alternate scenario for two flat branes without fine tuning appeared. Our mechanism is replaced there with an assumption about supersymmetry in the bulk and on the “Planck brane” [13].
with $\alpha = \frac{1}{4}\sqrt{\frac{5}{3}}$, $\beta = -\frac{3}{5}\alpha$. The ansätz for the self-dual 5-form is

$$H_{(5)} = 4me^{8\alpha\phi}\epsilon_{(5)} + 4m\epsilon_{(5)}(S^5),$$  \hspace{1cm} (1)

where $\epsilon_{(5)}$ and $\epsilon_{(5)}(S^5)$ are the volume forms of the non-compact and compact spaces respectively.

The effective five dimensional action is [4]

$$S = \int d^5x\sqrt{-G_5}[R - \frac{1}{2}(\partial\varphi)^2 + e^{16\alpha^2}R_5 - 8m^2e^{8\alpha\varphi}],$$  \hspace{1cm} (2)

where $R_5$ is the scalar curvature of the five sphere and we’ve set to zero all fields that are irrelevant to our discussion. Note [4] that this action allows the $AdS_5$ solution with constant breathing mode $\varphi = \varphi_0$ given by

$$e^{24\alpha\varphi_0} = \frac{R_5}{20m^2}$$  \hspace{1cm} (3)

and

$$R_{\mu\nu} = -4m^2e^{8\alpha\varphi_0}g_{\mu\nu}. \hspace{1cm} (4)$$

Now the fifth dimension is a $S^1/\mathbb{Z}_2$ orbifold. The fixed points would be orientifold planes and we place D-branes at one or other (or both) fixed points so as to have two (composite) branes at the ends of the fifth dimension. The picture is just like that in the ten dimensional type IA situation analyzed by Polchinski and Witten [14]. In the original ten dimensional framework, the self-dual five form field strength would satisfy in the presence of a collection of $n_0$ branes located near $x^4 = 0$ and extending in the $x^1, x^2, x^3$ directions (counting -16 for the orientifold fixed plane), the equation $dH_{(5)} = n_0\tau\delta(x^4) \wedge dx^4 \wedge \epsilon_5(S_5)$, where $\tau$ is the charge of a single brane. We take $x^0$ to be the time like direction and $x^1, x^2, \cdots, x^9$ the directions along the five sphere. Substituting expression (1) into this, gives $\partial_x m(x^4) = n_0\tau\delta(x^4)$. The solution for $H_{(5)}$ now takes the form as before, i.e. (1), but with $m$ being now a piece wise continuous function of $x^4$ with a jump $\Delta m = n_0\tau$ at $x^4 = 0$ and a similar jump at $x^4 = \pi R$. Using also the symmetry under $x^4 \to -x^4$, we find the sort of situations illustrated in figs. 1, 2, 3.

In the above, we should point out that we have assumed not only that there is a valid string description as in Berkovits et al [7] but also that the corresponding five dimensional orientifolds have charges $-16$. These values are just given for illustrative purposes and of course if the general scenario is valid with however different charges, then the examples would have to be modified accordingly. We should also point out that one might try to justify such a picture by considering the following procedure. First, compactify type I string theory on a five torus. Taking the T-dual picture one has a type IIB orientifold with $2^5$ orientifold 3-planes at the fixed points of the $\mathbb{Z}_2$ orbifold symmetry of the dual torus and 32 D3-branes to cancel the tadpoles. Now let the size of the dual torus go to infinity while keeping some number of the D-branes at the fixed point at the origin. Now adding the point at infinity should give presumably an $S^5/\mathbb{Z}_2$. It is not quite clear to us which compactification makes sense in the complete string theory but given that the 5D gauged supergravity models appear to come from five sphere or squashed sphere compactifications we will just consider these, assuming that a theory along the lines of Berkovits et al [7] will allow our scenario.

As we will discuss in more detail in the next section the bulk is taken to be at the $\mathcal{N} = 2$ critical point of the gauged supergravity potential. In the presence of the branes this is then broken down
to $N = 1$ so that one may have a phenomenologically acceptable model on one or the other brane. Let us pick the one at $x^4 = 0$ to situate the standard model. The other one can then play the role of a hidden sector (as in the Horava-Witten theory [15]) where supersymmetry can be broken and then communicated to the visible brane. The only new point here is that any cosmological constant that is generated as a result of the SUSY breaking is compensated by adjustment of integration constants and $R_5$.

It should be remarked here that $R_5$ is on the same footing as the other integration constants (which occur in the five dimensional theory) from the point of view of the ten dimensional theory. It is not a modulus. The relevant modulus is the field $\varphi$. $R_5$ is, from the point of view of the ten dimensional theory, and presumably of string theory, an integration constant appearing in the ansatz for the ten dimensional metric. For compactification on a Ricci flat metric such as a Calabi-Yau metric, this constant can be absorbed into $\phi$. In our case as long as $m$ is non-zero i.e. the potential is not runaway, $R_5$ cannot be absorbed into the modulus $\varphi$.

We stress again that this is not a mechanism that was available with standard compactification scenarios (Calabi-Yau, Orbifold etc).

## 3 The 5D Gauged Supergravity

The ungauged, $D = 5$, $N = 8$ supergravity [17], can be obtained by simple dimensional reduction of type IIB supergravity in $D = 10$. Its massless bosonic spectrum consists of a graviton, 27 Abelian gauge fields and 42 massless scalar fields, which parametrize the coset space $E_{6(6)}/USp(8)$ ($42 = 78 - 36$). This ungauged supergravity corresponds to toroidal compactification of type IIB string theory to five dimensions. A natural question is then what is the supergravity theory that corresponds at low energy to compactification of IIB string theory on a curved space. Probably the simplest possible case is when the curved space is the round five dimensional sphere $S^5$. The corresponding supergravity theory is the maximally supersymmetric ($N = 8$) gauged supergravity in $D = 5$, which was constructed some time ago [18]. Its construction requires gauging an $SO(6)$ subgroup of $USp(8)$, embedded in $E_6$. The 27 vector fields decompose as $15 + 6 + 6$ under $SO(6)$, with the $15$ being the adjoint of the gauge group and the other two sextets are dualized away in the way described in [18]. The ungauged action is, of course, of order $g^0$, with $g$ the gauge coupling constant. The gauging introduces terms of order $g^1$, in the form of minimal couplings in the gauge covariant derivatives and in the gauge transformations. From the point of view of the gauged supergravity, the value of $g$ is arbitrary. To preserve supersymmetry, it turns out that a term of order $g^2$ has to be added, which is just the potential for the scalar fields. From the point of view of the type IIB string theory, the bulk gauge coupling constant $g$ is the vacuum expectation value of a five dimensional modulus, associated with the curvature of the compact space, $R_5$. The only other massless scalar that has isometry preserving properties is the axion. In fact, the 42 massless scalar fields decompose under $SO(6)$ as $20 + 10 + 10 + 1 + 1$. The two singlets correspond to the five dimensional dilaton $(s)$ and axion $(a)$, and clearly they can take vacuum expectation values without breaking the isometry of the $S^5$ and consequently the maximal supersymmetry. Some of

\footnote{Recently, other warped compactifications, in the context of M/F theory, have appeared [16] where a RS type scenario with explicit branes is constructed. However for these Calabi-Yau compactifications with G-flux, it is not possible to use our mechanism, since the compact space is Ricci flat, so one does not have the required freedom in the choice of constants to cancel the CC.}
the nonsinglet fields are the volume preserving but isometry/supersymmetry breaking “squashing modes” $\theta_i$ [4]. When all the fields $\theta_i$ in $V$ vanish, the full gauge symmetry $SO(6)$ and the maximal $\mathcal{N} = 8$ supersymmetry are preserved. The string theory picture then is, type IIB compactified on a round $S^5$ [19]. When some of the fields $\theta_i$ take non zero values (yielding a non zero scalar potential), some of the isometry and supersymmetry breaks, corresponding to a deformed (squashed) sphere compactification of IIB. Actually, this was proven rigorously for the case of our interest only recently in [20]. In addition to these massless fields, there has to be one more scalar field that can take an arbitrary vev and preserve the isometry and the full supersymmetry. It is the field that corresponds to the radius of the sphere, the breathing mode $\varphi$. The group theory decomposition of the 42 massless scalars implies that it has to belong to a massive multiplet.

One striking observation is that some of these deformed sphere compactifications are possibly related to the round sphere compactification in an interesting way. It was already noted in [18] and more recently analyzed further in [21], that the potential $V(\theta_i)$ has more than one non-trivial critical points. Besides the above described maximally symmetric vacuum, there is another critical point, with $\mathcal{N} = 2$ supersymmetry and $SU(2) \times U(1)$ isometry and bulk gauge symmetry, which corresponds to a vacuum where two of the scalars ($\theta_1, \theta_2$) take specific, constant vacuum values [21].

Here, we will start from this picture and argue that the critical point with the reduced supersymmetry could be the place where our brane world sits. However, we would like, in addition, to keep other fields in this potential that preserve the $\mathcal{N} = 2$ supersymmetry. In other words, we would like to keep those fields that can take non zero values and do not move us out of the critical point. These are the dilaton $s$, the axion $a$ and the breathing mode $\varphi$. The compactification does not produce a potential for the dilaton and the axion, but it does produce a potential for $\theta_1, \theta_2$ and $\varphi$. To see this, recall that $s$ and $a$ are isometry preserving and therefore there should be no potentials generated for them, just as in the case of (toroidal) compactification on a flat space. For $\varphi$, on the other hand, there should be a potential generated when the compact space is curved, and this potential should be proportional to the compact space curvature, so that in the flat space limit the potential vanishes. Since the curvature is related to the gauge coupling associated to the gauging of the supergravity, we expect a potential that has a piece proportional to $g$. There is, in addition, the term (1) that also enters the potential, coming from the reduction ansatz of the five form $H_{(5)}$ of the IIB string theory and which will generate an additional contribution to the potential for $\varphi$, proportional to $m^2$. On general grounds, therefore, the form of the potential for the squashed sphere compactification with the squashing modes frozen to their critical values, is expected to be similar to the round sphere potential appearing in (2).

The straightforward way to find the potential with two squashing modes and one breathing mode would be a generalization of the method of [4]. They noticed that $S^5$ can be represented as the $U(1)$ fibration of $CP^2$ and therefore they were able to include one squashing mode to the breathing mode potential. To include a second squashing mode, clearly is not as simple, nevertheless we will argue that it is possible to guess the form of the potential.

Let us start from the Lagrangian with two non vanishing squashing modes, following [22]. In this action, the breathing mode $\varphi$ has been relaxed to a constant $\varphi = \varphi_0$, so it does not appear

---

6In this reference, a non trivial 3-form field strength was also kept in the compactification in addition to the 5-form, in which case there would be an additional contribution to the scalar potential. In this paper, for simplicity, we have ignored such a contribution.
explicitly:

\[
S(\theta_1, \theta_2) = \int d^5x \sqrt{-G_5} \left[ R - \frac{1}{2} (\partial \theta_1)^2 - \frac{1}{2} (\partial \theta_2)^2 - V(\theta_1, \theta_2) \right],
\]

\[
V(\theta_1, \theta_2) = -\frac{g^2}{4} \left[ e^{4 \theta_1/ \sqrt{6}} (1 - \sinh^4(\frac{\theta_2}{2})) + e^{-4 \theta_1/ \sqrt{6}} (1 + \cosh(\theta_2)) + \frac{1}{16} e^{-4 \theta_1/ \sqrt{6}} (1 - \cosh(2\theta_2)) \right].
\]

The \( \mathcal{N} = 2 \) ground state corresponds to a critical point of the potential (6), with respect to \( \theta_1 \) and \( \theta_2 \), and without loss of generality, it corresponds to

\[
\theta_1^0 = \ln 3 \quad \text{and} \quad \theta_2^0 = \frac{2}{\sqrt{6}} \ln 2.
\]

For these field values, the resulting cosmological constant is \(-\frac{2^{1/3}}{3} g^2 \) [21]. On the other hand, the action with the squashing modes relaxed to the above constant values but with arbitrary breathing mode, is the action (2) which has a potential

\[
V(\varphi) = -R_5 e^{a\varphi} + 8m^2 e^{b\varphi},
\]

with \( a = \frac{16}{9} \alpha, \) \( b = 8\alpha \) and \( R_5 \) is a constant, which in the round sphere limit \( (\theta_{1,2} = 0) \), would be the curvature of the round \( S^5 \). Now, it is a constant containing information about the curvature of the squashed sphere, which topologically is still an \( S^5 \). We use the same letter for it, since there is no possibility of confusion. As we mentioned in the introduction and we will shortly see explicitly, its value is related to that of the gauge coupling \( g \).

We now conjecture that the action with all three scalar fields, has to be of the form:

\[
S(\theta_1, \theta_2, \varphi) = \int d^5x \sqrt{-G_5} \left[ R - \frac{1}{2} (\partial \theta_1)^2 - \frac{1}{2} (\partial \theta_2)^2 - \frac{1}{2} (\partial \varphi)^2 - V(\theta_1, \theta_2, \varphi) \right],
\]

where the potential is:

\[
V(\theta_1, \theta_2, \varphi) = e^{a(\varphi - \varphi_0)} V(\theta_1, \theta_2) + 8m^2 e^{b\varphi}.
\]

To justify this, first notice that the above potential has the same critical points (with respect to \( \theta_1 \) and \( \theta_2 \)) (7), for any \( \varphi \). We would expect this from the fact that the breathing mode does not affect the supersymmetries of the vacuum. It just regulates the volume of the sphere. When \( \varphi \to \varphi_0 \), the part of potential (10) proportional to \( g^2 \), reduces to (6). The term proportional to \( m^2 \) is the contribution of the five form to the potential. When \( \theta_{1,2} \to \theta_{1,2}^0 \), (10) reduces to (8) if

\[
g^2 e^{-a\varphi_0} = \frac{3}{2^{4/3}} R_5,
\]

which is the equation that relates the curvature of the compact manifold to the bulk gauge coupling \( g \). The conclusion is that as far as the potential is concerned, the only difference from the maximally symmetric \( (\theta_1 = \theta_2 = 0) \) case is the value of \( R_5 \). The form of the potential is the same. Let us compare this to the case worked out in [4] with one squashing mode included in the potential. There, the potential was found to be

\[
V(\theta, \varphi) = e^{a\varphi} \left( -R_4 e^{\theta} + \mu^2 e^{\theta} \right) + 8m^2 e^{b\varphi},
\]
with \( p = 1/\sqrt{10} \) and \( q = 3\sqrt{2}/5 \). For \( \theta = 0 \), the above reduces to the form (8) with \( R_5 = R_4 - \mu^2 \), where \( R_5 \) is the curvature of the round \( S^5 \) and \( R_4 \) is the curvature of \( CP^2 \). If the squashing mode would take a constant nonzero value \( \theta = \theta_0 \), the potential would reduce again to the form (8) with the only difference being that \( R_5 \) would be the curvature associated with the squashed sphere. We do not see any reason for which this should not persist with a second squashing mode turned on. We finally note that the potential (10) does not reduce to the potential with one squashing mode (12), in the limit where one of \( \theta_{1,2} = 0 \). This is not surprising since our effective squashing does not correspond to the \( U(1) \) over \( CP^2 \) fibration.

We conclude this section by adding a comment. Our compact manifold is topologically a sphere. There are, however, other five dimensional compact spaces on which one could compactify IIB string theory which are not topologically spheres, still obtain \( \mathcal{N} = 2 \) supersymmetry and consequently implement a scenario similar to our proposed brane world one. One interesting alternative to the present proposal would be, perhaps, to compactify instead on \( S^5 \), on the space \( T^{1,1} = ((SU(2) \times SU(2))/U(1)) \), an \( S^3 \) over \( S^2 \) fibration, discussed in [24]. For work related to the RS scenario in a supergravity inspired context, see for example [25].

4 Flat Domain Wall Solutions - Brane Worlds

The equations of motion corresponding to the action (2) with \( \varphi = \varphi(x^4) \) only and the flat four dimensional domain wall 5D metric

\[
d s_5^2 = e^{2A(x^4)} \eta_{\mu\nu} d x^\mu d x^\nu + (d x^4)^2, \quad (13)
\]

where \( \eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1) \), are

\[
\varphi'' + 4A'\varphi' = \frac{\partial V(\varphi)}{\partial \varphi}, \quad (14)
\]

\[
A'' = -\frac{1}{6}\varphi'^2, \quad (15)
\]

\[
A^2 = -\frac{1}{12}V(\varphi) + \frac{1}{24}\varphi'^2. \quad (16)
\]

The prime denotes differentiation with respect to \( x^4 \). We emphasize that our domain wall ansatz is that we are looking for solutions with flat four dimensional slices of the five dimensional geometry.

It has been shown that by introducing a “superpotential” \( W \), the second order differential equations reduce to [8], [23]:

\[
\varphi' = \frac{\partial W(\varphi)}{\partial \varphi}, \quad (17)
\]

\[
A' = -\frac{1}{6}W, \quad (18)
\]

\[
V(\varphi) = \frac{1}{2}\left( \frac{\partial W}{\partial \varphi} \right)^2 - \frac{1}{3}W^2. \quad (19)
\]
This is a system of decoupled first order differential equations instead of the coupled second order differential equations that we had before. A particular ansatz for $W$ that satisfies the last condition for $V(\varphi)$ of the form (8), is

$$ W = p_1 e^{\frac{a}{2}\varphi} + p_2 e^{\frac{b}{2}\varphi}, \tag{20} $$

with

$$ p_1 = \pm 2 \sqrt{\frac{2 R_5}{a(b-a)}} \quad \text{and} \quad p_2 = \pm 2 \sqrt{\frac{16 m^2}{b(b-a)}}. \tag{21} $$

This ansatz essentially solves the system, but it seems that we have to pay a price. Namely, we have lost one integration constant because (20) is a particular solution of (19) and does not have the integration constant that a general solution should have.

Note that the original second order system apparently has four integration constants. However, (16) is a constraint equation which gives one relation between these constants. Also the zero mode of $A$ can always be absorbed in a rescaling of coordinates. Thus the original system has two independent integration constants. As stressed by [8], the first order system must be completely equivalent to the second order system and the parameter space should have the same dimension. This is of course possible only if we use the general solution for $W$ in which case the first order system will also have two independent integration constants, one in the solution for $W$ and one in the solution for $\varphi$. Of course if we work with the particular solution (20) we would be restricting ourselves to a one parameter subspace of the solution space. We will come back to the problem of the lost integration constant later.

For the moment, we have to solve the following system of first order differential equations:

$$ \varphi' = \frac{1}{2} a p_1 e^{\frac{a}{2}\varphi} + \frac{1}{2} b p_2 e^{\frac{b}{2}\varphi}, \tag{22} $$

$$ A' = -\frac{1}{6} p_1 e^{\frac{a}{2}\varphi} - \frac{1}{6} p_2 e^{\frac{b}{2}\varphi}. \tag{23} $$

Integrating (22) gives the Lerch transcendent [12] as the solution,

$$ r + c_1^\varphi = \frac{2}{a p_1} \sum_{n=0}^{\infty} \frac{(-\frac{bp_2}{ap_1})^n}{(a-b)n + a} \left[ 1 - e^{(a-b-a)\varphi} \right], \tag{24} $$

which is quite hard to invert for general $\varphi$, so as to obtain an exact solution for the warp factor. It is much easier to work in certain interesting limits:

- $\varphi \to \varphi_0$.

Let us first solve (22) and (23) for $\varphi = \varphi_0$, the value of $\varphi$ at the critical point of $V$. We have $\varphi' = 0$ and therefore $\frac{\partial V}{\partial \varphi} = 0$, which implies through (19) that this vacuum characterizes a critical point (in fact a minimum) of the scalar potential. Furthermore, from (23) we see that it corresponds to an exact AdS vacuum, since $A$ has a linear dependence in $x^4$. The condition, therefore, that determines $\varphi_0$, is

$$ a p_1 e^{\frac{a}{2}\varphi_0} + b p_2 e^{\frac{b}{2}\varphi_0} = 0. \tag{25} $$
Let us now solve the equations for a vacuum that is near the \( \varphi = \varphi_0 \) vacuum. For that, we assume
\[
\varphi = \varphi_0 + \epsilon f(x^4),
\] (26)
with \( \epsilon \) a small number. Substituting this ansatz into (22), we find that \( f \) satisfies the differential equation \( f' = 4kf \) and therefore
\[
\varphi = \varphi_0 + c_1^\varphi e^{4kx^4}, \quad k^2 = \frac{1}{32}a(b - a)R_5 e^{a\varphi_0},
\] (27)
where \( k \) and \( p_1 \) are of opposite signs and \( c_1^\varphi \) is the integration constant with \( \epsilon \) absorbed in it. Solving (23) for \( A \), yields the (almost AdS) warp factor:
\[
A(x^4) = k x^4 + \mathcal{O}(\epsilon^2).
\] (28)
The first term in the above corresponds to the AdS part. The solution (27) and (28), however, is valid only for large \(|x^4|\). In particular, if \( k < 0 \) (that is if \( p_1 > 0 \)) then the solution is valid when \( x^4 \rightarrow +\infty \) and if \( k > 0 \) \( (p_1 < 0) \) then the solution is valid when \( x^4 \rightarrow -\infty \). Finally, (25) implies that \( p_1 \) and \( p_2 \) must come with opposite signs.

- \( \varphi \rightarrow +\infty \).
  In this limit, we have
\[
\varphi' \simeq \frac{1}{2}b p_2 e^{\frac{1}{4}b\varphi},
\] (29)
which can be integrated to give
\[
\varphi = -\frac{2}{b} \left[ \ln \left( -\frac{b^2}{4} p_2 x^4 + c_1^\varphi \right) \right],
\] (30)
where \( c_1^\varphi \) is an integration constant. The above solution is valid for \( -\frac{b^2}{4} p_2 x^4 + c_1^\varphi \rightarrow 0^+ \). The solution for the warp factor turns out to be
\[
e^{2A(x^4)} = \left( -\frac{b^2}{4} p_2 x^4 + c_1^\varphi \right)^{\frac{1}{3b^2}},
\] (31)
up to an (irrelevant) overall integration constant. Thus in this limit, we have \( e^{2A} \rightarrow 0^+ \).

- \( \varphi \rightarrow -\infty \).
  In this limit, we have
\[
\varphi' \simeq \frac{1}{2}a p_1 e^{\frac{1}{4}a\varphi},
\] (32)
giving
\[
\varphi = -\frac{2}{a} \left[ \ln \left( -\frac{a^2}{4} p_1 x^4 + c_1^\varphi \right) \right],
\] (33)
which implies that \( -\frac{a^2}{4} p_1 x^4 + c_1^\varphi \rightarrow +\infty \) for any finite \( c_1^\varphi \).
The warp factor is
\[
e^{2A(x^4)} = \left( -\frac{a^2}{4} p_1 x^4 + c_1^\varphi \right)^{\frac{1}{3a^2}},
\] (34)
up to the usual overall constant. Thus, we have \( e^{2A} \rightarrow +\infty \) in this limit.
As was already noticed in [12], the above solution has two separate branches, one which corresponds to \( \varphi \in (\varphi_0, +\infty) \) for \( x^4 \in (-\infty, \frac{4}{b^2 p_2} c_1^p) \) and one which corresponds to \( \varphi \in (\varphi_0, -\infty) \) for \( x^4 \in (-\infty, +\infty) \). We will see later that the separation of the solution in two distinct branches is probably special to our specific choice (20) of \( W \) and that this choice is equivalent to choosing a gauge where \( c_2^p = 0 \). The first branch is singular at \( x^4 = \frac{4}{b^2 p_2} c_1^p \) because the warp factor vanishes, so we neglect it.

As an example, in fig. 4, we plot the first branch solutions (I and II) for the warp factor found in [12]. They correspond to \( p_1 < 0 \) and \( p_1 > 0 \) respectively, i.e. solution II can be obtained from solution I by a reflection around the origin \( x^4 = 0 \). Solution I corresponds to patching together in a smooth way the above found solutions for \( \varphi \rightarrow \varphi_0 \) and \( \varphi \rightarrow -\infty \) in which case the singularity encountered in the \( \varphi \rightarrow +\infty \) regime is avoided. Solutions I and II individually do not possess reflection symmetry around the origin, but they are the ones appropriate for constructing models with explicit branes inserted in the bulk. More specifically, we can construct a brane world scenario as follows: Consider the solutions of fig. 4 and choose a region in the \( x^4 \) coordinate, symmetric around \( x^4 \). The region is, say, the interval \([-\pi R, +\pi R]\). If \(-\pi R < x^4 < 0\), take solution I as it is and if \( 0 \leq x^4 \leq +\pi R \), take solution II to get a \( Z_2 \) symmetric situation. The orbifold will then be the region \( 0 < x^4 < \pi R \). A one brane world can be obtained by taking \( R \) to infinity. This construction, for the warp factor, is illustrated in fig. 5.

With the \( D3 \)-branes present, the 5 dimensional action is modified to:

\[
S(\varphi) = \int d^5 x \sqrt{-G_5} \left[ R - \frac{1}{2} (\partial \varphi)^2 - V(\varphi) \right] - \int d^4 x \sqrt{-G_4^{(1)}} T^{(1)}(\varphi) - \int d^4 x \sqrt{-G_4^{(2)}} T^{(2)}(\varphi),
\]

where \( G_{4\mu\nu}^{(i)} \) is the induced metric on the \((i)\)th brane by \( G_{5\mu\nu} \), and for our metric it is simply \( G_{4\mu\nu}^{(i)} = \eta_{\mu\nu} \), and \( T^{(1)}(\varphi) \) and \( T^{(2)}(\varphi) \) are the tensions of the branes. The dependence of \( T \) on \( \varphi \) is kept arbitrary since it will in general be affected by quantum effects on the brane. The action for the branes is written in the static gauge so that the embedding functions are \( x^\mu(\xi) = \xi^\mu, \mu = 0, \cdots, 3 \) and we ignore their fluctuations. Also of course we have set all other fields on the brane to the minimum of the quantum effective action so that what we have kept is just an effective description of the ground state of the theory on the brane.

Now, in addition to the equations of motion, we have to satisfy the jump conditions at \( x^4 = 0 \) (and \( x^4 = \pi R \) in the two brane case), which constitute the connection between the two solutions at opposite sides of the brane(s). They are:

\[
2A'(x^4) = +\frac{1}{6} T^{(1)}(\varphi(x^4)) \big|_{x^4=0},
\]

\[
2A'(x^4) = -\frac{1}{6} T^{(2)}(\varphi(x^4)) \big|_{x^4=\pi R},
\]

\[
2\varphi'(x^4) = -\frac{\partial T^{(1)}}{\partial \varphi}(\varphi(x^4)) \big|_{x^4=0},
\]

\[
2\varphi'(x^4) = +\frac{\partial T^{(2)}}{\partial \varphi}(\varphi(x^4)) \big|_{x^4=\pi R}.
\]

We are at this point ready to discuss one brane world scenarios.
4.1 One Brane World

An interesting alternative for brane-world scenarios [11], involves the possibility of a single 3-brane embedded in a $D = 5$ space time with an “infinite” fifth dimension. In the context of type IIB string theory in $D = 10$, such a 5/5 (non compact/compact) split seems natural owing to the presence of the RR self-dual 5-form. While string theory provides a natural route for confining gauge fields to sub manifolds (via open strings constrained to end on $D$-branes), gravity must necessarily be allowed to probe the full dimensionality of space time (closed string states propagating in the bulk). Consistency of a brane-world accompanied by an infinite fifth dimension with low energy four-dimensional physics requires a mechanism for effectively confining a massless gravitational mode to the sub manifold and suppressing any massive gravitational modes. Such a mechanism was outlined in [11]. An advantage to this construction as pointed out in [11], is the absence of a need to stabilize the modulus field associated with the fifth dimension. The modulus is allowed to simply ”relax” to infinity (a natural possibility usually regarded as disastrous). There will remain of course the problem of stabilizing various other moduli encountered in string compactifications, in particular the T-moduli associated with the size and shape of the, still requisite, $D = 5$ compact manifold.

The problem of stabilizing the modulus associated with the size of the fifth dimension in the two-brain scenario (with a compact fifth dimension) discussed in [10] was recast via the Goldberger-Wise mechanism [26]. A number of subsequent papers considered hybrid scenarios in which the distance between two brains embedded in an infinite fifth dimension could be stabilized via this mechanism. However, in these scenarios the motivation for an infinite fifth dimension is lost (leaving one to wonder why an infinite extra dimension was sought). Other papers took steps to extend the moduli ”relaxation” scenario by considering more than one non compact transverse dimension [27]. While this may (in no doubt a complex manner) ultimately eliminate the need for stabilizing the size of all six of the extra dimensions predicted by superstring theories, there will remain the problem of stabilizing the other moduli fields, e.g. the original $10D$ dilaton. It is the view of the authors of this paper that the advantages to an infinite extra dimension lie elsewhere, i.e. a natural 5/5 compactification split and the possibility of a vanishing $4D$ cosmological constant. Thus we will seek a realization of the scenario involving a single brane-world with one non compact transverse dimension.

In the previous section we maintained non triviality of the breathing mode $\varphi$ in order to allow a viable solution without fine tuning in the two-brane scenario (to be discussed in the next section). For the case of a single brane, the presence of the integration constant $R_5$ associated with the curvature of the five sphere is sufficient to allow for a solution without fine tuning. Though the non triviality of $\varphi$ is not particularly problematic for the one-brain scenario (see [12] for work along these lines), it appears to be an unnecessary complication. Thus, our starting point for the one-brain scenario will be the effective action (35) with a single brane tension term and the breathing mode $\varphi$ set to a constant value $\varphi_0$ which, we assume, has been frozen by string scale dynamics. These choices ensure a pure $AdS$ bulk geometry preserving $\mathcal{N} = 2$ supersymmetry. This “constant” potential will play the role of $\Lambda$ in the original RS action. We have therefore

$$S = \int d^5x \sqrt{-G_5}\left[ R + R_5 e^{\varphi_0} - 8m^2 e^{\varphi_0}\right] - \int d^4x \sqrt{-G_4}(\varphi_0). \quad (40)$$

The relevant bulk equations of motion are

$$A'' = 0, \quad (41)$$
\[
A'^2 = -\frac{1}{12}V(\phi_0) = \frac{1}{12}(R_5 e^{a\phi_0} - 8m^2 e^{b\phi_0}),
\]
which yield the \(AdS\) solution
\[
A(x^4) = kx^4,
\]
\[
k = \pm \sqrt{\frac{1}{12}(R_5 e^{a\phi_0} - 8m^2 e^{b\phi_0})},
\]
with \(1/|k|\) the radius of \(AdS\) and we have ignored the irrelevant additive constant in \(A\). It should be noted that the value of \(\phi_0\) in the present case need not necessarily be associated with a critical point of the potential since we are assuming it to be fixed by string scale dynamics (and therefore the value of \(k\) is also modified accordingly). However, we do assume that \(k^2\) in the above is positive.

The RS metric solution \(A(r) = -k|x^4|\) for \(k > 0\) can be obtained by taking the \(+\)(\(-\))\(k\) branch for \(- (+) x^4\). Patching together these solutions can be achieved via the brane source term which must satisfy a single non trivial jump condition (36)
\[
k^0_+ - k^0_- = -2\sqrt{\frac{1}{12}(R_5 e^{a\phi_0} - 8m^2 e^{b\phi_0})} = \frac{1}{6}T(\phi_0),
\]
Clearly, this relation can be satisfied for an appropriately chosen \(R_5\). This condition merely represents the relationship between the boundary and bulk cosmological terms required for flat brane solutions as first identified in \([11]\). Effecting this relationship was regarded as a fine tuning in the RS paper. In our case, the freedom of choosing \(R_5\) affords a more natural manner in which to satisfy the condition.

Having successfully reached the scenario proposed in \([11]\), we may take over all of the results and associated problems that follow from this and subsequent analyses. In particular, RS demonstrated through analysis of general linearized tensor fluctuations about the metric solution given in (13), that there exists a single normalizable zero-mass bound state Kaluza-Klein mode of the \(D = 5\) gravity. This mode was identified with the massless graviton of our \(D = 4\) brane-world. In addition they showed that, despite the presence of a continuum of higher mass Kaluza-Klein states (arbitrarily light), the resulting \(D = 4\) gravitational theory can easily fit within current experimental bounds. This is due to the suppression of all massive wave functions by the potential barrier surrounding the 3-brane. These two results are most easily recognized by picturing the volcano-like potential effecting graviton propagation in the fifth dimension \([28]\).

Cursory investigation of gravitational self-interactions exhibited strong coupling tendencies for zero-mode fluctuations extending to large values of the transverse coordinate. It was later shown, in a more careful analysis of linearized gravity in this background \([29]\), that the strong coupling tendencies of the zero-mode were due to treatment of this mode alone, and that inclusion of the continuum modes (which are the dominant ones at large values of the transverse coordinate) yields an overall falloff of the graviton propagator as one moves away from the brane.

A large volume of work \([8],[12],[30]\) subsequent to \([11]\) sought to realize the one-brain-world scenario as a classical solution to some supersymmetric theory, i.e. a supersymmetric domain-wall world. It was then demonstrated on rather general grounds that all attempts to obtain localized gravity on a BPS brane-world of this type suffered irreconcilable problems \([9]\). In particular, for most models the exponential warp factor was found to increase as one moved away from the brane. This criticism, however, does not apply to those models which, along the lines of the original RS
proposal, attempt to utilize true delta-function source terms to effect the required geometry, e.g. the last section of [12] and this paper.

As discussed in the introduction, we identify our singular source term with a stack of coincident string theoretic D-branes and/or orientifold planes. In addition to providing a singular source term (from the point of view of the low energy effective theory), D-branes afford a natural mechanism for the confinement of gauge fields. Regarding the type of gauge group which may reside on the brane-world we note two possible constructions:

- $x^4$ is the infinite radius limit of $S^1/\mathbb{Z}_2$

  In this case the arguments of section 2 regarding the cancellation of the RR charge induced by the orientifold planes would restrict us to certain D-brane configurations. A few of these are presented in fig.s 1,2,3. In each case one of the orientifold planes would be located at infinity.

- $x^4$ is infinite and exhibits a $\mathbb{Z}_2$ symmetry

  In this situation there are no charge cancellation restrictions, and we may stack any number of D-branes. The arbitrariness in this scenario makes it less satisfactory than the previous case.

4.2 Two Brane World

In the two brane world case, we need both nontrivial integration constants of the second order equations of motion (14), (15) and (16), because we have only $R_5$ and the size of the orbifold $R$ to satisfy four jump conditions. One might think with a solution with two branes and a pure $AdS$ bulk is possible [10], since there are only two non trivial jump conditions to satisfy (one for each brane) and two available constants, $R_5$ and $R$. However, as was pointed out in [3], the two branes have equal and opposite tensions at every point of the RG flow and the tension for a flat brane solution is fixed only in terms of $R_5$. The orbifold size does not enter the jump conditions and therefore it can not be used. In order to have a solution without fine tuning, we need a nontrivial scalar field in the bulk. But then, our solution based on the reduction of the second order equations of motion to first order equations does not provide us with the two required integration constants, since as we explained, we have lost one integration constant in the choice (20) for $W$. We therefore look for more general solutions to the equations of motion.

We make the ansatz

$$\varphi = \varphi_0 + \epsilon f_1(x^4) + \epsilon^2 f_2(x^4) + \epsilon^3 f_3(x^4) + \cdots$$

$$A = k x^4 + \epsilon g_1(x^4) + \epsilon^2 g_2(x^4) + \epsilon^3 g_3(x^4) + \cdots$$

with $k > 0$, and we substitute it into (14), (15) and (16). We obtain an infinite set of differential equations. We show the result for (14) and (15), up to order $\epsilon^3$:

$$f''_1 + 4k f'_1 + p f_1 = 0, \quad g''_1 = 0$$

$$f''_2 + 4k f'_2 + p f_2 - \frac{1}{2} a(a^2 - b^2) R_5 e^{4a\varphi_0} f_1^2 + 4 f'_1 g'_1 = 0, \quad g''_2 + \frac{1}{6} f_1'^2 = 0$$

14
\[ f''_3 + 4k f'_3 + p f_3 - a(a^2 - b^2) R_5 e^{4a\phi_0} f_1 f_2 - \frac{1}{6} a(a^3 - b^3) R_5 e^{4a\phi_0} f_1^3 + 4 f'_1 g'_2 + 4 f'_2 g'_1 = 0, \quad g'_3 + \frac{1}{3} f'_1 f'_2 = 0, \]

where \( p = -32k^2 \). The above pattern (that presumably continues), suggests that we can write the general solution for the field \( \phi \) as:

\[ \phi(x^4) = \phi_0 + c_1^\phi e^{4kx^4} + c_2^\phi e^{-8kx^4} + P(x^4, c_1^\phi, c_2^\phi), \]

where the part of the expression with the exponentials is the general solution to the equation \( f'' + 4k f' + pf = 0 \) and \( P(x^4, c_1^\phi, c_2^\phi) \) is some function of \( x^4 \) containing also the two integration constants and \( \epsilon \) has been absorbed into the integration constants. By looking at (48), (49) and (50), one can see that the solution is a small deviation from \( AdS(50) \), one can see that the solution is a small deviation from \( AdS \).

Then, the expansion in \( \epsilon \) makes sense only if \( c_2^\phi \neq 0 \) and therefore this case is just the solution (27). The second case is when \( x^4 \approx 0 \) and \( c_1^\phi, c_2^\phi \ll 1 \). This is a satisfactory solution with two explicit integration constants as long as the jump conditions allow them to be small. Another observation we can make here is that when \( c_2^\phi \neq 0 \), it is not clear that \( \phi \) has two separate branches. Nevertheless, we can still apply the method for the construction of the orbifold with the only caveat that if the jump conditions require integration constants of order one or larger, we still do not have an explicit solution.

We will now find a solution to the second order equations of motion in the \( \phi < \phi_0 \) region which will be valid for a larger range of the integration constants. Define \( H \equiv A' \) and denote \( \frac{\partial}{\partial \phi} \) with a dot. Also, for simplicity we will assume that \( R_5 = 8m^2 = 1 \), even though we have to keep in mind that \( R_5 \) is really a parameter determined by the integration constants. Combining equations (15) and (16), we obtain the equation \( \dot{H}^2 = \frac{1}{36} (24H^2 + 2V) \). Let us assume that \( 24H^2 > 2|V| \). We will verify soon that this is a valid assumption in the regime of interest. Then, we can easily solve for \( H \): \( H = e^{\sqrt{\frac{3}{2}} \phi} \), where we have chosen the positive branch of the square root. Using the value of \( \phi_0 \) from the minimization of \( V \), we deduce that \( 2V \) could be consistently dropped provided that

\[ \frac{1}{12} e^{a\phi} \left( 1 - \frac{a}{b} e^{(b-a)(\phi-\phi_0)} \right) \ll e^{3\sqrt{\phi}}. \]

Since our solution corresponds to \( \phi \in (-\infty, \phi_0) \), a numerical estimate tells us that the above is true if, approximately, \( -3 < \phi < -0.6 \) (for \( R_5 = 8m^2 = 1 \), we get from (25) \( \phi_0 \approx -0.6 \)). Next, using the above solution for \( H \), equation (14) becomes \( \phi'' + 4e^{\sqrt{3\phi}}\phi' - \dot{V} = 0 \). In the \( \phi < \phi_0 \) region the potential \( V \) approaches zero exponentially and its shape in this regime is rather flat, which means that \( \dot{V} \approx 0 \). Indeed, one can neglect \( \dot{V} \) in the equation for \( \phi \) provided that

\[ | - a e^{a\phi} + b e^{b\phi} | \ll 4e^{\sqrt{3\phi}} |\phi'| \] and \( |\phi''| \).

Then we can solve for \( \phi \) and we obtain

\[ \phi(x^4) = \frac{\sqrt{6}}{2} \ln \left( \frac{c_2^\phi}{1 + 12e^{x^4}(c_1^\phi + x^4)} \right) + c_2^\phi (c_1^\phi + x^4) + \frac{\sqrt{6}}{4} \ln 6. \]

By computing \( \phi' \) and \( \phi'' \) from the above expression, one can verify that (53) is true in the relevant range of \( \phi \) for a large range of \( c_1^\phi \) and \( c_2^\phi \), when \( x^4 \sim \mathcal{O}(0-10) \).\footnote{\( (53) \) is in fact true for any finite \( x^4 \) for appropriate choice of the order of magnitude of the integration constants, but the regime around \( x^4 = 0 \) is the interesting one since it is where the branes are located.}
For $\varphi >> \phi_0$, we can take similar steps. We can again assume for simplicity that $R_5 = 8m^2 = 1$. The derivative of the warp factor is obtained by solving the equation $\dot{H} = \sqrt{2} \frac{b}{6} e^{2\varphi}$, which yields $H = \frac{\sqrt{2}}{6} e^{\varphi/2}$ up to an irrelevant integration constant. The equation for $\varphi$ becomes $\varphi'' = be^{b\varphi}$, where we have ignored the smaller terms. The solution for $\varphi$ is then

$$\varphi(x^4) = \frac{\sqrt{15}}{10} \ln \left[ -\frac{c_1^2}{2} \left[ 1 + \tan \left( -\frac{c_1^2}{3} (x^4 + c_2^2)^2 \right) \right] \right]. \quad (55)$$

Finally, we note that one can solve the equations of motion in an analogous fashion for arbitrary $R_5$.

We can now construct the orbifold as in fig.5 and satisfy the four jump conditions. An interesting fact is that to do so, along with $c_1^2$ and $c_2^2$, we have to use $R_5$ and $R$, which provides us with a RG scale dependent determination of the 5D dilaton and the orbifold size.

We make a last comment concerning the presence of non trivial 3-form fields in the vacuum state we have discussed. As we mentioned, according to [20], the non triviality of $F_{(R)}(3)$ and $F_{(NS)}(3)$ is necessary. The effect of this on the scalar potential is a couple of additional terms [4]:

$$V = e^{\frac{16\pi}{3}} R_5 - 8m^2 e^{8\alpha \varphi} + \lambda_1 e^{-\varphi + c_1 \varphi} (F_{(R)}(3))^2 + \lambda_2 e^{\varphi + c_2 \varphi} (F_{(NS)}(3))^2, \quad (56)$$

where $\phi$ is the ten dimensional dilaton and $\lambda_1$, $\lambda_2$ and $c$ are fixed constants.$^8$ Clearly, this potential is more complicated from the one we have used to solve the equations of motion and in a complete treatment this is the potential that should be used. This, of course, would make solving the equations of motion even harder. However, by including the additional terms, we now have the possibility of stabilizing the 10D dilaton by minimizing the above potential with respect to $\phi$. The $\phi_0$ determined by the minimization of $V$ determines the four dimensional coupling constant $\alpha$:

$$e^{2\phi_0} = 4\pi \alpha = \frac{\lambda_1 (F_{(NS)}(3))^2}{\lambda_2 (F_{(R)}(3))^2} \quad (57)$$

and it is expected to be of order one. Once the dilaton is stabilized and we know the vacuum values of the 3-forms, we have an effective scalar potential which is a sum of four exponents instead of two, but still with $R_5$ as the only continuous adjustable parameter. Of course, in this case, eq. (11) will also be modified accordingly.

## 5 Conclusions

The main virtue of the originally proposed two brane world model of [10] was that it could provide an explanation of the “hierarchy problem” without introducing unnaturally large numbers. The large number necessary to explain the discrepancy between the electroweak scale and the Planck scale was generated by having a number of $O(40)$ in the exponent of the warp factor. In our case, it is not clear whether there is such a mechanism at work. To answer this question, a quantitative analysis would be necessary, which is possible only if the explicit form of the bulk solutions and

$^8$Further non trivial p-forms result in additional terms in the potential. Our remarks can then be generalized accordingly.
the brane tensions as functions of the breathing mode are known. However since we may have a supersymmetric four dimensional solution, it is not clear if we need such a mechanism at all.

There are many other issues that are probably worth investigating in the context of these two brane models. If the standard model does live on one of the branes, there should be an answer to questions such as what representations of matter fields live on the brane(s) and how do the three families of chiral fermions arise from the string compactification. Unlike in Calabi-Yau (or Orbifold) compactifications there does not seem to be any relation between the number of families and the topology of the compact manifold.

Also, the brane gauge group has to break somehow to the standard model gauge group. It is not at all clear that such a theory can be constructed from D-branes. In addition, there is the gauge group $SU(2) \times U(1)$ of gauged supergravity living in the bulk, which couples to the brane. One expects that the anomalies associated with all these gauge groups are zero since the starting point is an anomaly free string theory, but there are still non trivial issues, especially in the two brane case. Namely, the bulk gauge bosons couple to the massless chiral families of both branes and the anomalies could be canceled on each brane separately or across branes, yielding very different and distinctive phenomenology in each case. Another important issue is related to the mechanism of the supersymmetry breaking on the branes. Our two brane model has a natural “visible” and “hidden” brane, both with $\mathcal{N}=1$ world volume supersymmetry. In fact, our considerations would become those of [31] when the breathing mode is frozen\footnote{Note that the action used in the first of [31] is the same as the five dimensional action coming from sphere or squashed sphere compactifications with the scalar fields frozen at critical points and one of the two type IIB two form fields dualized with the other set to zero.}. This setup is ideal, for example, for a hidden sector supersymmetry breaking via gaugino condensation in the hidden sector and transmitted to the visible sector with gravitational or gauge (or both) interactions. The content of this paper is simply that even after supersymmetry breaking it is possible to obtain flat space on the brane by appropriately choosing integration constants and the curvature of an internal manifold. We believe that this is the first indication that such a flat four space solution is possible within a string theoretic framework.

Acknowledgements

This work is partially supported by the Department of Energy contract No. DE-FG02-91-ER-40672.
References


   E. Witten, “The cosmological constant from the viewpoint of string theory”, hep-th/0002297.


M. Gremm, “Four-dimensional gravity on a thick domain wall”, hep-th/9912060.


Figure 1: $IIB$ on $S^5$ brane world 1: $SO(16) \times SO(16)$

Figure 2: $IIB$ on $S^5$ brane world 2: $SO(32)$

Figure 3: $IIB$ on $S^5$ brane world 3: $SO(22) \times SO(10)$
Figure 4: Solutions for branes ([12]): Warp factor as a function of $x^4$; for solution $I$, the warp factor behaves as an exponential (AdS) at $-\infty$ and as a polynomial around $+\infty$. Solution $II$ can be obtained from solution $I$ by $x^4 \rightarrow -x^4$.

Figure 5: The orbifold construction.