CONSTRAINING THE ACCRETION RATE ONTO SAGITTARIUS A* USING LINEAR POLARIZATION

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ABSTRACT

Two possible explanations for the low luminosity of the supermassive black hole at the center of our galaxy are (1) an accretion rate of order the canonical Bondi value, but a very low radiative efficiency for the accreting gas or (2) an accretion rate much less than the Bondi rate. Both models can explain the broad-band spectrum of the Galactic Center. We show that they can be distinguished using the linear polarization of synchrotron radiation. Accretion at the Bondi rate predicts no linear polarization at any frequency due to Faraday depolarization. Low accretion rate models, on the other hand, have much lower gas densities and magnetic field strengths close to the black hole; polarization is therefore observable at high frequencies. If confirmed, a recent detection of linear polarization from Sgr A* at \( \gtrsim 150 \) GHz argues for an accretion rate \( \sim 10^{-8} M_\odot \text{yr}^{-1} \), much less than the Bondi rate. This test can be applied to other low-luminosity galactic nuclei.

Subject Headings: accretion, accretion disks — Galaxy: center — polarization

1. INTRODUCTION

In this paper we discuss the effect of Faraday depolarization on synchrotron radiation in spherical accretion flow models of low-luminosity galactic nuclei. We focus on the radio source Sgr A* at the Galactic Center, but our results can also be applied to other systems (see §4). This paper was motivated by a possible detection of linear polarization from Sgr A* (Aitken et al. 2000).

The Bondi accretion rate onto the supermassive black hole at the center of our galaxy is estimated to be \( \sim 10^{-4} - 10^{-5} M_\odot \text{yr}^{-1} \) (e.g., Coker & Melia 1997; Quataert, Narayan, & Reid 1999), implying a luminosity of \( \sim 10^{41} \) ergs s\(^{-1} \) if the radiative efficiency is \( \sim 10\% \). This is roughly 5 orders of magnitude larger than the observed luminosity (see Narayan et al. 1998 for a recent compilation). Comparable discrepancies are obtained for massive elliptical galaxies in nearby X-ray clusters (e.g., Fabian & Rees 1995).

One explanation for the low luminosity of nearby supermassive black holes is that they accrete via an advection-dominated accretion flow (ADAF), in which most of the dissipated turbulent energy is stored as thermal energy rather than being radiated (e.g., Rees et al. 1982; Narayan & Yi 1994, 1995; Abramowicz et al. 1995). In such models the accretion rate is of order the Bondi rate while the radiative efficiency is extremely small (\( \approx 10^{-6} \) for Sgr A*).

Another explanation for very low luminosity accreting systems is that the Bondi accretion rate estimate is inapplicable. Several physical mechanisms for reducing the accretion rate have been suggested: (1) Blandford & Begelman (1999) proposed that most of the mass supplied to non-radiating accretion flows is lost to an outflow/wind, rather than being accreted onto the black hole, (2) Gruzinov (1999) argued that thermal conduction can suppress the accretion rate by “over-heating” the outer parts of the accretion flow, (3) Numerical simulations of non-radiating accretion flows with small values of the dimensionless viscosity parameter \( \alpha \) find that the gas density scales with radius as \( \rho \propto r^{-3/2} \) rather than the canonical Bondi/ADAF scaling of \( \rho \propto r^{-3/2} \) (Stone, Pringle, & Begelman 1999; Igumenshchev & Abramowicz 1999, 2000; Igumenshchev, Abramowicz, & Narayan 2000). Narayan, Igumenshchev, & Abramowicz (2000) and Quataert & Gruzinov (2000a) explained these simulations in terms of a “convection-dominated accretion flow” (CDAF). In such a flow angular momentum is efficiently transported inwards by radial convection, nearly canceling the outward transport by magnetic fields.

In all three of the above models the accretion rate onto the black hole is suppressed with respect to the Bondi rate for a fixed density and temperature in the interstellar medium of the host galaxy. The low accretion rate is not due to a dearth of material at large radii. Rather, it is a consequence of the dynamics of quasi-spherical accretion (either because most of the gas is lost to an outflow/wind or because convection/conduction strongly suppresses the inflow velocity of the accreting gas at large radii).

Broad band spectra have thus far had difficulty distinguishing between these explanations for the low luminosity of nearby supermassive black holes. For example, Quataert & Narayan (1999; hereafter QN) showed that models with accretion rates much less than the Bondi rate could reproduce previously published ADAF spectra of low-luminosity galactic nuclei.

In this paper we show that the linear polarization of synchrotron radiation can be markedly different in low efficiency and low accretion rate models. This is because the gas density and magnetic field strength near the black hole are orders of magnitude higher in the former than in the latter; depolarization by Faraday rotation in thus significantly more important.

In the next section (§2) we present simple estimates of

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the physical parameters of the accretion flow relevant for our analysis. We then discuss Faraday depolarization in spherical accretion flow models of Sgr A* (§3). In §4 we compare these predictions with observational constraints on the linear polarization of Sgr A* and summarize our results. We also generalize our analysis to other low-luminosity galactic nuclei.

Throughout this paper, we focus on accretion models of Sgr A*. An unresolved jet or outflow may, however, contribute to the observed emission (e.g., Falcke, Mannheim, & Biermann 1993; Lo et al. 1998; Falcke 1999); this is briefly discussed in §4.

2. PLASMA PARAMETERS FOR SGR A*

Spherical kinematics show that there are \( \approx 2.6 \times 10^6 M_\odot \) within \( \approx 0.015 \) pc of the Galactic Center (Eckart & Genzel 1997, Ghez et al. 1998), centered on the radio source Sgr A* (Menten et al. 1997). The most plausible explanation is that Sgr A* is a \( \approx 2.6 \times 10^9 M_\odot \) accreting black hole. Sgr A* is believed to accrete the winds from nearby \( \sim 0.1 \) pc massive stars (Krabbe et al. 1991). The Bondi accretion rate of these winds onto the supermassive black hole is \( \approx 10^{-4} - 10^{-5} M_\odot \text{yr}^{-1} \) (e.g., Coker & Melia 1997; Quataert, Narayan, & Reid 1999).

Recent Chandra observations detect a point source coincident with the radio source Sgr A* to within \( \approx 0.5'' \approx 10^5 R_g \). Its \( 0.1 - 10 \) keV luminosity is \( L_X \approx 4 \times 10^{33} \text{ergs s}^{-1} \) (Baganoff et al. 2000). This emission can be modeled as thermal bremsstrahlung from large radii in a quasi-spherical accretion flow (e.g., Quataert & Gruzinov 2000b). The required density is \( n \approx 10^4 \text{cm}^{-3} \) at \( r \approx 10^8 \text{cm} \), where \( r \) is the radius in the flow in units of the Schwarzschild radius, \( R_g \).

If the accretion rate close to the black hole is of order the Bondi value the gas density near \( r \approx 1 \) is \( n \approx 10^9 - 10^{10} \text{cm}^{-3} \) (since \( v_r \approx c \) near the horizon); this is consistent with the gas density normalized to the Chandra observations, and an \( n \propto r^{-3/2} \) scaling as in Bondi and ADAF models. The corresponding magnetic field strength, assuming rough equipartition with the nearly relativistic protons, is \( B \approx 2 \times 10^3 \text{G} \). At such magnetic field strengths, relativistic electrons cooling by synchrotron radiation would have a cooling time much less than the infall time of the gas. In order to not overproduce the observed radio to sub-mm luminosity of Sgr A*, the bulk of the electrons must therefore be marginally relativistic, with \( T_e \approx 10^8 - 10^{10} \text{K} \). These plasma parameters \( (n, B, T_e) \) describe Bondi and ADAF models of Sgr A* (e.g., Melia 1992, 1994; Narayan, Yi, & Mahadevan 1995; Narayan et al. 1998). In such models the electrons are assumed to be adiabatically compressed from large radii in the accretion flow, with virtually no additional turbulent heating.

QN calculated spectral models of quasi-spherical accretion flows with strong outflows. Such spectra are relevant for any model in which the accretion rate close to the black hole is much less than the Bondi value. They showed that accretion at much less than the Bondi rate could produce the observed radio to sub-mm emission of Sgr A*, provided the electrons were much hotter than in standard ADAF models (see their Table 2 and Fig. 8b).

A simple explanation for this result can be obtained by applying the Burbidge (1958) estimate to Sgr A*. We consider synchrotron emission from a sphere of radius \( R \) containing relativistic electrons with a temperature \( kT_e = \gamma m_e c^2 \). We take the electron heating rate to be comparable to the net turbulent (magnetic) heating rate. As can be confirmed \( a \text{ posteriori} \), the synchrotron cooling time is \( \gg \) the inflow time of the gas. The electron energy density is then similar to the magnetic energy density

\[
n \gamma m_e c^2 \approx \frac{B^2}{8\pi},
\]

The frequency of peak synchrotron emission and the synchrotron luminosity are given by

\[
\nu \approx 0.1 \gamma^2 \frac{eB}{m_e c},
\]

and

\[
L \approx \sigma_T c B^2 \gamma^2 R_3 n,
\]

where \( \sigma_T \) is the Thomson cross section.

We express \( n, \gamma, \) and \( B \) in terms of \( R, \nu, \) and \( L \):

\[
\gamma \approx 3.2 \left( \frac{m_e}{c} \frac{\nu^3 R_3^4 L}{L} \right)^{1/7} \approx 100,
\]

and

\[
B \approx \sqrt{8 \pi n m_e c^2 \gamma n} \approx 45 \text{G},
\]

where \( \lambda = c/\nu \) is the wavelength and \( r_e = e^2/(m_e c^2) \) is the classical electron radius. For the numerical estimates in equations (4)-(6), we have used the observed values for Sgr A*. The peak synchrotron frequency is \( \nu \approx 10^3 \text{GHz} \) with a luminosity of \( L \approx 10^{36} \text{ergs s}^{-1} \) (e.g., Serabyn et al. 1997). In spherical accretion models, this high-frequency emission arises from very close to the black hole, so we have taken \( R \approx R_g \approx 10^{12} \text{cm} \).

For a density close to the black hole of \( n \approx 10^6 \text{cm}^{-3} \), the implied accretion rate is \( \approx 10^{-8} M_\odot \text{yr}^{-1} \), three to four orders of magnitude smaller than the Bondi value (this estimate is consistent with QN’s detailed spectral calculations). Despite this much smaller accretion rate, the radiative efficiency of the accreting gas is still rather small, \( \sim 10^{-3} \). This is because, even for \( \gamma \approx 100 \), the synchrotron cooling time is longer than the infall time of the gas at these low magnetic field strengths.

The thermal blackbody emission at frequency \( \nu \) from a sphere of radius \( R \) is

\[
L_\nu = 2\pi \nu^3 \gamma m_e 4\pi R^2 \approx 10^{37} \text{ergs s}^{-1},
\]

where the numerical estimate is for our fiducial parameters. This comparison shows that the synchrotron emission in our “maximal heating” model becomes optically thin below the peak frequency, near \( \nu \approx 300 \text{GHz} \). At lower frequencies the emission is self-absorbed.

The above considerations show that both low (\( \sim 10^{-8} M_\odot \text{yr}^{-1} \)) and high (\( \sim 10^{-5} - 10^{-4} M_\odot \text{yr}^{-1} \)) accretion rate models can explain the observed spectrum of

\[\text{X-ray luminosity from the accretion flow would exceed the observed value.}\]
Sgr A* (see QN). Both models are also consistent with the Chandra X-ray detection coincident with Sgr A*. If we normalize the gas density to be $n \approx 10^4$ cm$^{-3}$ at $r \approx 10^4$, an $n \propto r^{-3/2}$ scaling predicts a density near the black hole of $n \approx 10^{10}$ cm$^{-3}$, consistent with Bondi and ADAF models of the radio emission. Alternatively, if we use the CDAF scaling of $n \propto r^{-1/2}$, the density close to the black hole is $n \approx 10^6$ cm$^{-3}$, consistent with our “maximal heating” model.

Spectral models of Sgr A* can be distinguished by comparing the observed brightness temperature and/or radio image as a function of frequency with the theoretical predictions (see, e.g., Özel, Psaltis, & Narayan 2000). This has been difficult to implement because interstellar scattering spatially broadens the image of Sgr A*. In the next section we show that the linear polarization of Sgr A* at high frequencies provides an additional discriminant.

3. FARADAY DEPOLARIZATION

The anisotropic index of refraction of a magnetized plasma leads to a frequency-dependent rotation in the position angle, $\theta$, of linearly polarized electromagnetic waves,

$$\theta = RM \lambda^2,$$

where $RM$ is the rotation measure. This can lead to significant depolarization of intrinsically linearly polarized synchrotron emission.

For a “cold” non-relativistic plasma, $RM$ is given by (e.g., Rybicki & Lightman 1979)

$$RM = \frac{e^3}{2\pi m_e c^2} \int dl \cdot |B| n_{rad} \frac{rad}{m^2},$$

where $dl$ is the differential path length from the observer to the source. In the Appendix we show that the rotation measure for an ultrarelativistic thermal plasma is given by

$$CM = \frac{e^3}{2\pi m_e c^2} \int dl \cdot |B| n_{rad} \frac{log \gamma}{2\gamma^2} \frac{rad}{m^2},$$

where $\gamma = kT_e/m_e c^2$. A comparable expression is obtained for a power law distribution of relativistic electrons, with $\gamma$ replaced by $\gamma_{min}$, the minimum Lorentz factor of the electrons (Jones & O’Dell 1977). In what follows, we define $RM(r)$ to be the contribution to the net rotation measure from radii within $dr \approx r$ of radius $r$ in the accretion flow.

3.1. ADAF/Bondi Models

In spherical accretion flow models, radio emission arises from close to the black hole, where the electron temperature and magnetic field strengths are the largest; this is also generally true for jet models (e.g., Falcke 1999). Özel et al. (2000) show that in ADAF models of Sgr A* the synchrotron emission at frequency $\nu = 100 \nu_{100}$ GHz arises from a radius $r_\nu \approx 20 \nu_{100}^{-0.9}$ (see their Fig. 5). This radius defines the $\tau = 1$ surface of the synchrotron emission. For smaller radii the emission is self-absorbed while for larger radii it is optically thin. Faraday rotation is only important for $r > r_\nu$, where the photons “free stream” out of the accretion flow.

In fact, in Bondi/ADAF models Faraday rotation is so strong that the synchrotron emission is completely depolarized in the vicinity of the $\tau = 1$ surface where it is emitted. Taking $n \propto r^{-3/2}$ and $B \propto r^{-5/4}$, the rotation measure scales roughly as $RM \propto r^{-7/4} \gamma^{-2}$. The relativistic suppression of the rotation measure is small in all models which have an accretion rate comparable to the Bondi rate, because the electrons must then be at most marginally relativistic (§2). The rotation measure as a function of radius is thus given by

$$RM \approx 10^{13} r^{-7/4} \nu^{-2} \text{ rad m}^{-2}. \tag{11}$$

The normalization in equation (11) is set by the Chandra observations ($n \approx 10^4$ cm$^{-3}$ at $r \approx 10^4$); we have also assumed that the magnetic field is in rough equipartition with the gas pressure, is not finely tangled on scales of $\sim r$, and has a significant component along the line of sight. This $RM$ leads to a net rotation in the position angle of linearly polarized waves of

$$\theta_\nu \approx 10^8 \nu_{100}^{-2} \nu^{-7/4} \sim 10^6 \nu_{100}^{-0.43} \text{ rad}, \tag{12}$$

where the last estimate uses Özel et al.’s (2000) fit to $r_\nu(\nu)$.

These rotation angles are so large that the synchrotron emission in ADAF/Bondi models of Sgr A* is strongly depolarized by Faraday rotation. For example, in a simple uniform source model, the observed polarization is $\propto \theta_{\nu}^{-1}$ (Pacholczyk 1970). In general, the observed polarization depends on the rotation measure power spectrum, but is $\ll 1$ for $\theta_{\nu} \gg 1$ (e.g., Tribble 1991).

3.2. $\dot{M} \ll \dot{M}_{\text{Bondi}}$

If the accretion rate onto Sgr A* is much less than the Bondi rate, significant polarization may be observable at high frequencies. This is because the gas density and magnetic field strength are smaller than in ADAF/Bondi models, and because the electrons are relativistic; both effects substantially reduce the rotation measure. For the $M \sim 10^{-8} M_\odot \text{ yr}^{-1}$ model of §2, e.g., the rotation measure calculated using equation (10) is $RM \approx 10^9$ rad m$^{-2}$.
near \( r \sim 1 \). Moreover, if \( n \propto r^{-1/2} \), as in CDAF models, the magnetic field scales as \( B \propto r^{-3/4} \) and

\[
RM \approx 10^3 r^{-1/4} \left( \frac{\gamma}{100} \right)^{-2} \text{rad m}^{-2}. \tag{13}
\]

The variation of the electron Lorentz factor with radius is somewhat uncertain, but we expect roughly \( \gamma \propto r^{-1} \), so that the electrons become non-relativistic by \( r \sim 10^2 \). Equation (13) then shows that \( RM \) has its maximal value at large radii, \( r \sim 10^2 \), where \( RM \sim 3 \times 10^6 \) rad m\(^{-2} \).\(^6\)

Equation (13) demonstrates that there is no depolarization of synchrotron emission at small radii in models with accretion rates much less than the Bondi rate: \( RM \) is negligible in the region where the synchrotron emission is produced. Depolarization can still be important, however, if observed photons experience different Faraday rotation at large radii, \( r \gtrsim 10^2 \), on their way out of the accretion flow (see Bower et al. 1999ab). Note that Bower et al. (1999ab) have shown that depolarization in the interstellar medium is unlikely to be important.

Spatial variation in the rotation measure will depolarize Sgr A* at frequencies for which \( \delta \theta = \lambda^2 \delta RM \gtrsim \pi \), i.e., for

\[
\nu \lesssim 100 \left( \frac{\delta RM}{10^6 \text{rad m}^{-2}} \right)^{1/2} \text{GHz}, \tag{14}
\]

where \( \delta RM \) is the difference in the rotation measure for photons of a given frequency which travel through different parts of the accretion flow. Quantitative calculations of depolarization by differential Faraday rotation are uncertain because \( \delta RM \) depends on the size of the emitting region as a function of frequency and on the poorly understood density and magnetic field structure of the accretion flow at \( r \sim 10^2 \). Two points are, however, clear: (1) At low frequencies, \( \ll 100 \) GHz, Sgr A* is easily depolarized at \( r \gtrsim 10^2 \). The required \( \delta RM \) is \( \ll 10^6 \) rad m\(^{-2} \), orders of magnitudes smaller than the values of \( RM \) obtained at \( r \sim 10^2 - 10^4 \). Moreover, both the intrinsic and scatter broadened sizes of Sgr A* increase at low frequencies. Large values of \( \delta RM / RM \) can thus be obtained. (2) Emission above \( \sim 100 \) GHz can plausibly be linearly polarized if the accretion rate onto Sgr A* is much less than the Bondi rate. In particular, equations (13) and (14) show that for \( \dot{M} \ll \dot{M}_{\text{Bondi}} \), emission above \( \approx 100 \) GHz is not depolarized propagating out of the accretion flow.

4. DISCUSSION

ADAF/Bondi models assume that the accretion rate onto Sgr A* is of order the Bondi rate \( (\sim 10^{-4} - 10^{-5} M_\odot \text{ yr}^{-1}) \) and that the radio to infrared emission is produced by synchrotron emission from marginally relativistic electrons \((T_e \approx 10^9 - 10^{10} \text{ K})\). In such models the rotation measure is \( \gtrsim 10^{40} \) rad m\(^{-2} \) inside \( \approx 100 \) Schwarzschild radii where the synchrotron emission is produced. ADAF/Bondi models thus predict that Sgr A* should be depolarized by Faraday rotation over the entire radio to infrared spectrum, and should have nearly zero linear polarization.

The theoretical arguments summarized in §1 propose that the accretion rate onto Sgr A* is much less than the Bondi rate. We have described one such model, in which the electron heating rate is of order the rate of change of the magnetic energy density. For an accretion rate \( \sim 10^3 \) times smaller than the Bondi rate, i.e., \( \sim 10^{-5} M_\odot \text{ yr}^{-1} \), and with relativistic electrons with \( \gamma \approx 100 \), this model can explain the observed characteristics of Sgr A* (see QN for spectral models). Moreover, it predicts that the rotation measure in the accretion flow is much smaller than in ADAF/Bondi models. This is because the gas density and magnetic field strength close to the black hole are much smaller, and because the electrons are relativistic \( (RM \propto \gamma^{-4} \log \gamma \text{ for } \gamma \gg 1) \); see §3 and the Appendix). The maximal contribution to the rotation measure comes from \( \sim 10^2 - 10^3 \) Schwarzschild radii, where \( RM \sim 10^6 \) rad m\(^{-2} \).

Rotation measures of \( \sim 10^6 \) rad m\(^{-2} \) can depolarize Sgr A* at \( \nu \ll 100 \) GHz by differential Faraday rotation; photons of a given frequency travel through different rotation measures on their way out of the accretion flow. Following Bower et al. (1999ab), we believe that this accounts for the \( 0.2\% \) linear polarization of Sgr A* at low frequencies \((\approx 4 \text{ to } 23 \text{ GHz}; \text{ see Bower et al. 1999ab}); it is less clear, however, that it can account for Bower et al.’s \( (1999b) \) limit of \( \approx 1\% \) linear polarization at 86 GHz (see below). In fact, rotation measures of \( \approx 10^6 \) rad m\(^{-2} \) are insufficient to depolarize emission above \( \approx 100 \) GHz. As a result, in models with accretion rates much less than the Bondi rate, \( \gtrsim 100 \) GHz emission is not depolarized propagating out of the accretion flow; intrinsically polarized synchrotron emission may therefore be observable at high frequencies.

The above considerations show that the linear polarization of Sgr A* at high frequencies provides a means of distinguishing between accretion at the Bondi rate, and accretion at a much smaller rate. In fact, Aitken et al. (2000) report a possible detection of \( \sim 10\% \) linear polarization from Sgr A* between 150 and 400 GHz. These observations cannot be readily explained by ADAF/Bondi models. Instead, the most straightforward interpretation is that the accretion rate onto Sgr A* is much less than the Bondi rate (our specific model has \( \dot{M} \approx 10^{-8} M_\odot \text{ yr}^{-1} \)).

One difficulty in interpreting Aitken et al.’s results is the large beam \((\approx 20\')\) of the SCUBA camera on the JCMT. This large beam forced Aitken et al. to subtract out free-free and dust emission in order to isolate the flux and polarization of Sgr A*. Although we believe that they have likely detected polarized flux from Sgr A*, future high resolution polarimetry at mm wavelengths is necessary to further address this important issue.

Aitken et al. find that the position angle of Sgr A* changes by \( \lesssim 10^6 \) between \( \lambda = 0.135 \text{ cm} \) and \( \lambda = 0.2 \text{ cm} \); at face value this implies \( RM \lesssim 10^5 \text{ rad m}^{-2} \), somewhat smaller than the values of \( \sim 10^6 \) rad m\(^{-2} \) in our model. This assumes, however, that the intrinsic position angle of Sgr A* is the same at \( \lambda = 0.135 \text{ cm} \) and \( \lambda = 0.2 \text{ cm} \), which need not be the case. Moreover, our estimates of \( RM \) are actually upper limits, since they assume \( (1) \) equipartition magnetic fields aligned along the line of sight and \( (2) \) that our line of sight passes through the equatorial plane of the accretion flow.

\(^6\)Although we have used the CDAF scaling in equation (13), the relativistic suppression of Faraday rotation ensures that \( RM \) will be dominated by \( r \sim 10^2 - 10^3 \) in any model with an accretion rate much less than the Bondi rate.
One interesting feature of the polarization observations of Sgr A* is the $10^{-3}$% polarization found by Aitken et al. (2000) at 150 GHz and the $\lesssim 1$% polarization found by Bower et al. (1999b) at 86 GHz, an apparently abrupt change in polarization over a factor of $\approx 2$ in frequency. Note, however, that Aitken et al’s error bars are only 1σ, so the difference may not be so large.

A rapid change in polarization with frequency could be due to the fact that different frequencies probe different radii in the accretion flow. In particular, the highest frequency emission from Sgr A* arises from very close to the black hole in our models, where one might plausibly expect a change in the magnetic field structure, electron distribution function, etc.; this possibility is also suggested by the change in the spectrum of Sgr A* above $\approx$ 100 GHz (e.g., Serabyn et al. 1997). Furthermore, if the electrons are thermal, linear polarization is exponentially suppressed below the self-absorption frequency; even power law electrons show a factor of $\approx 10$ decrease in polarization from the optically thin to the optically thick limit (e.g., Ginzburg & Syrovatskii 1969). This could contribute to a rapid decline in polarization at low frequencies.

Our analysis of depolarization is likely applicable even if the radio emission from Sgr A* is dominated by a jet/outflow, rather than the accretion flow as we have assumed. In jet models, it is still natural for the highest frequency emission to originate very close to the black hole; in Falcke’s model, e.g., the $\gtrsim$ 100 GHz emission arises from $\lesssim 10 R_g$, in what is really a “transition region” between the accretion flow and the jet (Falcke 1999). In order for this emission to not be depolarized (either in situ or propagating through the accretion flow), our constraints on the rotation measure and the plasma conditions close to the black hole still apply.

Two scenarios in which accretion at the Bondi rate could be consistent with observed linear polarization at high frequencies are (1) if the high frequency emission arises close to the black hole, but in a nearly empty funnel pointed directly towards us (e.g., along the rotation axis of an ADAF) or (2) if the high frequency emission from Sgr A* is produced at very large distances from the black hole, $r \gtrsim 10^3$. The former possibility requires a rather special geometry and the latter is likely ruled out by the VLBI source size of $\approx 10 R_g$ (Krichbaum et al. 1998) and the variability of Sgr A* at $\approx$ 100 GHz (Tsuboi, Miyazaki, & Tsutsumi 1999).

4.1. Application to Other Systems

Although we have have focused our analysis on Sgr A* at the Galactic Center, linear polarization of high frequency radio emission can be used as a probe of the accretion physics in other low-luminosity galactic nuclei. For a black hole of mass $M = m_9 10^9 M_\odot$ accreting (spherically) at a rate $\dot{M} = 10^{-4} \dot{m}_{-4} \dot{M}_{\text{edd}} \approx 10^{23} \dot{m}_{-4} m_9 \text{ g s}^{-1}$, the density, magnetic field strength, and rotation measure in ADAF models are

$$n \approx 3 \times 10^6 \dot{m}_{-4} m_9^{-1} r^{-3/2} \text{ cm}^{-3},$$  \hspace{1cm} (15)

$$B \approx 100 \dot{m}_{-4}^{1/2} m_9^{-1/2} r^{-5/4} \text{ G},$$ \hspace{1cm} (16)

and

$$RM \approx 3 \times 10^{10} \dot{m}_{-4}^{3/2} m_9^{-1/2} r^{-7/4} \text{ rad m}^{-2}.$$ \hspace{1cm} (17)

Equation (17) shows that large rotation measures and the associated depolarization of synchrotron emission by Faraday rotation are generic features of ADAF models (unless $\dot{m}_{-4} \ll 1$).

The absence of observed linear polarization in the radio spectrum of a low-luminosity galactic nucleus would be consistent with ADAF models. By contrast, detected linear polarization would argue against an ADAF as the source of the observed radio emission. Although particular systems should be analyzed on an individual basis, we expect that in many cases both a jet and a CDAF-like model could account for observed linear polarization. Distinguishing between these two possibilities requires high resolution observations.

A particularly interesting class of systems for future polarization are elliptical galaxies in nearby X-ray clusters (e.g., NGC 4649, 4472, and 4636 in the Virgo cluster). As discussed by, e.g., Fabian & Canizares (1988), Fabian & Rees (1995), and Di Matteo et al. (1999, 2000), many of these galaxies have extremely dim nuclei given the inferred black hole masses ($\sim 10^9 M_\odot$) and Bondi accretion rates. Linear polarization may shed important light on the physics of these systems.

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APPENDIX

FARADAY ROTATION IN AN ULTRA-RELATIVISTIC MAXWELLIAN PLASMA

Faraday rotation in a cold plasma is described by a change in polarization angle given by

$$\frac{d\theta}{dl} = \frac{k_{||} \omega_p \omega_B}{2 \omega^3}, \quad (A1)$$

where $\omega = ck$ is the frequency of the radio wave, $k_{||}$ is the projection of the wavenumber along the magnetic field, $\omega_p^2 = 4\pi ne^2/m_e$ is the plasma frequency, and $\omega_B = eB/(m_e c)$ is the cyclotron frequency. This corresponds to the usual rotation measure

$$RM \equiv \frac{\theta}{\lambda^2} = \frac{e^3}{2\pi m_e^2 c^4} \int dl \cdot Bn = 2.63 \times 10^{-13} \times \int dl \cdot Bn \frac{\log \gamma}{2\gamma^2} \frac{\text{rad}}{m^2}. \quad (A2)$$

Here we derive the rotation measure for an ultrarelativistic Maxwellian plasma:

$$RM_\gamma = \frac{e^3}{2\pi m_e^2 c^4} \int dl \cdot Bn \frac{\log \gamma}{2\gamma^2} = 2.63 \times 10^{-13} \times \int dl \cdot Bn \frac{\log \gamma}{2\gamma^2} \frac{\text{rad}}{m^2}, \quad (A3)$$

where we have defined $\gamma \equiv kT_e/(m_e c^2)$. The dominant correction to the non-relativistic expression is the relativistic mass: $m_e \rightarrow \gamma m_e$.

We use the Vlasov equations to calculate the plasma permittivity and hence the dispersion relation for electromagnetic waves. For a magnetic field and wavenumber along the $z$ axis, the first-order (in the unperturbed magnetic field) permittivity is given by

$$\epsilon_{xy}^{(1)} = \frac{-i 4\pi e^2}{2\omega m_e m_e c} \int d^3p \frac{1}{(\omega - kv_z)^2} \frac{p_z^2}{p} dF \frac{m_e^2 c^2}{p^2 + m_e^2 c^2}, \quad (A4)$$

where the unperturbed distribution function is normalized by $\int d^3p F(p) = n$, and $p_z^2 \equiv p_x^2 + p_y^2$. For a cold plasma, equation (A4) gives

$$\epsilon_{xy} = \frac{i\omega_p^2 \omega_B}{\omega^3}. \quad (A5)$$

Using standard arguments (e.g., Rybicki & Lightman 1979) this leads to the RM for a cold plasma given by equation (A2). For an ultra-relativistic plasma, equation (A4) gives

$$\epsilon_{xy} = \frac{i\omega_p^2 \omega_B \log \gamma}{\omega^3 \frac{2\gamma^2}{m^2}}, \quad (A6)$$

where we have not changed the definition of $\omega_p$ and $\omega_B$ in the ultra-relativistic regime. Equation (A6) for the permittivity gives the ultra-relativistic RM in equation (A3).
REFERENCES