Galactic Gamma-Ray Background Radiation from Supernova Remnants

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ABSTRACT

The contribution of the Source Cosmic Rays (SCRs), confined in Supernova Remnants, to the diffuse high energy $\gamma$-ray emission above 1 GeV from the Galactic disk is studied. $\gamma$-rays produced by the SCRs have a much harder spectrum compared with those generated by the Galactic Cosmic Rays which occupy a much larger residence volume uniformly. SCRs contribute less than 10% at GeV energies and become dominant at $\gamma$-ray energies above 100 GeV. The contributions from $\pi^0$-decay and Inverse Compton $\gamma$-rays have comparable magnitude and spectral shape, whereas the Bremsstrahlung component is negligible. At TeV energies the contribution from SCRs increases the expected diffuse $\gamma$-ray flux almost by an order of magnitude. It is shown that for the inner Galaxy the discrepancy between the observed diffuse intensity and previous model predictions at energies above a few GeV can be attributed to the SCR contribution.

Subject headings: gamma-rays – background radiation – cosmic rays – supernova remnants
1. Introduction

Observations of the diffuse Galactic $\gamma$-ray emission give information about the Galactic Cosmic Rays (GCRs), the interstellar gas and diffuse photon fields, and about the interactions between them. The observational results obtained with the Energetic Gamma Ray Experiment Telescope (EGRET) on the Compton Gamma Ray Observatory can be described fairly well by a suitable model for the diffuse interstellar gas, Cosmic Ray (CR) and photon distributions (e.g. Hunter et al. 1997a; Hunter et al. 1997b).

However, above 1 GeV the observed average diffuse $\gamma$-ray intensity in the inner Galaxy, $300^\circ < l < 60^\circ, |b| \leq 10^\circ$, exceeds the model prediction significantly. There are at least two possible explanations for this discrepancy (e.g. Hunter et al. 1997b; Weekes et al. 1997). The high-energy $\gamma$-ray excess may indicate that the GCR spectrum observed in the local neighborhood is not representative of the diffuse CR population in the Galactic disk; a harder average diffuse proton spectrum is required to explain the $\gamma$-ray excess if it is due to $\pi^0$-decay. An unresolved distribution of CR sources is the other possibility.

The physical picture which we consider in this paper corresponds to the second possibility. The idea that CRs, after leaving their sources, could in principle produce $\gamma$-rays in ambient dense clouds with a harder spectrum than those produced by the average GCRs, was proposed by Aharonian and Atoyan (1996). Another proposed possibility, also invoking transport effects, is the local hardening of the CR energy spectrum in the direction perpendicular to the Galactic disk above strong CR sources, especially above the inner region of the Galaxy, due to faster CR convection in a faster Galactic Wind (Völk 1999). This contributes a principally observable harder than average $\gamma$-ray component in such regions. In contrast, we consider here the accelerating particles inside their sources, where they are much more strongly scattered than in the ISM, neglecting the contributions invoked by Aharonian and Atoyan, and by Völk (see also below).
We assume that SNRs are the dominant sources of the Galactic CRs. On this premise we find that CRs, accelerated and confined in SNRs, give an important contribution to the high-energy $\gamma$-ray emission from the Galactic disk. Since the CR energy spectrum inside SNRs is much harder than on average in the Galaxy — the average spectrum being softened by rigidity-dependent escape from the Galaxy in the diffusion region above the disk — the relative SNR contribution increases with energy and becomes in fact dominant at $\gamma$-ray energies $\epsilon_\gamma \gtrsim 100$ GeV. It may substantially increase the diffuse TeV $\gamma$-ray emission from the Galactic disk so as to constitute a significant and spatially variable observational background which must also be taken into account in the search for spatially extended Galactic CR sources in this energy region. A physically analogous problem is the contribution of CR electrons in SNRs to the radio synchrotron spectrum of normal galaxies, without an Active Galactic Nucleus. It has been recently discussed by Lisenfeld & Völk (1999).

In this paper we shall investigate pion-decay $\gamma$-ray emission from CR nuclei as well as Inverse Compton (IC) radiation and Bremsstrahlung due to CR electrons. It will be shown that the average IC $\gamma$-ray background from SNRs is comparable in magnitude and spectral form to the pion-decay background at high energies, whereas the corresponding Bremsstrahlung component is negligible.

2. Gamma-ray luminosity of old SNRs

The majority of the GCRs, at least up to kinetic energies $\epsilon \sim 10^{14}$ eV, is presumably accelerated in SNRs. According to modern theory a significant part of the hydrodynamic Supernova (SN) explosion energy $E_{SN} \sim 10^{51}$ erg is converted into CRs already in the early Sedov phase of the evolution, due to diffusive shock acceleration (e.g. Berezhko et al. 1996; Berezhko & Völk 1997). Later on, the CR energy content and the high-energy $\gamma$-ray
production slowly decrease with time. This is at least true as long as the progenitor star is not so massive as to have a strong wind which significantly modifies the circumstellar medium (Berezhko & Völk, 2000). The total number of SNRs $N_{SN} = \nu_{SN} T_{SN}$ is an increasing function of their assumed life time $T_{SN}$, i.e. the time until which they can confine the accelerated particles; here $\nu_{SN}$ is the Galactic SN rate. Therefore we conclude that the population of the oldest SNRs dominates the total $\gamma$-ray luminosity of the ensemble of Galactic SNRs. Thus we consider only old SNRs which nevertheless are still strong enough to confine most of the CRs produced during the prior evolutionary stages.

We then have the situation that the CRs in the Galaxy are represented by two basically different populations. The first one consists of the ordinary GCRs and presumably occupies a large Galactic residence volume quasi-uniformly. This residence volume exceeds that occupied by the CR sources by far (e.g. Ptuskin et al. 1997; for an earlier review, see Berezinsky et al. 1990). The second CR population, which we call Source Cosmic Rays (SCRs), is represented by shock accelerated CRs that are still confined in the localized SNRs. During the initial, active period of SNR evolution of about $t \lesssim 10^5$ yr when the SN shock is relatively strong, the volume occupied by the accelerated CRs practically coincides with the shock volume. In later stages the shock becomes weak and CRs begin to leave the SNR acceleration region. After some period of time $T_{SN}$ the escaping SCRs become very well mixed with the “sea” of GCRs. We shall assume that the transitory period during which SCRs are transformed into GCRs is much shorter than the preceding confinement time $T_{SN}$. The most important factor for CR confinement is the shock strength. Even in the phase where radiative cooling would formally become important, this remains true since CRs prevent cooling compression due to their pressure. Thus the assumed mean life time $T_{SN} \lesssim 10^5$ yr is determined by the shock dynamics more than by anything else.

Since the $\gamma$-ray production due to GCRs is quite well studied (e.g. Hunter et al.
1997b; Mori 1997), it is primarily important to find the relative contribution of the SCR population.

2.1. Gamma rays from $\pi^0$-decay

The production rate of $\pi^0$-decay $\gamma$-rays from inelastic CR-gas collisions, primarily p-p collisions, may be written in the form (Drury et al. 1994)

$$Q_\gamma(\epsilon) = Z_\gamma \sigma_{pp} c N_g n(\epsilon),$$

where $N_g$ is the local gas number density, $\sigma_{pp}$ is the inelastic p-p cross-section, $Z_\gamma$ is the so-called spectrum-weighted moment of the inelastic cross-section, $n(\epsilon)d\epsilon$ is the CR spatial number density of CRs in the kinetic energy interval $d\epsilon$, and $c$ is the speed of light. Thus we have to primarily calculate $n(\epsilon)$ for the two CR populations.

The quasi-uniform GCR population in the gas disk is assumed to have roughly a power law spectrum in the relativistic range

$$n_{GCR}(\epsilon) = \frac{n_{GCR}^0 (\gamma_{GCR} - 1)}{m c^2} \left( \frac{\epsilon}{m c^2} \right)^{-\gamma_{GCR}},$$

where $m$ is the proton mass. For simplicity we restrict our consideration here to the proton component which is energetically dominant in both the GCR and the SCR populations.

The total number $n_{GCR}^0$ of relativistic GCRs with $\epsilon > m c^2$, per unit volume, can be expressed in terms of the CR energy density $e_{GCR}$:

$$n_{GCR}^0 = \frac{(\gamma_{GCR} - 2) e_{GCR}}{m c^2},$$

where $m$ is the proton mass. For simplicity we restrict our consideration here to the proton component which is energetically dominant in both the GCR and the SCR populations.

In contrast to the GCR population, the SCRs are confined inside a discrete number $N_{SN}$ of SNRs. These are assumed to be predominantly located in the Galactic gas disk, of volume $V_g$. Spatially averaged over the Galactic disk volume their $\gamma$-ray production
rate is determined by an expression analogous to eq. (1), where instead of \( n(\epsilon) \) one should substitute the SCR distribution

\[
n_{\text{SCR}}(\epsilon) = N_{\text{SCR}}(\epsilon)N_{\text{SN}}/V_g,
\]

with \( N_{\text{SCR}}(\epsilon)d\epsilon \) being the overall (i.e. integrated over the SNR volume) SCR number in the energy interval \( d\epsilon \), and should use an appropriate mean local interstellar medium (ISM) number density \( N_{g}^{\text{SCR}} \) into which the SNe explode.

Since the CRs produced inside SNRs have also a power-law spectrum \( N_{\text{SCR}} \propto \epsilon^{-\gamma_{\text{SCR}}} \) in the relativistic range, \( n_{\text{SCR}}(\epsilon) \) can be expressed in the same forms (2) and (3), with

\[
e_{\text{SCR}} = N_{\text{SN}}\delta E_{\text{SN}}/V_g,
\]

and putting \( \gamma_{\text{SCR}} > 2 \). Here \( \delta \) is the fraction of the SN explosion energy \( E_{\text{SN}} \) converted into SCRs.

However, according to the prediction from nonlinear kinetic theory (Berezhko et al. 1996), diffusive shock acceleration produces an extremely hard spectrum of SCRs at the early Sedov phase which is characterized by a power law index \( \gamma_{\text{SCR}} = 2 \). In this case we have

\[
n_{0}^{\text{SCR}} = \frac{e_{\text{SCR}}}{m c^2 \ln(\epsilon_{\text{max}}/mc^2)},
\]

where \( \epsilon_{\text{max}} \) is the maximum SCR energy.

For the ratio \( R = Q_{\gamma}^{\text{SCR}}/Q_{\gamma}^{\text{GCR}} \) of the \( \gamma \)-ray production rates due to SCRs and GCRs, we have

\[
R(\epsilon) = \frac{Z_{\gamma}^{\text{SCR}}N_{\text{SN}}\delta E_{\text{SN}}}{Z_{\gamma}^{\text{GCR}}(\gamma_{\text{GCR}} - 2)\ln(\epsilon_{\text{max}}/mc^2)V_g e_{\text{GCR}}} \times \zeta \left( \frac{\epsilon_{\gamma}}{mc^2} \right)^{\gamma_{\text{GCR}} - 2},
\]

where \( \zeta \) is the ratio \( N_{g}^{\text{SCR}}/N_{g}^{\text{GCR}} \), and \( N_{g}^{\text{GCR}} \) denotes the average gas density in the disk. In fact we assume that the gas and the CRs are distributed uniformly inside each SNR.
which is approximately true for the old SNRs which we consider here. The parameter $\zeta$ describes a possible spatial correlation between SN occurrence and local ISM density. If on average Supernovae explode in a denser than average medium in the Galactic disk, then $\zeta > 1$, whereas $\zeta < 1$ in the opposite case.

The main mass of the ISM $M_g = 4 \times 10^9 M_\odot$ is contained in a Galactic disk region of a thickness of about 240 pc, corresponding to the thickness of the HI gas (Dickey and Lockman 1990) which has the volume $V_g = 2.5 \times 10^{66}$ cm$^3$ referred to before. Here we take a disk radius of about 10 kpc which implies an average gas density $N_{GCR}^g = 2$ cm$^{-3}$. Taking the relativistic part of the GCR spectrum to be characterized by $e_{GCR} \simeq 10^{-12}$ erg/cm$^3$, and $\gamma_{GCR} = 2.75$ which results in $Z_{\gamma}^{SCR}/Z_{\gamma}^{GCR} = 10$ (Drury et al. 1994), we obtain for the standard set of SN parameters $E_{SN} = 10^{51}$ erg, $\nu_{SN} = 1/30$ yr$^{-1}$:

$$R(\epsilon_\gamma) = 0.16\zeta \left( \frac{T_{SN}}{10^5 \text{ yr}} \right) \left( \frac{\epsilon_\gamma}{1 \text{ GeV}} \right)^{0.75},$$

in addition using the rather moderate parameter values $\delta = 0.1$ and $\epsilon_{max} = 10^5 m c^2$ to characterize CR acceleration inside SNRs (e.g. Berezhko et al. 1996). One can see from this expression that for $T_{SN} \sim 10^5$ yr the $\gamma$ ray production due to SCRs becomes dominant already at energies $\epsilon_\gamma \gtrsim 10$ GeV.

Note that the quantity $\delta E_{SN} N_{SN} / (V_g e_{GCR})$ represents the ratio of currently existing total SCR energy and GCR energy inside $V_g$. For the above set of parameters it is about 0.1. Despite the fact that the SCRs represent only a relatively small fraction of the total CR energy content even in the disk, they may dominate the $\gamma$-ray production at sufficiently high energies due to their much harder spectrum.

It is clear that the quantity $R(\epsilon_\gamma)$ determines the average ratio

$$(dN_{\gamma}^{SCR}/d\epsilon_\gamma)/(dN_{\gamma}^{GCR}/d\epsilon_\gamma)$$

of $\gamma$-ray spectra produced in any region of the disk, by SCRs and GCRs, respectively. Therefore the total $\gamma$-ray spectrum measured from an arbitrary
Galactic disk volume is expected to be

\[ \frac{dN_{\gamma}}{d\epsilon_{\gamma}} = \frac{dN_{\gamma}^{GCR}}{d\epsilon_{\gamma}} [1.4 + R(\epsilon_{\gamma})], \]  

(9)

where the additional factor 0.4 is introduced to approximately take into account the contribution of GCR electron component to the diffuse \( \gamma \)-ray emission at GeV energies (e.g. Hunter et al. 1997b). In Fig.1 we present the expected differential flux of \( \gamma \)-rays from the inner Galaxy, calculated for \( \zeta = 1 \) and \( T_{SN} = 10^5 \) yr, with the spectrum \( dN_{\gamma}^{GCR}/d\epsilon_{\gamma} \) taken from the paper by Hunter et al. (1997b), and extended into the region \( \epsilon_{\gamma} > 30 \) GeV according to the law \( \epsilon_{\gamma}^{-2.75} \). One can see that after inclusion of the \( \gamma \)-rays produced by SCRs, the calculated flux even exceeds the EGRET flux for \( \epsilon_{\gamma} \gtrsim 20 \) GeV. This suggests that expression (7) overestimates the \( \gamma \)-ray production by SCRs.

It is possible that the overall source spectral index \( \gamma_{SCR} \) is somewhat larger than 2 due to very late accumulation of only low-energy particles. It can also not be excluded that the confinement time \( T_{SN} \) of the SCRs depends on energy. Due to their high mobility, the highest energy particles may leave the vicinity of parent SNR earlier and also more rapidly. This process of SCR escape into the ISM starts for the most energetic particles already at the early Sedov phase of SNR evolution (e.g. Berezhko et al. 1996; Berezhko & Völk 1997). Therefore at time \( T_{SN} \), when the main part of SCRs are released from the SNR, their spectrum may be somewhat steeper than a \( \gamma_{SCR} = 2 \) spectrum.

Due to the importance of the problem we derive the relation between \( n_{SCR} \) and \( n_{GCR} \) in a different form. It leads to the same results if SNRs are the GCR source. We start from the usual leaky box balance equation

\[ \frac{n_{GCR}(\epsilon)}{\tau_c} = \frac{N_{SCR}}{V_c} \nu_{SN}, \]  

(10)

where \( V_c(\epsilon) \) is the energy-dependent residence volume occupied by GCRs that reach the gas disk during their mean residence time \( \tau_c \) in \( V_c \). In the case of an extended Galactic halo
due to a Galactic wind driven by the GCRs themselves, and for energies much larger than a few GeV, $V_c$ can be much greater than $V_g$. In fact $V_c(\epsilon) \propto \epsilon^{0.55}$ in such a selfconsistent halo model (Ptuskin at al. 1997).

Note that the leaky box model deals with a CR distribution $n_{GCR}(\epsilon)$ averaged over the residence volume $V_c$. Therefore it can only be applied to the GCRs which can be assumed to be almost uniformly distributed in the residence volume. It is not valid for the SCRs, because their behavior is determined not only by large-scale transport but also by other physical factors which, for example, lead to their acceleration. Technically the volume $V_{SCR}$, occupied by the SCRs, should be excluded from the residence volume $V_c$ and the SCRs appear in the balance equation (10) for the GCRs only in the form of a source term $N_{SCR}v_{SN}/V_c$ as a CR population released from the source region into the ISM after some unspecified evolutionary period $T_{SN}$. Therefore eq.(10) does not depend upon $T_{SN}$. However, the effects produced by the SCRs confined inside the ensemble of simultaneously existing SNRs, for example the additional $\gamma$-ray production, essentially depends on $T_{SN}$, since it directly determines the total number $N_{SN}$ of simultaneously existing SNRs.

Using eq. (4) we can write

$$\frac{n_{SCR}}{n_{GCR}} = \frac{V_c T_{SN}}{V_g \tau_c} = \frac{T_{SN}}{\tau_g}.$$  

(11)

The GCR residence time in the disk volume, $\tau_g = \frac{\tau_c V_g}{V_c}$, can be derived from the measured grammage $x$, which is the mean mass of Interstellar matter traversed by GCRs of speed $v$ in the course of their random walk in the Galaxy:

$$\tau_g = \frac{x V_g}{v M_g}.$$  

(12)

The measured grammage at high energies $\epsilon \geq \epsilon_0 = 4.4$ GeV is

$$x = 14 \left(\frac{v}{c}\right) (\epsilon/4.4 \text{ GeV})^{-0.60} \text{ g/cm}^2; \text{ for } \epsilon < \epsilon_0,$$

$$x = 14 v/c \text{ g/cm}^2 \text{ (Engelman et al. 1990).}$$
Therefore the residence time in the gas disk can be written in the form

\[ \tau_g = \tau_0 (\epsilon / \epsilon_0)^{-\mu}, \]  

(13)

where \( \tau_0 = 4.6 \times 10^6 \text{ yr} \), \( \epsilon_0 = 4.4 \text{ GeV} \), \( \mu = 0.6 \), and \( \epsilon \geq \epsilon_0 \). This experimentally inferred value for \( \mu \) closely agrees with the theoretical result of Ptuskin et al. (1997).

We note that at relativistic energies \( \epsilon > mc^2 \), according to the initial balance eq.(10), the GCR spectrum \( n_{GCR} \propto \epsilon^{-\gamma_{GCR}} \) and the overall SCR spectrum \( N_{SCR} \propto \epsilon^{-\gamma_{SCR}} \) should be connected by the relation

\[ \gamma_{SCR} = \gamma_{GCR} - \mu. \]  

(14)

For \( \mu = 0.60 \) the source should produce a SCR spectrum with \( \gamma_{SCR} = 2.15 \), while for the case \( \gamma_{SCR} = 2 \) one would need \( \mu = 0.75 \).

At the same time the SCR distribution \( n_{SCR}(\epsilon) \propto \epsilon^{-\gamma'_{SCR}} \), averaged over the gas disk, can have a different shape compared to \( N_{SCR}(\epsilon) \) in the case of an energy dependent SNR confinement time \( T_{SN}(\epsilon) \), according to eq. (11). It can be steeper, \( \gamma'_{SCR} = \gamma_{GCR} - \mu + \beta \), if \( T_{SN} \propto \epsilon^{-\beta} \) is a decreasing function of energy. It would mean that the highest energy SCRs leave the parent SNR faster than the lower energy SCRs.

Eq.(11) leads to a simple expression for the \( \gamma \)-ray production ratio

\[ R(\epsilon_\gamma) = \zeta Z^{SCR}_{\gamma} T_{SN}^{\gamma} Z^{GCR}_{\gamma} \tau_g, \]  

(15)

independently of \( \nu_{SN} \). Substituting the residence time in the form (13), and taking \( \gamma_{SCR} = 2.15 \), which leads to \( Z^{SCR}_{\gamma} / Z^{GCR}_{\gamma} = 7.5 \) (Drury et al. 1994) in eq.(15), we obtain for \( \epsilon_\gamma \geq 4.4 \text{ GeV} \):

\[ R(\epsilon_\gamma) = 0.07 \zeta \left( \frac{T_{SN}}{10^5 \text{ yr}} \right) \left( \frac{\epsilon_\gamma}{1 \text{ GeV}} \right)^{0.6}. \]  

(16)

We shall only consider \( \gamma \)-ray energies \( \epsilon_\gamma \geq 4.4 \text{ GeV} \). One can see here again that the SCR contribution is determined by the value of the confinement time inside SNRs, \( T_{SN} \).
Unfortunately there is no detailed description of when and how CRs, accelerated in SNR, are released into the ISM. Nevertheless one can give some constraints on this process.

According to the standard theory, the expanding SNR shock produces a power law CR spectrum up to the maximum energy (Berezhko 1996; Berezhko et al. 1996; Berezhko & Völk 1997)

$$\epsilon_m \propto R_s V_s,$$

which is determined by the radius $R_s$ and speed $V_s$ of the shock. The CRs with the highest energy $\epsilon_{\text{max}}$ are produced at the very beginning of the Sedov phase $t \sim t_0$ when the product $R_s V_s$ has its maximum $R_0 V_0$, where

$$t_0 = \frac{R_0}{V_0}, \quad R_0 = \left(\frac{3M_{\text{ej}}}{4\pi \rho_g}\right)^\frac{1}{4}, \quad V_0 = \sqrt{\frac{2E_{\text{sn}}}{M_{\text{ej}}}}$$

are the sweep-up time, sweep-up radius and initial mean ejecta speed respectively; $M_{\text{ej}}$ denotes the ejecta mass, and $\rho_g = N_g m$ the ISM density. Subsequently, the product $R_s V_s$ decreases with time as $t^{-1/5}$ and the SNR shock produces CRs with progressively lower cutoff energy $\epsilon_m(t) < \epsilon_{\text{max}} = \epsilon_m(t_0)$. During that phase those CRs that were previously produced with energies $\epsilon_m < \epsilon < \epsilon_{\text{max}}$ now propagate outward diffusively without significant influence of the SNR shock. If their expansion is still governed by the Bohm diffusion coefficient as during their acceleration, the expansion rate is only slightly higher than the SNR expansion rate and these particles should be considered as confined inside the source (i.e. the SNR).

The opposite extreme case corresponds to the assumption that particles with energies $\epsilon > \epsilon_m$ do no more produce a high level of turbulence. Let us consider this pessimistic scenario in terms of confinement here. In this case the propagation of these very high energy particles is governed by the mean Galactic diffusion coefficient which is very much larger than the Bohm diffusion coefficient. In this situation particles with energy $\epsilon$ should be considered as released from the source at the moment $t \geq t_0$ when $\epsilon_m(t)$ drops below $\epsilon$. 
Since the particles with maximum energy are produced at $t \approx t_0$, one can write

$$T_{SN}(\epsilon) = t_0 \left( \frac{\epsilon}{\epsilon_{\text{max}}} \right)^{-5}. \quad (19)$$

Due to this strong dependence it is clear that $T_{SN}(\epsilon)$ will still deviate from the overall gas dynamic life time $T_{SN}^{\text{tot}}$ only for large energies $\epsilon$ near $\epsilon_{\text{max}}$. All particles with

$$\epsilon/\epsilon_{\text{max}} < \left( T_{SN}^{\text{tot}}/t_0 \right)^{-1/5}$$

will remain confined until $T_{SN}^{\text{tot}}$.

Since the majority of Galactic SNe are core collapse SNe from stars with masses exceeding about $8M_\odot$, we shall use $M_{ej} = 10M_\odot$ for purposes of estimate. Except for progenitor masses exceeding $15M_\odot$, the progenitors have only a weak stellar wind. Therefore the assumption of a uniform circumstellar medium remains reasonable for the average properties of the SNR population.

For the main fraction of CRs the confinement in SNRs terminates when the SNR shock becomes weak and produces CRs with a very steep spectrum that cannot anymore excite a high level of turbulence near the shock front. If we take $M = 4$ as a critical Mach number, the corresponding SNR age will be $t = (M_0/M)^{5/3} = 1.5 \times 10^3 t_0$, where $M_0 = V_0/c_{S0}$ is the initial shock Mach number and $c_{S0}$ is the ISM sound speed. For an ISM with number density $N_g = 1 \text{ cm}^{-3}$, $c_{S0} \simeq 4 \text{ km/sec}$, and then $t_0 \simeq 1.5 \times 10^3 \text{ yr}$ which gives $T_{SN} \simeq 2 \times 10^6 \text{ yr}$ for $M_{ej} = 10M_\odot$ and $E_{SN} = 10^{51} \text{ erg}$. This estimate shows that the SNR shock remains rather strong during a very long period of time.

Another physical factor which can restrict the SCR confinement is the radiative cooling of the postshock gas. Approximately it becomes important when the postshock temperature drops below $\sim 10^6 \text{ K}$, or when the postshock sound speed drops below $c_{S2} \sim 100 \text{ km/s}$. In the case of a strong shock, with Mach number $M \gg 1$, the postshock sound speed is determined by the shock speed $c_{S2} \sim \sqrt{5}V_s/3$. During the Sedov phase the shock speed
decreases with time according to the law \( V_s = 0.4 \times V_0 \left(\frac{t}{t_0}\right)^{-3/5} \). Therefore the shock speed drops to the value \( V_c = 100 \) km/s at the age \( t_c = t_0 \left(0.4 \frac{V_0}{V_c}\right)^{5/3} \). The above set of SN and ISM parameters gives \( t_c \approx 6 \times 10^4 \) yr. Since gas clumping as a result of cooling may lead to effective SCR leakage from the SNR, a value of the confinement time \( T_{SN} = 10^5 \) yr is reasonable for \( N_g = 1 \) cm\(^{-3} \). (Fig. 1).

In Fig.1 we present a calculated \( \gamma \)-ray spectrum based on the above expression (16) with

\[
T_{SN} = \min\{10^5, 10^3(\epsilon/\epsilon_{max})^{-5}\} \text{ yr,}
\]

\( \epsilon_{max} = 10^5 \) GeV, and \( \epsilon_\gamma = 0.1\epsilon \), which is roughly valid for the hadronic considered \( \gamma \)-ray production process considered. At GeV energies in this case SCRs contribute about 10\% of the total \( \gamma \)-ray flux. Due to their hard spectrum this contribution progressively increases with energy and becomes dominant at \( \epsilon_\gamma \gtrsim 100 \) GeV. It increases the expected TeV \( \gamma \)-ray flux by about a factor of ten. Note that the actual SCR contribution from the inner part of the Galaxy is somewhat higher than the above estimate due to the larger SNR concentration in this region.

As one can see from Fig. 1, the SCR contribution in the case \( \gamma_{SCR} = 2 \), using eqs. (8) and (9), is for all energies larger than that for \( \gamma_{SCR} = 2.15 \), using eq. (16). This difference is related to the different SCR acceleration efficiencies. In the first case it is characterized by the parameter \( \delta = 0.1 \) which, according to eq. (5), directly determines the \( \gamma \)-ray production rate. In the second case the SCR acceleration efficiency \( \delta = \left[\int_{m_c^2}^{\epsilon_{max}} \epsilon N_{SCR}(\epsilon) d\epsilon\right]/E_{SN} \) is not contained in the final expression (15), but one can derive it easily from the balance equation (10) which gives:

\[
\delta \nu_{SN} E_{SN} = \int_{m_c^2}^{\epsilon_{max}} \frac{n_{GCR}V_0}{\tau_g} \epsilon d\epsilon.
\]

\( \text{(21)} \)
To determine $\delta$ we use the (demodulated) GCR distribution

$$n_{GCR}(\epsilon) = 8.1 \times 10^{-10} \times \left(\frac{\epsilon}{1 \text{ GeV}} + \frac{mc^2}{1 \text{ GeV}}\right)^{-2.75} \text{ cm}^{-3} \text{ (GeV)}^{-1}. \quad (22)$$

(Ryan et al. 1972; Perko 1987).

Substituting the values of $\tau_g(\epsilon)$, $V_g$, $E_{SN}$, and $\nu_{SN}$, we obtain $\delta \approx 0.05$ for the relativistic part of the GCR spectrum. The required acceleration efficiency is two times lower compared with the case $\gamma_{SCR} = 2$ due to the essentially steeper spectrum. Note, that in both cases the SCR spectra with $\gamma_{SCR} = 2$, $\delta = 0.1$ and $\gamma_{SCR} = 2.15$, $\delta = 0.05$ contain about the same number of relativistic CRs. In the first case the required GCR residence time is $\tau_g \propto \epsilon^{-0.75}$, whereas the second case with $\tau_g \propto \epsilon^{-0.6}$ is close to the experiment (Engelman et al. 1990). Therefore we believe that the dashed line in Fig.1 represents the most reliable estimate for the expected diffuse $\pi^0$-decay $\gamma$-ray emission, especially at high energies $\epsilon_\gamma \gtrsim 100$ GeV.

Note also that the acceleration efficiency required by eq. (21), using an assumed Galactic SN rate $\nu_{SN} = 1/30 \text{ yr}^{-1}$ and a mean SN explosion energy $E_{SN} = 10^{51}$ erg, is considerably lower than that predicted by shock acceleration theory, which gives $\delta = 0.2 \div 0.5$ (Berezhko et al. 1996). Yet, in contrast to the acceleration models which determine $\delta$ from the injection rate and the nonlinear acceleration theory selfconsistently, eq. (21) determines only the product $\delta E_{SN} \nu_{SN}$ from observed quantities. The observationally inferred SN explosion energies $E_{SN}$ can be at least by a factor 2 smaller than $10^{51}$ erg, and from comparisons with galaxies similar to our own $\nu_{SN}$ could vary between $1/30 \div 1/100 \text{ yr}^{-1}$. Therefore the empirical value of $\delta$ from eq. (21) can vary between 0.05 and 1/3. Nevertheless, the theoretically determined efficiencies appear systematically too high.

As a possible solution for this discrepancy one might assume that, just before being
released, the SCRs lose an important part of their energy by adiabatic expansion so that the SCRs’ energy content inside a SNR is higher than the energy contained in the released spectrum $N_{SCR}(\epsilon)$. However, there is little dynamical basis for such an assumption. Much more likely is that the very efficient CR acceleration inside SNRs predicted by the nonlinear kinetic theory, assuming spherical symmetry, takes place in reality only at some fraction of the SN shock surface, because suprathermal positive ion injection into the acceleration process on the highly oblique part of the shock can be significantly suppressed (Bennet & Ellison 1995; Malkov & Völk 1995). In this case the acceleration efficiency, calculated for a spherical SNR shock, should be reduced by a factor of a few.

The actual SNR distribution can in fact be rather nonuniform within the disk volume, contrary to what we have assumed implicitly up to now. In this case the estimated value of $R(\epsilon, \gamma)$, which describes the relative SCR contribution to the diffuse $\gamma$-ray emission, should be corrected by a factor $N_{SN}^a/N_{SN}$, where $N_{SN}^a$ is the expected number of SNRs in the observed region and $N_{SN}$ represents this number in the case of uniformly distributed SNRs. It is clear that the expected value of $R(\epsilon, \gamma)$ is almost independent of the actual SNR distribution if the observed region is an essential part of the whole disk volume $V_g$.

2.2. Inverse Compton and Bremsstrahlung gamma-rays from SCR electrons

Electrons, once being injected into the diffusive shock acceleration process, will be as efficiently accelerated in SNRs as are the protons. Even though there exist theoretical concepts (e.g. Levinson 1994; Galeev et al. 1995; McClements et al. 1997; Bykov & Uvarov 1999), electron injection is not completely understood. However, there is no doubt that electrons undergo continuous acceleration during SNR evolution. The spectral shape of accelerated electrons $N_{SCR}^e(\epsilon)$ inside SNRs deduced from radio-observations on average agrees with what is expected from shock acceleration. Since relativistic electrons
with energies $\epsilon > 1$ GeV are dynamically indistinguishable from protons, their source spectrum $N_{SCR}(\epsilon) = K_{ep}N_{SCR}(\epsilon)$ can differ from that of the protons $N_{SCR}(\epsilon)$ only by some energy independent factor $K_{ep}$ that is determined by the injection process. High energy electrons produce $\gamma$-ray emission due to IC scattering, especially on the Cosmic Microwave Background (CMB) and by Bremsstrahlung on the interstellar gas. We shall first consider the IC contribution here.

### 2.2.1. Inverse Compton contribution

In an approximate form, valid if the generating electron energy distribution is smoothly varying, like in the case of a power law considered here, the IC $\gamma$-ray emissivity $Q_{\gamma}^{IC}(\epsilon_{\gamma})$ can be written as (e.g. Longair 1981, Berezinsky et al. 1990)

$$Q_{\gamma}^{IC}(\epsilon_{\gamma}) = \sigma_T c N_{ph} n_{e}^{e} SCR(\epsilon_{e}) \frac{d\epsilon_e}{d\epsilon_{\gamma}}, \quad (23)$$

where

$$\epsilon_{e} = m_e c^2 \sqrt{3\epsilon_{\gamma}/(4\epsilon_{ph})} \quad (24)$$

is the energy of electrons which produce an IC photon with mean energy $\epsilon_{\gamma}$, $\sigma_T = 6.65 \times 10^{-25}$ cm$^2$ denotes the Thomson cross section, $\epsilon_{ph} = 6.7 \times 10^{-4}$ eV and $N_{ph} = 400$ cm$^{-3}$ are the mean energy and number density of the CMB photons, respectively. Finally $n_{e}^{e} SCR = N_{e}^{e} SCR N_{SN}^{e} / V_g$ denotes the average spatial electron SCR number density.

Thus the ratio of the IC to the $\pi^0$-decay $\gamma$-ray production rate reads as

$$\frac{Q_{\gamma}^{IC}}{Q_{\gamma}^{SCR}} = 1028K_{ep} \left( \frac{1 \text{ cm}^{-3}}{N_{g}^{SCR}} \right) \left( \frac{\epsilon_{\gamma}}{1 \text{ TeV}} \right)^{1/2} \frac{N_{e}^{e} SCR}{N_{SN}} \frac{N_{SN}^{e}}{N_{SN}}, \quad (25)$$

where we have used $\gamma_{SCR} = 2$ and $\sigma_{pp} = 4 \times 10^{-26}$ cm$^{-2}$; $N_{SN}^{e}$ denotes the number of SNRs which contribute to the IC emission at energy $\epsilon_{\gamma}$ from CR electron sources.
From this expression it appears as if the IC $\gamma$-ray contribution would be dominant at TeV-energies if $K_{ep}$ is as large as $10^{-2}$ and if the mean gas number density inside SNRs is about $N^{SCR}_g = 1 \text{ cm}^{-3}$. However, the radiative cooling time $\tau_e(\epsilon_e)$ of electrons, which produce $\gamma$-rays with energy $\epsilon_\gamma = (\epsilon_e/17.1 \text{ TeV})^2$ TeV,

$$\tau_e = 7.3 \times 10^3 \left( \frac{10 \mu G}{B} \right)^2 \left( \frac{1 \text{ TeV}}{\epsilon_\gamma} \right)^{1/2} \text{ yr}, \quad (26)$$

reaches the above assumed overall SNR confinement time $T_{SN} = 10^5 \text{ yr}$ for $\gamma$-ray energies $\epsilon_\gamma < \epsilon^*_\gamma = 5.4(B/10 \mu G)^{-4} \text{ GeV}$. Here $B$ is the magnetic field strength inside the source, whose typical value inside SNRs is about $10 \mu G$. Therefore, taking into account the obvious relation $N_{SN}^e/N_{SN} = \tau_e/T_{SN}$, on average the relative IC contribution of the electron component of SCRs in TeV $\gamma$-ray can be written as

$$\frac{Q^\xi_{IC}}{Q^{SCR}_{\gamma}} = 75.5K_{ep} \left( \frac{1 \text{ cm}^{-3}}{N^{SCR}_g} \right) \left( \frac{10^5 \text{ yr}}{T_{SN}} \right) \left( \frac{10 \mu G}{B} \right)^2. \quad (27)$$

It is independent of the $\gamma$-ray energy for $\epsilon_\gamma > \epsilon^*_\gamma$, and only given by eq. (25) with $N_{SN}^e/N_{SN} = 1$ for $\epsilon_\gamma < \epsilon^*_\gamma$. This consideration shows that for the parameters assumed, and for $K_{ep}$ of the order of $10^{-2}$, we have an IC contribution to the average $\gamma$-ray background which is comparable to the hadronic background for all energies above a few GeV.

### 2.2.2. Bremsstrahlung contribution

At high energies, $\epsilon_e, \epsilon_\gamma \gg m_e c^2$, we have for the Bremsstrahlung $\gamma$-ray emissivity $Q^{Br}_{\gamma}$

$$Q^{Br}_{\gamma}(\epsilon_\gamma) = 2 \int_{\epsilon_{min}}^{\infty} d\epsilon_e \frac{d\sigma_{ep}^{Br}}{d\epsilon_\gamma} cN^{SCR}_g n_{SCR}^e(\epsilon_e), \quad (28)$$

where $\epsilon_{min}$ is the minimum electron energy necessary to produce a Bremsstrahlung $\gamma$-ray of energy $\epsilon_\gamma$, the factor 2 takes into account the contributions of electron-electron and electron-proton collisions, and where the differential electron-proton Bremsstrahlung
cross-section is given by (e.g. Berezinsky et al. 1990)

\[
\frac{d\sigma^{Br}(\epsilon_e, \epsilon_\gamma)}{d\epsilon_\gamma} = \frac{4\alpha r_0^2}{\epsilon_\gamma} \left[ \frac{4}{3} - \frac{4}{3} \frac{\epsilon_\gamma}{\epsilon_e} + \left( \frac{\epsilon_\gamma}{\epsilon_e} \right)^2 \right] \left[ \ln \left( \frac{2\epsilon_e}{mc^2} \frac{\epsilon_e}{\epsilon_\gamma} \right) - \frac{1}{2} \right].
\]  

(29)

Here \( \alpha \approx 1/137 \) and \( r_0 = 2.818 \times 10^{-13} \) cm denote the fine structure constant and the classical electron radius, respectively.

For our chosen value \( \gamma_{SCR} = 2 \), the integral for \( Q^{Br}_\gamma(\epsilon_\gamma) \) can be calculated in closed form. In the limit \( \ln(\epsilon_\gamma/m_e c^2) \gg 1 \), of interest here, it reduces to the asymptotic form

\[
Q^{Br}_\gamma(\epsilon_\gamma) = 8\alpha r_0^2 c N^e_{SCR} n^e_{SCR}(\epsilon_\gamma) \ln \left( \frac{\epsilon_\gamma}{m_e c^2} \right).
\]

(30)

Thus, finally, we obtain

\[
\frac{Q^{Br}_\gamma}{Q^{SCR}_\gamma} = \frac{8\alpha r_0^2 K_{ep} N^e_{SN}}{Z^e_{SCR} \sigma_{pp} N^e_{SN}} \ln \left( \frac{\epsilon_\gamma}{m_e c^2} \right) = 9.3 K_{ep} \frac{N^e_{SN}}{N^e_{SN}} \left[ 1 + 0.066 \ln \left( \frac{\epsilon_\gamma}{1 \text{ TeV}} \right) \right]
\]

(31)

This small ratio implies that Bremsstrahlung \( \gamma \)-rays play no role for the average \( \gamma \)-ray background above GeV energies, if \( K_{ep} \ll 0.1 \), taking into account, that \( N^e_{SN}/N_{SN} \) is always smaller than 1.

3. Discussion

We note that of order ten SNRs of age younger than \( 10^5 \) yr can on average lie within a 1 degree field of view of a detector directed towards the Galactic Center. Therefore a moderate fluctuation of the measured \( \gamma \)-ray intensity is expected due to variations of the actual number of SNRs within such a detector’s field of view. At the same time, for directions perpendicular to the Galactic plane, the chance to observe the contribution of SCR \( \gamma \)-ray emission is quite negligible. A question is then how one might best study the nonuniformities of this background experimentally. Clearly this is an investigation of its own. Therefore we would like to restrict ourselves to a few comments here. Due to the
spectral form of the background its graininess is most pronounced at high $\gamma$-ray energies. This is even more true due to the fact that for individual sources with a very low magnetic field the IC emission could be much stronger than the $\gamma$-ray emission due to hadronic interactions; in addition the IC emission has a harder spectrum. This suggests the use of imaging atmospheric Cherenkov telescopes. Their resolution in angle and energy is as good or better than that of other ground-based detectors. However, the study of extended sources is not an easy task with imaging telescopes which have a very limited field of view, even employing the stereoscopic method. For low brightness extended sources a satellite instrument like the future GLAST detector is well suited since it does not have to deal with the charged CR background due to its use of an anticoincidence shield. On the other hand, the statistics achievable with a small area space detector gets very low above a few tens of GeV. Thus one should probably attempt such a study with both types of instruments due to their complementary properties.

A different question concerns the limitations of our approach due to its concentration on SNRs as the sources of the GCRs. In fact the considerations in this paper can be applied to any alternative class of dominant GCR sources. The most important aspect, which leads to the dominance of SCRs in high-energy $\gamma$-ray production, is that the GCR sources should generate SCRs with a spectrum that is significantly harder than the GCR spectrum. Eq. (15) is valid for an arbitrary class of CR sources if we substitute some other value of the SCR confinement time $T_S$ instead of $T_{SN}$, since the grammage $x$ is an experimentally fixed quantity. Let us then assume that some class of compact CR sources produces an energy $E_C$ in the form of CRs with spectrum $N_{SCR} \propto \epsilon^{-\gamma_{SCR}}$ with average frequency $\nu_S$, and let us further assume that this spectrum remains unchanged inside the source regions for some period of time $T_S$ after which these CRs are released into the ISM as the GCRs. It is obvious that due to the general energy requirement the production rate $\nu_S E_C$ should be about the same as $\nu_{SN} \delta E_{SN}$. The SCR energy $E_C$, deposited in some initial volume $V$, will
produce a dynamically significant disturbance in the background ISM if we assume that
the initial SCR energy density $E_C/V$ is much greater than the thermal ISM energy density
which in turn is of order $e_{GCR}$. This will inevitably lead to the confinement of these SCRs
inside an expanding, disturbed volume for some period of time $T_S$ before the SCRs will be
released. It is difficult to give a general relation between $E_C$ and $T_S$ and there may exist
only lower bounds on $T_S$, given $E_C$. Therefore, we cannot exclude speculative source classes
with many weak but long-lived sources. The opposite case of many weak and short-lived
sources is excluded to the extent that the present explanation of the hard $\gamma$-ray spectrum
by the contribution of the SCRs is unique.

4. Summary

Our considerations suggest that the SCRs can provide an essential contribution to the
high-energy Galactic $\gamma$-ray flux. According to our estimates, depending on the parameters,
the SCR contribution is less than 10% of the GCR contribution at GeV energies and it
dominates at energies greater than 100 GeV due to its essentially harder spectrum. This
 conclusion is confirmed by calculations performed for the case when SNRs are the main
source of GCRs. At TeV energies the SCRs increase the expected $\gamma$-ray flux from the
Galactic disk by almost an order of magnitude.

The single physical parameter which determines the SCR contribution due to hadronic
interactions is the SCR confinement time $T_{SN}$. As far as the $\gamma$-ray emission due to $\pi_0$-decay
is concerned, the above conclusions are valid for $T_{SN} \sim 10^5$ yr. Since this SCR contribution
is proportional to $T_{SN}$, it would be negligible at TeV energies if $T_{SN} \lesssim 10^4$ yr. A SNR age
of $10^4$ yr typically corresponds to the intermediate Sedov phase, when the SNR shock is
still quite strong. Therefore it seems to be quite improbable that the GCRs are replenished
from SNRs at such an early phase. For the IC contribution even a ten times shorter source
life time would be sufficient at TeV energies. In fact, for the TeV IC emission the relevant
time scale is the life time $\tau_e(\epsilon_e)$ of parent SCR electrons due to their synchrotron losses,
which is indeed about $10^4$ yr. For decreasing $\gamma$-ray energies $\tau_e(\epsilon_e)$ increases beyond $10^4$ yrs,
and therefore a source life time of this magnitude would become a limiting factor.

Our conclusions remain valid for alternative classes of possible GCR sources with
comparable overall energy release and comparable individual confinement times. We note
that this contribution of the dominant GCR sources necessarily exists. As we argue, it may
be sufficient by itself to explain the observed $\gamma$-ray excess, at least in the inner Galaxy
where it is well documented, without a need to invoke additional particle populations (e.g.

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Fig. 1.— The average diffuse γ-ray spectrum of inner Galaxy (300° < l < 60°, |b| ≤ 10°). The full (dashed) line represents the calculation with SCR spectral index $\gamma_{SCR} = 2$ ($\gamma_{SCR} = 2.15$), and the dash-dotted line corresponds to the $\pi^0$ decay γ-ray spectrum produced by GCRs (Hunter et al. 1997b). EGRET data are also taken from the review paper by Hunter et al. (1997b).