We demonstrate that two spatially separated parties (Alice and Bob) can utilize shared prior quantum entanglement, and classical communications, to establish a synchronized pair of atomic clocks. In contrast to classical synchronization schemes, the accuracy of our protocol is independent of Alice or Bob’s knowledge of their relative locations or of the properties of the intervening medium.

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In the Special Theory of Relativity, there are two methods for synchronizing a pair of spatially separated clocks, A and B, which are at rest in a common inertial frame. The usual procedure is Einstein Synchronization (ES), which involves an operational line-of-sight exchange of light pulses between two observers, say Alice and Bob, who are co-located with their clocks A and B, respectively [1]. A less commonly used Clock Synchronization Protocol (CSP) is that of Eddington, namely, Slow Clock Transport (SCT). In the SCT scheme, the two clocks A and B are first synchronized locally, and then they are transported adiabatically (infinitesimally slowly) to their final separate locations in the common inertial frame [2,3]. In this paper we propose a third CSP that utilizes the resource of shared prior entanglement between the two synchronizing parties.

Our proposed method of Quantum Clock Synchronization (QCS) has features in common with Ekert’s entanglement-based quantum key-distribution protocol [4] in which Alice and Bob initially share only prior-entangled qubit pairs. The key does not exist initially but is created from the ensemble of entangled pairs through a series of measurements and classical messages. Similarly for our QCS protocol below, no actual clocks exist initially but rather only “entangled clocks” in a global state which does not evolve in time. The synchronized clocks are then extracted via the measurements and classical communications performed by Alice and Bob. In this way our QCS scheme establishes synchrony without having to transport timing information between Alice and Bob. In contrast, for the classical ES and SCT synchronization schemes, synchrony information must transmitted from Alice to Bob over some classical channel, which can limit the accuracy of the synchronization.

We first review how an atomic clock operates, in the language of quantum information theory. An atomic clock consists of an ensemble of identical two-level systems (qubits) whose temporal evolution rate is taken as the time standard. For example, in the International System of Units (SI), the unit of time is defined as the second, which is the duration of exactly 9,192,631,770 periods of oscillation corresponding to the hyperfine (radio) transition frequency for the ground-state of the Cs\(^{133}\) atom [5]. The fact that this frequency is identical for all Cs\(^{133}\) atoms, which are sufficiently isolated from the environment, allows anyone to establish a Cs\(^{133}\) time standard of comparable accuracy.

In general, any set of identical qubits may be used as the time standard in a temporal interferometer, which employs the Ramsey method of separated oscillatory fields [6]. In the language of quantum information theory this Ramsey interferometer corresponds to a simple quantum circuit with just two gates, acting on one qubit. Specifically, let us suppose that the qubit has energy eigenstates \(|0\rangle\) and \(|1\rangle\) with energy eigenvalues \(E_0 < E_1\) respectively. We introduce the dual basis \(|\text{pos}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\) and \(|\text{neg}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\) and write \(\Omega = \frac{1}{E_1 - E_0}\) which will define our unit of time. The Hadamard transform (or \(\pi/2\) pulse) is defined by the operation \(|\text{pos}\rangle \rightarrow |\text{pos}\rangle\) and \(|\text{neg}\rangle \rightarrow |\text{neg}\rangle\). The dual basis evolves in time (up to an overall unobservable phase) as

\[
|\text{pos}(t)\rangle = \frac{1}{\sqrt{2}} (e^{-i\Omega t/2}|0\rangle + e^{i\Omega t/2}|1\rangle)
\]

\[
|\text{neg}(t)\rangle = \frac{1}{\sqrt{2}} (e^{-i\Omega t/2}|0\rangle - e^{i\Omega t/2}|1\rangle)
\]

To construct a qubit atomic clock via the Ramsey method [6], we apply a Hadamard transform to an ensemble of \(N\) identical qubits all in state \(|0\rangle\) at some time \(t = 0\). This generates an ensemble of \(|\text{pos}\rangle\) states which then evolve as in eq. (1). After a time \(t\), we apply a second Hadamard transform to the evolving ensemble of qubits. This gives a final state for each qubit:

\[
|\psi_{\text{pos}}(t)\rangle = \cos(\Omega t/2)|0\rangle - i \sin(\Omega t/2)|1\rangle
\]

(For later purposes we also note that if we had started with an ensemble of \(|\text{neg}\rangle\) states we would finally get the states

\[
|\psi_{\text{neg}}(t)\rangle = -i \sin(\Omega t/2)|0\rangle + \cos(\Omega t/2)|1\rangle
\]

which is just \(\pi/2\) out of phase with \(|\psi_{\text{pos}}(t)\rangle\)). At this point we simultaneously measure each qubit in the \(|\{0\}, |1\}\rangle\) basis. The probabilities of obtaining \(|0\rangle\) or \(|1\rangle\) are given by

\[
P_0 = \frac{1}{2} (1 + \cos(\Omega t)) \quad \text{and} \quad P_1 = \frac{1}{2} (1 - \cos(\Omega t))
\]
By monitoring the oscillations of either $P_0$ or $P_1$ as a function of time we get an estimate of the clock phase $\Omega \mod 2\pi$ and hence of $t$.

We now describe our proposed QCS scheme. A fundamental ingredient will be the entangled singlet state
$$\ket{\psi_{0,0}} = \frac{1}{\sqrt{2}} (\ket{0}_A\ket{1}_B - \ket{1}_A\ket{0}_B)$$
where the subscripts refer to particles held by Alice and Bob. This singlet state is a “dark state” that does not evolve in time provided $A$ and $B$ undergo identical unitary evolutions. Indeed for any 1-qubit unitary $U$ we have $(U \otimes U)\ket{\psi_{0,0}} = (\text{det} U)\ket{\psi_{0,0}}$ so that $\ket{\psi_{0,0}}$ changes only by an overall unobservable phase. Our protocol below (slightly modified) would work equally well using the state
$$\ket{\psi_{0,0}(\delta)} = \frac{1}{\sqrt{2}} (\ket{0}_A|1\rangle_B + e^{i\delta}\ket{1}_A|0\rangle_B)$$
for any fixed $\delta$. This state still has the essential property of being constant in time i.e. invariant under $U \otimes U$ where $U$ is time evolution, diagonal in the $\{|0\rangle, |1\rangle\}$ basis (but unlike the singlet i.e. $\delta = \pi$, it is not invariant under more general $U$’s).

We imagine that Alice and Bob share an ensemble of a large number of such pairs, labelled $n = 1, 2, 3, \ldots$ where the labels are known to both Alice and Bob. We will refer to a pair of clocks in state $\ket{\psi_{0,0}}$ as an entangled pair of pre-clocks. Since $\ket{\psi_{0,0}}$ is constant in time the pre-clock pairs could be said to be “idling” – they can provide no direct timing information. We may also write $\ket{\psi_{0,0}}$ as
$$\ket{\psi_{0,0}} = \frac{1}{\sqrt{2}} (\ket{\text{pos}}_A\ket{\text{neg}}_B - \ket{\text{neg}}_A\ket{\text{pos}}_B)$$

To start the clocks at some time $t_A$, which we take to be $t = t_A = 0$ in Alice’s and Bob’s shared inertial rest frame, Alice simultaneously measures all of her pre-clock pairs in the dual basis $\{|\text{pos}\rangle, |\text{neg}\rangle\}$. Thus each pair collapses randomly and simultaneously at $A$ and $B$ into one of the following states:
$$
\begin{align*}
\ket{\psi^I} & = \ket{\text{pos}}_A\ket{\text{neg}}_B \\
\ket{\psi^II} & = \ket{\text{neg}}_A\ket{\text{pos}}_B \\
\end{align*}
$$
with equal probability $\frac{1}{2}$. Alice’s measurement has removed the entangled state’s invariance under temporal evolution, and thus the $A$ and $B$ clocks begin to evolve in time, in accordance with Eq. (1) – all starting synchronously at a time of $t = 0$ in Alice and Bob’s shared inertial frame. Comparing the clock states in $\ket{\psi^I}$ and $\ket{\psi^II}$ with the evolution in Eq. (1), we see that Alice’s measurement effectively reproduces the result of the first one-clock Hadamard transform in the Ramsey scheme. However the result here is a mixture of two equally weighted ensembles I and II. For Alice and Bob these subensembles are internally synchronised. In addition, each of Bob’s sub-ensembles is running in antisynchrony with corresponding subensemble of Alice. As a result of her measurement, Alice knows the labels belonging to the subensembles I and II but Bob is unable to distinguish them.

As the next step in our QCS protocol, Bob performs a Hadamard transform on each of his qubits, at some time $t = t_B$. Thus he will get an equal mixture of the states $\ket{\psi_{\text{neg}}(t)}$ and $\ket{\psi_{\text{pos}}(t)}$ in eqs. (2) and (3). The corresponding density matrix is $\frac{1}{2}I$, independent of $t$, so no measurement statistic can provide Bob with any timing information. For Bob to extract a clock, a classical message from Alice is required. So let us now suppose that Alice post-selects from her entire ensemble the subensemble of, say, Type-I qubits. Since the qubits are labelled, she can then tell Bob which subset of his qubits are also Type-I by broadcasting their ordinal labels (say, $n = 3, 5, 13, \ldots$) via any form of classical communiqué. Bob is then able to extract his own Type-I and Type-II subensembles. Choosing the Type-II subensemble, Bob will have a clock exactly in phase with a Type-I clock that Alice started at $t = 0$, the time of her initial measurement. Bob may either wait for Alice’s message before performing his final Hadamard transform and measurements, or alternatively, he may do these operations earlier and record the outcomes for each label value $n$. In other words, Alice and Bob now have clocks that are “ticking” in unison.

For some applications, such as satellite-based Very Long Baseline Interferometry (VLBI) [7], the fact that Alice and Bob’s clocks are phase locked up to only modulo $2\pi$ is sufficient. However, there are other applications, such as the synchronization of satellite-borne atomic clocks in the Global Positioning System (GPS) [8], where it is important to have a shared origin of time. For such applications, it is a simple matter to adapt our QCS protocol to construct a common temporal point of reference. Let us suppose that, in addition to the standard clock qubits that run at the frequency $\Omega$, Alice and Bob have an additional set of identical qubits all with a slightly shifted frequency $\Omega' = \Omega + \Delta\Omega$. For example, if $\Omega$ corresponds to the two-level hyperfine transition of the standard Cs$^{133}$ clock atom, then $\Omega'$ could correspond to the same transition in the long-lived radioactive isotope Cs$^{135}$, which is slightly different from $\Omega$ due to the very small isotope shift [9].

So now Alice and Bob have two ensembles of entangled pre-clocks – one with frequency $\Omega$ and the other with $\Omega'$ – and the entire protocol is carried out on both ensembles simultaneously. Bob is then able to extract both a Type-I and a Type-$I'$ clock of his own and observe the two evolution probabilities, $P_B = \frac{1}{2}(1 + \cos \Omega t)$ and $P'_B = \frac{1}{2}(1 + \cos \Omega' t)$. He may subtract these two signals to get a difference function $f(t)$ for $t_B = P_B(t) - P'_B(t)$, namely,
$$f(t) = \sin \left( \frac{1}{2} \Delta\Omega (t - t_B) \right) \sin \left( \frac{1}{2} (\Omega + \Omega')(t - t_B) \right)$$
having a slowly varying beat envelope of the form $e(t) = \sin(\Delta \Omega (t-t_B))/2$. If $\Delta \Omega$ is sufficiently small, Bob can now determine an origin of time in coincidence with Alice’s. Indeed if they have arranged in advance that the whole protocol is completed in time $T$ satisfying $\frac{1}{2} \Delta \Omega T < \frac{\pi}{4}$ then the slowly varying beat envelope will be in its first quarter-cycle of oscillation and Bob can uniquely determine Alice’s origin $t = t_A = 0$ by a measurement of the beat intensity.

There are several immediate applications and advantages of our QCS protocol. For example, in the GPS satellite constellation, the ability of the space-borne atomic clocks to synchronize with a master atomic clock on the ground is limited by the fluctuating refractive index of the atmosphere. The GPS essentially uses the classical ES protocol for establishing synchrony, that is, by exchanging light signals (radio waves) with the ground station. However, from Earth to Space the fluctuating index of refraction of the atmosphere causes the speed of light to vary randomly, limiting one’s ability to establish absolute distance and the resultant timing information with high accuracy. This index fluctuation error is the current limiting factor of GPS precision [8]. With our QCS prescription, the properties of the atmosphere do not matter – synchrony is established instantaneously via the quantum-entangled channel. The uncertainties induced by sending timing information through the atmosphere simply do not arise in the QCS procedure. In fact, Alice and Bob need not have exact knowledge of their relative locations. So long as they share prior entanglement, any broadcast classical message is sufficient to establish synchrony.

There is another fundamental advantage of using QCS over ES. Classical ES requires the exchange and timing of light pulses, but light is actually a quantum field. Hence the arrival time of a light pulse is itself subject to quantum fluctuations, limiting the accuracy of the ES protocol. In contrast, our QCS scheme is unaffected by this kind of noise.

We also mention that the Ramsey two-pulse temporal interferometer is isomorphic, via the SU(2) algebra, to an optical or matter-wave Mach-Zehnder interferometer (MZI). In this case, the qubit states $|0\rangle$ and $|1\rangle$ now represent the presence or absence of a single-particle Fock state incident on either the lower or upper input port of the MZI, respectively [11]. Hence, the QCS protocol may be immediately adapted to the task of phase locking a pair of spatially separated optical or atom interferometers, if we identify $\Omega t \leftrightarrow kx$, where $x$ is the interferometric path-length difference, and $k = 2\pi/\lambda$ is the particle wavenumber. We may then use essentially the same procedure given above to develop a nonlocal, Quantum Interferometer Phase-Locking protocol, allowing us to phase lock two spatially separated interferometers, which may reside at unknown relative locations. This result too has obvious applications to VLBI, as well as other forms of interferometry.

Finally we wish to point out some intriguing shortcomings and avenues for further development of our QCS protocol. We have assumed throughout that Alice and Bob are relatively at rest in a common inertial frame. But in most applications we would expect some relative motion to have taken place. If Alice and Bob initially share some $|\psi_{0,0}\rangle$ states then after some relative motions we would expect this state to develop a phase (via differing time dilation effects on the evolutions of the energy eigenstates at A and B), becoming $|\psi_{0,0}(\delta)\rangle$ with $\delta$ depending on the entire history of the relative motion. Our protocol would need to keep track of this phase too, which emerges as an extra time offset between Alice and Bob’s determined synchronisation. More fundamentally, we know of no general treatment of the concept of entanglement for particles in (relativistic) motion (e.g. for solutions of the Dirac equation) and an associated theory of local measurements. A treatment that is sufficiently encompassing for our purposes, including both internal and spacetime degrees of freedom, may need the full machinery of a relativistic quantum field theory. It would also be of great interest to consider other basic protocols of quantum information theory (such as teleportation, dense coding and entanglement purification) in the context of a fully relativistic theory allowing general relative motions. But our QCS protocol may have an especially exciting status here – the atomic clocks used in the GPS are already sufficiently accurate so that both special and general relativistic corrections must be made, and our protocol may provide an especially good starting point for the experimental investigation of the “tension” between the fundamentals of quantum measurement theory and relativity [12–14].

The current QCS protocol is also limited by the fact that it does not specify the method by which the shared prior entanglement between Alice and Bob is established. One possibility is for Alice and Bob to meet at a common location, create an ensemble of $N$ identical EPR pairs each in the state $\frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B)$ and then go their separate ways. This method might appear to be equivalent to that of classical slow clock transport (CSCT) (in which a pair of (non-entangled) clocks are synchronized at a common location at time $t = 0$ and transported slowly to remote locations) but there are essential differences. In CSCT, the clocks begin ticking in synchrony but subsequently, because they are non-entangled, their times drift with respect to one another necessitating periodic re-synchronization, after some characteristic time $t = t_{\text{drift}}$ via the Einstein synchronization (ES) protocol. Unfortunately, ES is an imperfect procedure because of the inability to measure the time of arrival of a photon precisely and the uncertainty in the refractive index of the medium separating the clocks (inducing uncertainty in the speed of light be-
to establish and maintain the necessary shared prior entanglement and consider further the problem of how these results to incorporate both special and general relativity. In future work we intend to generalize technique for phase locking remote optical or matter-wave interferometers. The procedure can also be mapped directly into a new technique for phase locking remote optical or matter-wave interferometers and our protocol has direct applications for use in very long baseline inter-referometry or noisy on the classical channel. Our protocol of the time transfer is not affected by the distance of separation or by noise on the classical channel. The two synchronizing parties may be at far-distant and unknown relative locations and the accuracy of the time transfer is not affected by the distance of separation or by noise on the classical channel. Our protocol has direct applications for use in very long baseline interferometry and remote satellite synchronization, and our protocol can also be mapped directly into a new technique for phase locking remote optical or matter-wave interferometers. In future work we intend to generalize these results to incorporate both special and general relativistic effects and consider further the problem of how to establish and maintain the necessary shared prior entanglement.

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