Visualizations of the QCD Vacuum

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Action and topological charge densities of the Euclidean-space QCD vacuum are visualized in three-dimensional slices of a $24^3 \times 36$ space-time lattice and an $O(a^2)$-improved $16^3 \times 32$ lattice. Features include instanton anti-instanton annihilation and a comparison of standard and over-improved actions used in the smoothing of the gauge fields.

1 Overview

The numerical approach to resolving the features of QCD is occasionally criticized via statements suggesting the knowledge of QCD lies “in the silicon.” What these speakers are recognizing is that there are massive amounts of data processed by today’s supercomputers in arriving at the final few bytes of information reported as QCD observables. The focus of this investigation is to further probe the features of QCD by using visualization techniques which efficiently convey the content of these massive amounts of data.

Here we will examine the properties of typical pure-gauge QCD-vacuum field configurations created on a $24^3 \times 36$ space-time lattice using the standard Wilson action at $\beta = 6.0$, which provides a lattice spacing of 0.1 fm. In addition, we consider an $O(a^2)$-improved action on a $16^3 \times 32$ lattice at $\beta = 4.38$ providing a lattice spacing of 0.17 fm. To remove the short-range noisy perturbative fluctuations, the field configurations are smoothed by a local algorithm designed to minimize the gauge action at each update. This algorithm known as cooling is a well established method for locally suppressing quantum fluctuations. The locality of the method allows topologically nontrivial field configurations to survive numerous iterations of the cooling algorithm.

Visualization techniques may be used to answer questions such as the following: To what extent do the QCD vacuum field configurations resemble a

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bSuch statements overlook the fact that one can map out the properties of QCD by exploring the quark-mass and temperature dependence of observables and by resolving the individual quark sector contributions to various observables.
randomly oriented multi-(anti)instanton field configuration. Do the instantons display a remnant of spherical symmetry expected for isolated instantons, or do the nonperturbative interactions completely distort this picture? While it has been established that anti-instanton instanton pairs have an attractive interaction and will annihilate during cooling, it is not clear how this process takes place. Do the pairs simply come together or do they wrap around each other in the process of annihilation? One can also search for visual evidence of a polarization phenomena where large sized instantons tend to have on average the same sign and are over screened by smaller instantons which tend to have the opposite topological charge of the larger instantons. Visualization techniques have also been used to examine the microscopic effects of various smoothing algorithms.

2 Visualizations

Fig. 1 illustrates a three-dimensional slice of the QCD action density from the $24^3 \times 36$ lattice after 30 cooling sweeps. Here the blue isosurface connects all points having the same action density. Tri-linear interpolation is used to smooth the surface. Volume rendering is done within the surface to illustrate changes in the action density. Outside the surface at low action densities, no volume rendering is done in order to allow one to see within the field configuration. Sharp peaks in the action density have been clipped to aid in the illustration.

The regions in which the action density has survived have non-trivial topological charge density as illustrated in Fig. 2. Here the yellow (green) isosurface connects points having equal positive (negative) topological charge density. Volume rendering of the topological charge density inside the isosurfaces illustrates the changes in the positive (negative) topological charge densities by red (blue) extremes in colour.

Here we see that the (anti)instantons that are somewhat isolated do show a remnant of spherical symmetry in the action and topological charge densities. Indeed similar plots in which the isosurface explores very high action or topological charge density show an elliptically shaped isosurface.

However, closely spaced (anti)instantons can show extreme deviations from spherical symmetry. The action and topological charge densities illustrated in the upper center of Figs. 1 and 2 correspond to instanton anti-instanton pairs in the process of annihilation. By 100 iterations of the cooling algorithm, the action and topological charge densities have largely vanished from this region.

In the process of annihilation, one can see regions in which fingers of significant

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3 Every link on the lattice is updated once during a single sweep.
Figure 1: The action density after 30 cooling sweeps.

Figure 2: The topological charge density after 30 cooling sweeps.
topological charge density are wrapping around negative topological charge density. Between these isosurfaces, the topological charge density is rapidly going to zero and therefore is not rendered. The action density simply shows a region of significant interaction.

Animations (in animated-gif format suitable for viewing within a browser) illustrating this process of annihilation are available on the web at http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/QCDvacuum/

The process of cooling smooths and extends the size of (anti)instantons such that the action and charge densities decrease. If one produces an animation with a fixed isosurface value one will see the rendered regions shrink in size and eventually disappear as the local density drops below the isosurface threshold. One would incorrectly reach the conclusion that instantons shrink during cooling and eventually fall through the lattice. These animations illustrate the local action and charge densities relative to the total action and topological charge to more correctly illustrate the distribution of action and charge on the lattice.

3 Over-Improved Cooling

It is well known that errors in the standard Wilson action eventually destroy (anti)instanton configurations. Errors in the standard Wilson action underestimate the action of a field configuration such that application of a cooling sweep on a single instanton configuration will result in an action less than the one-instanton bound. The instanton is spoiled by the cooling process and eventually the remaining action will be eliminated via the cooling procedure.

Hence we also examine the five-loop over-improved action of De Forcrand et al. designed to render instantons stable over several hundreds of sweeps. This approach includes extended planar paths combined to reproduce the classical action with no $O(a^2)$ nor $O(a^4)$ discretization errors and with coefficients fine-tuned to stabilize instantons over many applications of the algorithm.

To investigate these algorithms based on improved actions, we consider pure-gauge configurations from an $O(a^2)$-improved $16^3 \times 32$ lattice with a lattice spacing of 0.17 fm. Fig. 3 illustrates a three-dimensional slice of the action density after 30 sweeps of cooling using the standard one-loop Wilson action. The population of (anti)instantons is very sparse and the isolated instantons are quite spherical.

Fig. 4 illustrates the same gauge-field configuration, this time cooled with a parallel implementation of the five-loop over-improved action. While there is a correspondence between the instantons surviving 30 sweeps of the one-loop action and those of the five-loop action, it is clear that many more
Figure 3: The action density after 30 cooling sweeps using the standard one-loop Wilson action.

Figure 4: The action density after 30 cooling sweeps using the five-loop over-improved action.⁴
(anti)instantons have survived the improved cooling algorithm. Hence one can see the difficulty in cooling with the standard one-loop Wilson action. The density and size of (anti)instantons is dependent on the number of cooling sweeps. The true vacuum is much denser as suggested by the five-loop action density.

In short, the visualizations reveal rich structure in typical field configurations of the QCD vacuum. The action and topological charge densities presented here display long-range non-perturbative correlations between strongly interacting instantons and anti-instantons.

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References


While these facts are fairly well known, we find the visualizations illustrating these difficulties very compelling.