Cosmological constant vs. quintessence

Pierre Binétruy, LPT, Université Paris-Sud, France

Abstract

There is some evidence that the Universe is presently undergoing accelerating expansion. This has restored some credit to the scenarios with a non-vanishing cosmological constant. From the point of view of a theory of fundamental interactions, one may argue that a dynamical component with negative pressure is easier to achieve. As an illustration, the quintessence scenario is described and its shortcomings are discussed in connection with the nagging “cosmological constant problem”.

1 Cosmological constant

As is well known, the cosmological constant appears as a constant in the Einstein equations:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -8\pi G_N T_{\mu\nu} + \lambda g_{\mu\nu},$$

where $G_N$ is Newton’s constant and $T_{\mu\nu}$ is the energy-momentum tensor. The cosmological constant $\lambda$ is thus of the dimension of an inverse length.

---

1Lectures given at Les Houches summer school “The early Universe”, July 1999 and Peyresq 4 meeting.
It was introduced by Einstein [1, 2] in order to build a static universe model, its repulsive effect compensating the gravitational attraction, but, as we now see, constraints on the expansion of the Universe impose for it a very small upper value.

It is more convenient to work in the specific context of a Friedmann universe, with a Robertson metric:

\[ ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right], \]

where \( a(t) \) is the cosmic scale factor. Implementing energy conservation into the Einstein equations then leads to the Friedmann equation which gives an expression for the Hubble parameter \( H \):

\[ H^2 = \frac{\dot{a}^2(t)}{a^2(t)} = \frac{1}{3} (\lambda + 8\pi G_N \rho) - \frac{k}{a^2}, \]

where, using standard notations, \( \dot{a} \) is the time derivative of the cosmic scale factor, \( \rho = T^{00}_0 \) is the energy density and the term proportional to \( k \) is a spatial curvature term (see (2)). Note that the cosmological constant appears as a constant contribution to the Hubble parameter.

Evaluating each term of the Friedmann equation at present time allows for a rapid estimation of an upper limit on \( \lambda \). Indeed, we have \( H_0 = h_0 \times 100 \text{ km.s}^{-1}\text{Mpc}^{-1} \) with \( h_0 \) of order one, whereas the present energy density \( \rho_0 \) is certainly within one order of magnitude of the critical energy density \( \rho_c = 3H_0^2/(8\pi G_N) = h_0^2 \times 2 \times 10^{-26} \text{ kg.m}^{-3} \); moreover the spatial curvature term certainly does not represent presently a dominant contribution to the expansion of the Universe. Thus, (3) implies the following constraint on \( \lambda \):

\[ |\lambda| \leq H_0^2. \]

In other words, the length scale \( \ell_\Lambda = |\lambda|^{-1/2} \) associated with the cosmological constant must be larger than \( H_0^{-1} = h_0^{-1} \times 10^{26} \text{ m} \), and thus a macroscopic distance.

This is not a problem as long as one remains classical. Indeed, \( H_0^{-1} \) provides a natural macroscopic scale for our present Universe. The problem arises when one tries to combine gravity with the quantum theory. Indeed, from the Newton’s constant \textit{and} the Planck constant \( \hbar \) one can construct a mass scale or a length scale

\[ m_P = \sqrt{\frac{\hbar c}{8\pi G_N}} = 2.4 \times 10^{18} \text{ GeV/c}^2, \]

\[ \ell_P = \frac{\hbar}{m_P c} = 8.1 \times 10^{-35} \text{ m} \]
The above constraint now reads:
\[ \ell_\Lambda \equiv |\lambda|^{-1/2} \geq \frac{1}{H_0} \sim 10^{60} \ell_P. \] (5)

In other words, there are more than sixty orders of magnitude between the scale associated with the cosmological constant and the scale of quantum gravity.

A rather obvious solution is to take \( \lambda = 0 \). This is as valid a choice as any other in a pure gravity theory. Unfortunately, it is an unnatural one when one introduces any kind of matter. Indeed, set \( \lambda \) to zero but assume that there is a non-vanishing vacuum (i.e., groundstate) energy: \( < T_{\mu\nu} > = - \rho g_{\mu\nu} \), then the Einstein equations (1) read
\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G_N T_{\mu\nu} + 8\pi G_N < \rho > g_{\mu\nu}, \] (6)

The last term is interpreted as an effective cosmological constant:
\[ \lambda_{eff} = 8\pi G_N < \rho > \equiv \frac{\Lambda^4}{m_P^2}. \] (7)

Generically, \( < \rho > \) receives a non-zero contribution from symmetry breaking: for instance, the scale \( \Lambda \) would be typically of the order of 100 GeV in the case of the gauge symmetry breaking of the Standard Model or 1 TeV in the case of supersymmetry breaking. But the constraint (5) now reads:
\[ \Lambda \leq 10^{-30} m_P \sim 10^{-3} \text{ eV}. \] (8)

It is this very unnatural fine-tuning of parameters (in explicit cases \( < \rho > \) and thus \( \Lambda \) are functions of the parameters of the theory) that is referred to as the *cosmological constant problem*, or more accurately the vacuum energy problem.

## 2 The role of supersymmetry

If the vacuum energy is to be small, it may be easier to have it vanishing through some symmetry argument. Global supersymmetry is the obvious candidate. Indeed, the supersymmetry algebra
\[ \{ Q_r, \tilde{Q}_s \} = 2\gamma_\mu P_\mu \] (9)
yields the following relation between the Hamiltonian $H = P_0$ and the supersymmetry generators $Q_r$:

$$H = \frac{1}{4} \sum_r Q_r^2,$$

and thus the vacuum energy $\langle 0 | H | 0 \rangle$ is vanishing if the vacuum is supersymmetric ($Q_r | 0 \rangle = 0$).

Unfortunately, supersymmetry has to be broken at some scale since its prediction of equal mass for bosons and fermions is not observed in the physical spectrum. Then $\Lambda$ is of the order of the supersymmetry breaking scale, that is a few hundred GeV to a TeV.

However, the right framework to discuss these issues is supergravity i.e. local supersymmetry since locality implies here, through the algebra (9), invariance under “local” translations that are the diffeomorphisms of general relativity. In this theory, the graviton, described by the linear perturbations of the metric tensor $g_{\mu\nu}(x)$, is associated through supersymmetry with a spin 3/2 field, the gravitino $\psi_\mu$. One may write a supersymmetric invariant combination of terms in the action:

$$S = \int d^4x \sqrt{g} \left[ 3m_P^2 m_{3/2}^2 - m_{3/2} \bar{\psi}_\mu \sigma^{\mu\nu} \psi_\nu \right],$$

where $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/4$. If the first term is made to cancel the vacuum energy, then the second term is interpreted as a mass term for the gravitino. We thus see that the criterion for spontaneous symmetry breaking changes from global supersymmetry (non-vanishing vacuum energy) to local supersymmetry or supergravity (non-vanishing gravitino mass). It is somewhat a welcome news that a vanishing vacuum energy is not tied to a supersymmetric spectrum. On the other hand, we have lost the only rationale that we had to explain a zero cosmological constant.

Let us recall for future use that, in supergravity, the potential for a set of scalar fields $\phi^i$ is written in terms of the Kähler potential $K(\phi^i, \bar{\phi}^\dagger)$ (the normalisation of the scalar field kinetic terms is simply given by the Kähler metric $K_{ij} = \partial^2 K / \partial \phi^i \partial \bar{\phi}^j$) and of the superpotential $W(\phi^i)$, an holomorphic function of the fields:

$$V = e^{K/m_P^2} \left[ \left( W_i + \frac{K_i}{m_P^2} W \right) K_{i\jmath} \left( \bar{W}_j + \frac{\bar{K}_j}{m_P^2} \bar{W} \right) - \frac{3 |W|^2}{m_P^2} \right] + D \text{ terms}$$

where $K_i = \partial K / \partial \phi^i$, etc. and $K_{i\jmath}$ is the inverse metric of $K_{ij}$. Obviously, the positive definiteness of the global supersymmetry scalar potential is lost in supergravity.
3 Observational results

Over the last years, there has been an increasing number of indications that the Universe is presently undergoing accelerated expansion. This appears to be a strong departure from the standard picture of a matter-dominated Universe. Indeed, the standard equation for the conservation of energy,

$$\dot{\rho} = -3(p + \rho)H,$$

allows to derive from the Friedmann equation (3), written in the case of a universe dominated by a component with energy density $\rho$ and pressure $p$:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p).$$

(14)

Obviously, a matter-dominated ($\rho \sim 0$) universe is decelerating. One needs instead a component with a negative pressure.

A cosmological constant is associated with a contribution to the energy-momentum tensor as in (6)(7):

$$T^\mu_\nu = -\Lambda^4 \delta^\mu_\nu = (-\rho, p, p, p).$$

(15)

The associated equation of motion is therefore

$$p = -\rho.$$  

(16)

It follows from (14) that a cosmological constant tends to accelerate expansion.

The discussion of data is thus often expressed in terms of the energy density $\Lambda^4$ stored in the vacuum versus the energy density $\rho_M$ in matter fields (baryons, neutrinos, hidden matter,...). It is customary to normalize with the critical density (corresponding to a flat Universe):

$$\Omega_\Lambda = \frac{\Lambda^4}{\rho_c}, \quad \Omega_M = \frac{\rho_M}{\rho_c}, \quad \rho_c = \frac{3H^2}{8\pi G_N}.$$  

(17)

The relation

$$\Omega_M + \Omega_\Lambda = 1,$$  

(18)

a prediction of many inflation scenarios, is found to be compatible with recent Cosmic Microwave Background measurements [4]². It is striking that

²This follows from the fact that the first acoustic peak is expected at an “angular” scale $\ell \sim 200/\sqrt{\Omega_M + \Omega_\Lambda}$ [5].
independent methods based on the measurement of different observables on rich clusters of galaxies all point towards a low value of $\Omega_M \sim 1/3$ \cite{6}: mass-to-light ratio, baryon mass to total cluster mass ratio (the total baryon density in the Universe being fixed by primordial nucleosynthesis), cluster abundance. This necessarily implies a non-vanishing $\Omega_{\Lambda}$ (non-vanishing cosmological constant or a similar dynamical component).

There are indeed some indications going in this direction from several types of observational data. One which has been much discussed lately uses supernovae of type Ia as standard candles\footnote{by calibrating them according to the timescale of their brightening and fading.}. Two groups, the Supernova Cosmology Project \cite{7} and the High-$z$ Supernova Search \cite{8} have found that distant supernovae appear to be fainter than expected in a flat matter-dominated Universe. If this is to have a cosmological origin, this means that, at fixed redshift, they are at larger distances than expected in such a context and thus that the Universe is accelerating.

More precisely, the relation between the flux $f$ received on earth and the absolute luminosity $\mathcal{L}$ of the supernova depends on its redshift $z$, but also on the geometry of spacetime. Traditionally, flux and absolute luminosity are expressed on a log scale as apparent magnitude $m_B$ and absolute magnitude $M$ (magnitude is $-2.5 \log_{10}$ luminosity + constant). The relation then reads

$$m_B = 5 \log(H_0 d_L) + M - 5 \log H_0 + 25. \hspace{1cm} (19)$$

The last terms are $z$-independent, \textit{if one assumes that supernovae of type Ia are standard candles}; they are then measured by using low $z$ supernovae. The first term, which involves the luminosity distance $d_L$, varies logarithmically with $z$ up to corrections which depend on the geometry. Expanding in $z$\footnote{Of course, since supernovae of redshift $z \sim 1$ are now being observed, an exact expression \cite{9} must be used to analyze data. The more transparent form of (20) gives the general trend.}, one obtains \cite{10}:

$$H_0 d_L = cz \left[ 1 + \frac{1-q_0}{2} z + \cdots \right], \hspace{1cm} (20)$$

where $q_0 \equiv -a\ddot{a}/a^2$ is the deceleration parameter. This parameter is easily obtained from (14): in a spatially flat Universe with only matter and a cosmological constant (cf. (16)), $\rho = \rho_M + \Lambda^4$ and $p = -\Lambda^4$ which gives

$$q_0 = \Omega_M/2 - \Omega_{\Lambda}. \hspace{1cm} (21)$$

This allows to put some limit on $\Omega_{\Lambda}$ on the model considered here (see Fig. 1).
Let us note that the combination (21) is ‘orthogonal’ to the combination \( \Omega_T \equiv \Omega_M + \Omega_\Lambda \) measured in CMB experiments (see footnote preceding page). The two measurements are therefore complementary: this is sometimes referred to as ‘cosmic complementarity’.

Of course, such type of measurement is sensitive to many possible systematic effects (evolution besides the light-curve timescale correction, etc.), and this has fueled a healthy debate on the significance of present data. This obviously requires more statistics and improved quality of spectral measurements. A particular tricky systematic effect is the possible presence of dust that would dimmer supernovae at large redshift.

Other results come from gravitational lensing. The deviation of light rays by an accumulation of matter along the line of sight depends on the distance to the source [10]

\[
 r = \int_t^{t_0} \frac{dt}{a(t)} = \frac{1}{a(t_0)H_0} \left( z - \frac{1}{2} (1 + q_0)z^2 + \cdots \right) \tag{22}
\]

and thus on the cosmological parameters \( \Omega_M \) and \( \Omega_\Lambda \). As \( q_0 \) decreases (i.e. as the Universe accelerates), there is more volume and more lenses between the observer and the object at redshift \( z \). Several methods are used: abundance of multiply-imaged quasar sources [11], strong lensing by massive clusters of galaxies (providing multiple images or arcs) [12], weak lensing [13].

4 Quintessence

From the point of view of high energy physics, it is however difficult to imagine a rationale for a pure cosmological constant, especially if it is nonzero but small compared to the typical fundamental scales (electroweak, strong, grand unified or Planck scale). There should be physics associated with this form of energy and therefore dynamics. For example, in the context of string models, any dimensionful parameter is expressed in terms of the fundamental string scale \( M_s \) and vacuum expectation values of scalar fields. The physics of the cosmological constant is then the physics of the corresponding scalar fields.

Introducing dynamics generally modifies the equation of state (16) to the more general form with negative pressure:

\[
 p = w \rho, \quad w < 0. \tag{23}
\]
Let us recall that $w = 0$ corresponds to non-relativistic matter (dust) whereas $w = 1/3$ corresponds to radiation. A network of light, nonintercommuting topological defects [14, 15] on the other hand gives $w = -n/3$ where $n$ is the dimension of the defect \textit{i.e.} 1 for a string and 2 for a domain wall. Finally, the equation of state for a minimally coupled scalar field necessarily satisfies the condition $w \geq -1$.

Experimental data may constrain such a dynamical component just as it did with the cosmological constant. For example, in a spatially flat Universe with only matter and an unknown component $X$ with equation of state $p_X = w_X \rho_X$, one obtains from (14) with $\rho = \rho_M + \rho_X$, $p = w_X \rho_X$ the following form for the deceleration parameter

$$g_0 = \frac{\Omega_M}{2} + (1 + 3w_X) \frac{\Omega_X}{2},$$

(24)

where $\Omega_X = \rho_X/\rho_c$. Supernovae results give a constraint on the parameter $w_X$ as shown in Fig. 2. Similarly, gravitational lensing effects are sensitive to this new component through (22).

A particularly interesting candidate in the context of fundamental theories is the case of a scalar field $\phi$ slowly evolving in a runaway potential which decreases monotonically to zero as $\phi$ goes to infinity [16, 17, 18]. This is often referred to as \textit{quintessence}. This can be extended to the case of a very light field (pseudo-Goldstone boson) which is presently relaxing to its vacuum state [19]. We will discuss the two situations in turn.

### 4.1 Runaway quintessence

A runaway potential is frequently present in models where supersymmetry is dynamically broken. Indeed, supersymmetric theories are characterized by a scalar potential with many flat directions, \textit{i.e.} directions $\phi$ in field space for which the potential vanishes. The corresponding degeneracy is lifted through dynamical supersymmetry breaking, that is supersymmetry breaking through strong interaction effects. In some instances (dilaton or compactification radius), the field expectation value $\left< \phi \right>$ actually provides the value of the strong interaction coupling. Then at infinite $\phi$ value, the coupling effectively goes to zero together with the supersymmetry breaking effects and the flat direction is restored: the potential decreases monotonically to zero as $\phi$ goes to infinity.

\footnote{A vector field or any field which is not a Lorentz scalar must have settled down to a vanishing value. Otherwise, Lorentz invariance would be spontaneously broken.}
Dynamical supersymmetry breaking scenarios are often advocated because they easily yield the large scale hierarchies necessary in grand unified or superstring theories in order to explain the smallness of the electroweak scale with respect to the fundamental scale. Let us take the example of supersymmetry breaking by gaugino condensation in effective superstring theories. The value $g_0$ of the gauge coupling at the string scale $M_s$ is provided by the vacuum expectation value of the dilaton field $s$ (taken to be dimensionless by dividing by $m_P$) present among the massless string modes: $g_0^2 = <s>^{-1}$. If the gauge group has a one-loop beta function coefficient $b$, then the running gauge coupling becomes strong at the scale

$$A \sim M_s e^{-1/2b g_0^2} = M_s e^{-s/2b}.$$ (25)

At this scale, the gaugino fields are expected to condense. Through dimensional analysis, the gaugino condensate $<\lambda \lambda>$ is expected to be of order $A^3$. Terms quadratic in the gaugino fields thus yield in the effective theory below condensation scale a potential for the dilaton:

$$V \sim |<\lambda \lambda>|^2 \propto e^{-3s/b}.$$ (26)

The $s$-dependence of the potential is of course more complicated and one usually looks for stable minima with vanishing cosmological constant. But the behavior (26) is characteristic of the large $s$ region and provides a potential slopping down to zero at infinity as required in the quintessence solution. A similar behavior is observed for moduli fields whose $vev$ describes the radius of the compact manifolds which appear from the compactification from 10 or 11 dimensions to 4 in superstring theories [20].

Let us take therefore the example of an exponentially decreasing potential. More explicitly, we consider the following action

$$S = \int d^4 x \sqrt{g} \left[ -\frac{m_P^2}{2} R - \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi) \right],$$ (27)

which describes a real scalar field $\phi$ minimally coupled with gravity and the self-interactions of which are described by the potential:

$$V(\phi) = V_0 e^{-\lambda \phi/m_P},$$ (28)

where $V_0$ is a positive constant.

The energy density and pressure stored in the scalar field are respectively:

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi).$$ (29)
We will assume that the background (matter and radiation) energy density $\rho_B$ and pressure $p_B$ obey a standard equation of state

$$p_B = w_B \rho_B.$$  

(30)

If one neglects the spatial curvature ($k \sim 0$), the equation of motion for $\phi$ simply reads

$$\ddot{\phi} + 3H \dot{\phi} = -\frac{dV}{d\phi},$$  

(31)

with

$$H^2 = \frac{1}{3m_P^2}(\rho_B + \rho_\phi).$$  

(32)

This can be rewritten as

$$\dot{\rho}_\phi = -3H \dot{\phi}^2.$$  

(33)

We are looking for scaling solutions i.e. solutions where the $\phi$ energy density scales as a power of the cosmic scale factor: $\rho_\phi \propto a^{-n_\phi}$ or $\dot{\rho}_\phi/\rho_\phi = -n_\phi H$.

In this case, one easily obtains from (29) and (33) that the $\phi$ field obeys a standard equation of state

$$p_\phi = w_\phi \rho_\phi,$$  

(34)

with

$$w_\phi = \frac{n_\phi}{3} - 1.$$  

(35)

Hence

$$\rho_\phi \propto a^{-3(1+w_\phi)}.$$  

(36)

If one can neglect the background energy $\rho_B$, then (32) yields a simple differential equation for $a(t)$ which is solved as:

$$a \propto t^{2/[3(1+w_\phi)]}.$$  

(37)

Since $\dot{\phi}^2 = (1 + w_\phi)\rho_\phi$, one deduces that $\phi$ varies logarithmically with time. One then easily obtains from (31,32) that

$$\phi = \phi_0 + \frac{2}{\lambda} \ln(t/t_0).$$  

(38)

and$^6$

$$w_\phi = \frac{\lambda^2}{3} - 1,$$  

(39)

$^6$under the condition $\lambda^2 \leq 6$ ($w_\phi \leq 1$ since $V(\phi) \geq 0$).
It is clear from (39) that, for \( \lambda \) sufficiently small, the field \( \phi \) can play the role of quintessence.

But the successes of the standard big-bang scenario indicate that clearly \( \rho_\phi \) cannot have always dominated: it must have emerged from the background energy density \( \rho_B \). Let us thus now consider the case where \( \rho_B \) dominates. It turns out that the solution just discussed with \( \rho_\phi \gg \rho_B \) and (39) is a late time attractor \([21]\) only if \( \lambda^2 < 3(1 + w_B) \). If \( \lambda^2 > 3(1 + w_B) \), the global attractor turns out to be a scaling solution \([16, 22, 23]\) with the following properties:\footnote{See ref.[24] for the case where the scalar field is non-minimally coupled to gravity.}

\[
\Omega_\phi \equiv \frac{\rho_\phi}{\rho_\phi + \rho_B} = \frac{3}{\lambda^2}(1 + w_B) \tag{40}
\]

\[
w_\phi = w_B \tag{41}
\]

The second equation (41) clearly indicates that this does not correspond to a quintessence solution (23).

The semi-realistic models discussed earlier tend to give large values of \( \lambda \) and thus the latter scaling solution as an attractor. For example, in the case (26) where the scalar field is the dilaton, \( \lambda = 3/b \) with \( b = C/(16\pi^2) \) and \( C = 90 \) for a \( E_8 \) gauge symmetry down to \( C = 9 \) for \( SU(3) \). Moreover [23], on the observational side, the condition that \( \rho_\phi \) should be subdominant during nucleosynthesis (in the radiation-dominated era) imposes to take rather large values of \( \lambda \). Typically requiring \( \rho_\phi/(\rho_\phi + \rho_B) \) to be then smaller than 0.2 imposes \( \lambda^2 > 20 \).

Although not quintessence, such attractor models with a fixed fraction \( \Omega_\phi \) as in (40) have interest of their own [23], in particular for structure formation if \( \lambda \in [5, 6] \). It has been proposed recently [25] to make the prefactor \( V_0 \) in (28) a trigonometric function in \( \phi \). This allows for some modulation around the previous attractor in an approximately oscillatory way: \( \Omega_\phi \) could then have been small at the time of nucleosynthesis and be much larger at present times. Finally, very recently [26], such models have been coupled to a system of a Brans-Dicke field and a dynamical field characterizing the cosmological constant, with a diverging kinetic term, to provide a relaxation mechanism for the cosmological constant [27].

Ways to obtain a quintessence component have been proposed however. Let me sketch some of them in turn.

One is the notion of \textit{tracker field}\footnote{Somewhat of a misnomer since in this solution, as we see below, the field \( \phi \) energy density tracks the radiation-matter energy density before overcoming it (in contradistinction with (40)). One should rather describe it as a \textit{transient tracker field}.} [28]. This idea also rests on the ex-
istence of scaling solutions of the equations of motion which play the role of late time attractors, as illustrated above. An example is provided by a scalar field described by the action (27) with a potential

$$V(\phi) = \lambda \frac{\Lambda^{4+\alpha}}{\phi^\alpha}$$

with \(\alpha > 0\). In the case where the background density dominates, one finds an attractor scaling solution \([17, 29, 30, 24]\) \(\phi \propto a^{3(1+w_B)/(2+\alpha)}\), \(\rho_\phi \propto a^{-3\alpha(1+w_B)/(2+\alpha)}\). Thus \(\rho_\phi\) decreases at a slower rate than the background density \((\rho_B \propto a^{-3(1+w_B)})\) and tracks it until it becomes of the same order at a given value \(a_Q\). More precisely \([20]\):

$$\phi = m_P \sqrt{\frac{\alpha(2+\alpha)}{3(1+w_B)}} \left( \frac{a}{a_Q} \right)^{3(1+w_B)/(2+\alpha)},$$

$$\rho_\phi \sim \frac{\lambda \Lambda^{4+\alpha}}{m_P^2} \left( \frac{a}{a_Q} \right)^{-3\alpha(1+w_B)/(2+\alpha)}.$$  \(44\)

One finds

$$w_\phi = -1 + \frac{\alpha(1+w_B)}{2+\alpha}.$$  \(45\)

Shortly after \(\phi\) has reached for \(a = a_Q\) a value of order \(m_P\), it satisfies the standard slowroll conditions:

$$m_P |V'|/V| \ll 1,$$  \(46\)

$$m_P^2 |V''/V| \ll 1,$$  \(47\)

and therefore (45) provides a good approximation to the present value of \(w_\phi\). Thus, at the end of the matter-dominated era, this field may provide the quintessence component that we are looking for.

Two features are particularly interesting in this respect. One is that this scaling solution is reached for rather general initial conditions, i.e. whether \(\rho_\phi\) starts of the same order or much smaller than the background energy density \([28]\). The second deals with the central question in this game: why is the \(\phi\) energy density (or in the case of a cosmological constant, the vacuum energy density) emerging now? Since \(\phi\) is of order \(m_P\) in this scenario, it can be rephrased here into the following: why is \(V(m_P)\) of the order of the critical energy density \(\rho_c\)? Using (44), this amounts to a constraint on the parameters of the theory:

$$\Lambda \sim \left( H_0^2 m_P^{2+\alpha} \right)^{1/(4+\alpha)}.$$  \(48\)
For example, this gives for $\alpha = 2$, $\Lambda \sim 10$ MeV, not such an unnatural value.

Let us note here the key difference between this tracking scenario and the preceding one\(^9\). Whereas the exponential potential model accounts for a fixed fraction $\Omega_\phi$ in the attractor solution (and thus $\phi$ is a tracker in the strict sense), the final attractor in the tracker field solution corresponds to scalar field dominance ($\Omega_\phi \sim 1$). It is the scale $\Lambda$ which allows to regulate the time at which the scalar field starts to emerge and makes it coincide with present time. The welcome property is that the required value for $\Lambda$ falls in a reasonable range from a high energy physics point of view. On the other hand, we will see below that the fact that the present value for $\phi$ is of order $m_P$ is a source of problems.

Models of dynamical supersymmetry breaking easily provide a model of the type just discussed [20]. Let us consider supersymmetric QCD with gauge group $SU(N_c)$ and $N_f < N_c$ flavors, i.e. $N_f$ quarks $Q^i$ (resp. antiquarks $\bar{Q}_i$), $i = 1 \cdots N_f$, in the fundamental $N_c$ (resp. anti-fundamental $\bar{N}_c$ of $SU(N_c)$). At the scale of dynamical symmetry breaking $\Lambda$ where the gauge coupling becomes strong\(^{10}\), boundstates of the meson type form: $\Pi_{ij} = Q_i \bar{Q}_j$. The dynamics is described by a superpotential which can be computed non-perturbatively using standard methods:

$$W = (N_c - N_f) \frac{\Lambda^{(3N_c-N_f)/(N_c-N_f)}}{(\det \Pi)^{1/(N_c-N_f)}}. \quad (49)$$

Such a superpotential has been used in the past but with the addition of a mass or interaction term (i.e. a positive power of $\Pi$) in order to stabilize the condensate. One does not wish to do that here if $\Pi$ is to be interpreted as a runaway quintessence component. For illustration purpose, let us consider a condensate diagonal in flavor space: $\Pi_{ij} \equiv 2 \delta_{ij}$ (see [31] for a more complete analysis). Then the potential for $\phi$ has the form (42), with $\alpha = 2(N_c + N_f)/(N_c - N_f)$. Thus,

$$w_\phi = -1 + \frac{N_c + N_f}{2N_c}(1 + w_B), \quad (50)$$

which clearly indicates that the meson condensate is a potential candidate for a quintessence component.

Another possibility for the emergence of the quintessence component out of the background energy density might be attributed to the presence

---

\(^{9}\)I wish to thank M. Joyce for discussions on this point.

\(^{10}\)It is given by an expression such as (25) where $g_0$ is the value of the gauge coupling at the large scale $M_s$ and $b$ the one-loop beta function coefficient for gauge group $SU(N_c)$.
of a local minimum (a “bump”) in the potential $V(\phi)$: when the field $\phi$ approaches it, it slows down and $\rho_\phi$ decreases more slowly ($n_\phi$ being much smaller as $w_\phi$ temporarily becomes closer to $-1$, cf. (35)). If the parameters of the potential are chosen carefully enough, this allows the background energy density, which scales as $a^{-3(1+w_B)}$ to become subdominant. The value of $\phi$ at the local minimum provides the scale which allows to regulate the time at which this happens. This approach can be traced back to the earlier work of Waiterich [16] and has recently been advocated by Albrecht and Skordis [32] in the context of an exponential potential. They argue quite sensibly that, in a “realistic” string model, $V_0$ in (28) is $\phi$-dependent: $V_0(\phi)$. This new field dependence might be such as to generate a bump in the scalar potential and thus a local minimum. Since

$$
\frac{1}{V} \frac{dV}{d\phi} = \frac{V_0'(\phi)}{V_0(\phi)} - \frac{\lambda}{m_P},
$$

it suffices that $m_P V_0'(\phi)/V_0(\phi)$ becomes temporarily larger than $\lambda$ in order to slowdown the redshift of $\rho_\phi$: once $\rho_\phi$ dominates, an attractor scaling solution of the type (38,39) is within reach, if $\lambda$ is not too large. As pointed out by Albrecht and Skordis, the success of this scheme does not require very small couplings.

One may note that, in the preceding model, one could arrange the local minimum in such a way as to completely stop the scalar field, allowing for a period of true inflation [33]. The last possibility that I will discuss goes in this direction one step further. It is known under several names: deflation [34], kination [35, 36], quintessential inflation [37]. It is based on the remark that, if a field $\phi$ is to provide a dynamical cosmological constant under the form of quintessence, it is a good candidate to account for an inflationary era where the evolution is dominated by the vacuum energy. In other words, are the quintessence component and the inflaton the same unique field?

In this kind of scenario, inflation (where the energy density of the Universe is dominated by the $\phi$ field potential energy) is followed by reheating where matter-radiation is created by gravitational coupling during an era where the evolution is driven by the $\phi$ field kinetic energy (which decreases as $a^{-6}$). Since matter-radiation energy density is decreasing more slowly, this turns into a radiation-dominated era until the $\phi$ energy density eventually emerges as in the quintessence scenarios described above.

Finally, it is worth mentioning that, even though the models discussed above all have $w_\phi \geq -1$, models with lower values of $w_\phi$ may easily be constructed. One may cite models with non-normalized scalar field kinetic terms [38], or simply models with non-minimally coupled scalar fields [39].
Indeed, it has been argued by Caldwell [40] that such a “phantom” energy density component fits well the present observational data.

4.2 Pseudo-Goldstone boson

There exists a class of models [19] very close in spirit to the case of runaway quintessence: they correspond to a situation where a scalar field has not yet reached its stable groundstate and is still evolving in its potential.

More specifically, let us consider a potential of the form:

$$V(\phi) = M^4 v \left( \frac{\phi}{f} \right),$$

where $M$ is the overall scale, $f$ is the vacuum expectation value $<\phi>$ and the function $v$ is expected to have coefficients of order one. If we want the potential energy of the field (assumed to be close to its vev $f$) to give a substantial fraction of the energy density at present time, we must set

$$M^4 \sim \rho_c \sim H_0^2 m_P^2.$$  \hspace{1cm} (53)

However, requiring that the evolution of the field $\phi$ around its minimum has been overdamped by the expansion of the Universe until recently imposes

$$m_\phi^2 = \frac{1}{2} V''(f) \sim \frac{M^4}{f^2} \leq H_0^2.$$ \hspace{1cm} (54)

Let us note that this is one of the slowroll conditions familiar to the inflation scenarios.

From (53) and (54), we conclude that $f$ is of order $m_P$ (as the value of the field $\phi$ in runaway quintessence) and that $M \sim 10^{-3}$ eV (not surprisingly, this is the scale $\Lambda$ typical of the cosmological constant, see (8)). The field $\phi$ must be very light: $m_\phi \sim h_0 \times 10^{-60} m_P \sim h_0 \times 10^{-33}$ eV. Such a small value is only natural in the context of an approximate symmetry: the field $\phi$ is then a pseudo-Goldstone boson.

A typical example of such a field is provided by the axion field (QCD axion [41] or string axion [42]). In this case, the potential simply reads:

$$V(\phi) = M^4 [1 + \cos(\phi/f)].$$ \hspace{1cm} (55)

5 Quintessential problems

However appealing, the quintessence idea is difficult to implement in the context of realistic models [43, 44]. The main problem lies in the fact that
the quintessence field must be extremely weakly coupled to ordinary matter. This problem can take several forms:

- we have assumed until now that the quintessence potential monotonically decreases to zero at infinity. In realistic cases, this is difficult to achieve because the couplings of the field to ordinary matter generate higher order corrections that are increasing with larger field values, unless forbidden by a symmetry argument. For example, in the case of the potential (42), the generation of a correction term \( \lambda d \, m_P^{4-d} \phi^d \) puts in jeopardy the slowroll constraints on the quintessence field, unless very stringent constraints are imposed on the coupling \( \lambda d \). But one typically expects from supersymmetry breaking \( \lambda d \sim M_S^2 / m_P^4 \) where \( M_S \) is the supersymmetry breaking scale [44].

Similarly, because the vev of \( \phi \) is of order \( m_P \), one must take into account the full supergravity corrections. One may then argue [45] that this could put in jeopardy the positive definiteness of the scalar potential, a key property of the quintessence potential. This may point towards models where \( < W > = 0 \) (but not its derivatives, see (12)) or to no-scale type models: in the latter case, the presence of 3 moduli fields \( T^i \) with Kähler potential \( K = -\sum_i \ln(T^i + T^i) \) cancels the negative contribution \(-3|W|^2\) in (12).11

- the quintessence field must be very light [43]. If we return to our example of supersymmetric QCD in (42), \( V''(m_P) \) provides an order of magnitude for the mass-squared of the quintessence component:

\[
m_{\phi} \sim \Lambda \left( \frac{\Lambda}{m_P} \right)^{1+\alpha/2} \sim H_0 \sim 10^{-33} \text{ eV}. \tag{56}
\]

using (48). Similarly, we have seen that the mass of a pseudo-Goldstone boson that could play the rôle of quintessence is typically of the same order. This field must therefore be very weakly coupled to matter; otherwise its exchange would generate observable long range forces. Eötvös-type experiments put very severe constraints on such couplings.

Again, for the case of supersymmetric QCD, higher order corrections to the Kähler potential of the type

\[
\kappa(\phi, \phi^\dagger) \left[ \beta_{i\bar{j}} \left( \frac{Q_i Q^\dagger}{m_P^2} \right) + \bar{\beta}_{i\bar{j}} \left( \frac{Q^\dagger \bar{Q}}{m_P^2} \right) \right] \tag{57}
\]

11 Moreover, supergravity corrections may modify some of the results. For example, the presence of a (flat) Kähler potential in (12) induces exponential field-dependent factors. A more adequate form for the inverse power law potential (42) is thus [45] \( V(\phi) = \lambda e^{\phi^2/2m_P^2} \Lambda^{4+\alpha/\phi^\alpha} \). The exponential factor is not expected to change much the late time evolution of the quintessence energy density. Brax and Martin [45] argue that it changes the equation of state through the value of \( w_\phi \).
will generate couplings of order 1 to the standard matter fields $\phi_i$, $\phi_j^\dagger$ since $<Q>$ is of order $m_P$. In order to alleviate this problem, Masiero, Pietroni and Rosati [31] have proposed a solution much in the spirit of the least coupling principle of Damour and Polyakov [46]: the different functions $\beta_{ij}$ have a common minimum close to the value $<Q>$, which is most easily obtained by assuming “flavor” independence of the functions $\beta_{ij}$. This obviously eases the Eötvös experiment constraints. In the early stages of the evolution of the Universe, when $Q \ll m_P$, couplings of the type (57) generate a contribution to the mass of the $Q$ field which, being proportional to $H$, does not spoil the tracker solution.

- it is difficult to find a symmetry that would prevent any coupling of the form $\beta(\phi/m_P)^n F_{\mu\nu} F_{\mu\nu}$ to the gauge field kinetic term. Since the quintessence behavior is associated with time-dependent values of the field of order $m_P$, this would generate, in the absence of fine tuning, corrections of order one to the gauge coupling. But the time dependence of the fine structure constant for example is very strongly constrained [47]: $|\dot{\alpha}/\alpha| < 5 \times 10^{-17}\text{yr}^{-1}$. This yields a limit [43]:

$$|\beta| \leq 10^{-6} m_P H_0 < \phi >,$$

(58)

where $< \dot{\phi} >$ is the average over the last $2 \times 10^9$ years.

A possible solution is to implement an approximate continuous symmetry of the type: $\phi \rightarrow \phi^+ + \text{constant}$ [43]. This symmetry must be approximate since it must allow for a potential $V(\phi)$. Such a symmetry would only allow derivative couplings, an example of which is an axion-type coupling $\tilde{\beta}(\phi/m_P) F_{\mu\nu} \tilde{F}_{\mu\nu}$. If $F_{\mu\nu}$ is the color $SU(3)$ field strength, QCD instantons yield a mass of order $\tilde{\beta} \Lambda_{QCD}^2 / m_P$, much too large to satisfy the preceding constraint. In any case, supersymmetry would relate such a coupling to the coupling $\beta(\phi/m_P) F_{\mu\nu} F_{\mu\nu}$ that we started out to forbid.

The very light mass of the quintessence component points towards scalar-tensor theories of gravity, where such a dilaton-type (Brans-Dicke) scalar field is found. This has triggered some recent interest for this type of theories. Attractor scaling solutions have been found for non-minimally coupled fields [24, 48]. However, as discussed above, one problem is that scalar-tensor theories lead to time-varying constants of nature. One may either put some limit on the couplings of the scalar field [49] or use the attractor mechanism towards General Relativity that was found by Damour and Nordtvedt [50, 46]. This mechanism exploits the stabilisation of the dilaton-type scalar
through its conformal coupling to matter. Indeed, assuming that this scalar field $\phi$ couples to matter through an action term $S_m(\psi_m, f(\phi)g_{\mu\nu})$, then its equation of motion takes the form:

$$\frac{2}{3 - \phi^2} \phi'' + (1 - w_B)\phi' = -(1 - 3w_B)\frac{d\ln f(\phi)}{d\phi},$$

(59)

where $\phi' = d\phi/d\ln a$. This equation can be interpreted [50] as the motion of a particle of velocity-dependent mass $2/(3 - \phi^2)$ subject to a damping force $(1 - w_B)\phi'$ in an external force deriving from a potential $v_{\text{eff}}(\phi) = (1 - 3w_B)\ln f(\phi)$. If this effective potential has a minimum, the field quickly settles there. Bartolo and Pietroni [51] have recently proposed a model of quintessence (they add a potential $V(\phi)$) using this mechanism: the quintessence component is first attracted to General Relativity and then to a standard tracker solution.

Scalar-tensor theories of gravity naturally arise in the context of higher-dimensional theories and we will return to such scenarios in the next section where we discuss these theories.

All the preceding shows that there is extreme fine tuning in the couplings of the quintessence field to matter, unless they are forbidden by some symmetry. This is somewhat reminiscent of the fine tuning associated with the cosmological constant. In fact, the quintessence solution does not claim to solve the cosmological constant (vacuum energy) problem described above. If we take the example of a supersymmetric theory, the dynamical cosmological constant provided by the quintessence component clearly does not provide enough amount of supersymmetry breaking to account for the mass difference between scalars (sfermions) and fermions (quarks and leptons): at least 100 GeV. There must be other sources of supersymmetry breaking and one must fine tune the parameters of the theory in order not to generate a vacuum energy that would completely drown $\rho_\phi$.

In any case, the quintessence solution shows that, once this fundamental problem is solved, one can find explicit fundamental models that effectively provide the small amount of cosmological constant that seems required by experimental data.

6 Extra spacetime dimensions

The old idea of Kaluza and Klein about compact extra dimensions has received a new twist with the realisation, motivated by string theory [52], that such extra dimensions may only be felt by gravitational interactions [53]. In
other words, our 4-dimensional world of quarks, leptons and gauge interactions may constitute a hypersurface in a higher-dimensional Universe. Such a hypersurface is called a brane in modern jargon: certain types of branes (Dirichlet branes) appear as solitons in open string theories [54]. In what follows, we will mainly consider 4-dimensional branes to which are confined observable matter as well as standard non-gravitational gauge interactions. The part of the Universe which is not confined to the brane is called the bulk (which for simplicity we will take to be 5-dimensional).

In this framework, the very notion of a cosmological constant takes a new meaning and there has been recently a lot of activity to try to unravel it. The hope is that the cosmological constant problem itself may receive a different formulation, easier to deal with.

If we think of the cosmological constant as some vacuum energy, one has the choice to add it to the brane or to the bulk. The consequences are quite different:

- If we introduce a vacuum energy $\lambda_5 > 0$ on the brane, it creates a repulsive gravitational force outside (i.e. in the bulk). Indeed, a result originally obtained by Ipser and Sikivie [55] in the case of a domain wall may be adapted here as follows: let $p$ and $\rho$ be the pressure and energy density on the brane, then if $\rho + 3p$ is positive (resp. negative), a test body may remain in the bulk stationary to the brane if it accelerates away from (resp. towards) the brane.\(^{12}\) In the case of a positive cosmological constant, $\rho = -p = \lambda_5$ and $\rho + 3p = -2\lambda_5 < 0$.

Projected back to our 4-dimensional brane-world, this yields a different behaviour from the one seen in a standard 4-dimensional world. For example, the vacuum energy contributes to the Hubble parameter describing the expansion of the brane world in a (non-standard) quadratic way [56]: $H^2 = \frac{\lambda_5^2}{(36M^6)} + \cdots$, where $M$ is the fundamental 5-dimensional scale.

- If we introduce a vacuum energy $\lambda_B$ in the 5-dimensional bulk (this $\lambda_B$ is then of mass dimension 5), this will induce a potential for the modulus field whose vev measures the radius of the compact dimension. Let us call for simplicity $R$ this modulus, which is often referred to as the radion. Then in the case of a single compact dimension, $V(R) = \lambda_B R$ [57]. The contribution of this bulk vacuum energy to the square of the Hubble parameter on the brane is standard (linear) : $H^2 = \frac{\lambda_B}{(6M^3)} + \cdots$

\(^{12}\)One may note that, if the expansion in the brane is standard, then, according to (14), the expansion in the brane is decelerating (resp. accelerating).
Allowing both types of vacuum energies allows to construct static solutions with a cancelling effect in the bulk. Indeed, if one imposes the condition:

$$\lambda_B = -\frac{\lambda_b^2}{6M^3}, \quad (60)$$

the effective 4-dimensional cosmological constant, i.e. the constant term in the Hubble parameter $H$, vanishes.

A striking property of this type of configuration is that it allows to localize gravity on the brane. This is the so-called Randall-Sundrum scenario [58] (see also [59] for earlier works). The 5-dimensional Einstein equations are found to allow for a 4-dimensionally flat solution with a warp factor (i.e. an overall fifth dimension-dependent factor in front of the four-dimensional metric):

$$ds^2 = e^{-y/3M^3} \eta_{\mu \nu} dx^\mu dx^\nu + dy^2 \quad (61)$$

if the condition (60) is satisfied. Let us note that this condition ensures that the bulk is anti-de Sitter since $\lambda_B < 0$. If $\lambda_b > 0$, one finds a single non-normalisable massless mode of the metric which is interpreted as the massless 4-dimensional graviton. The wave function of this mode turns out to be localized close to the brane, which gives an explicit realisation of 4-dimensional gravity trapping. There is also a continuum of non-normalisable massive modes (starting from zero mass) which are interpreted as the Kaluza-Klein graviton modes.

Of course, the Randall-Sundrum condition (60) is another version of the standard fine tuning associated with the cosmological constant. One would like to find a dynamical justification to it.

Some progress has recently been made in this direction [60, 61]. The presence of a scalar field in the bulk, conformally coupled to the matter on the brane allows for some relaxation mechanism that screens the 4-dimensional cosmological constant from corrections to the brane vacuum energy. Let us indeed consider such a scalar field, of the type discussed above in the context of scalar-tensor theories. The action is of the following form:

$$S = \int d^5x \sqrt{g^{(5)}} \left[ \frac{M^3}{2} R^{(5)} - \frac{1}{2} \partial^N \phi \partial_N \phi - V(\phi) \right] + S_m(\psi_m, g_{\mu \nu} f(\phi)) \quad (62)$$

where the fields $\psi_m$ are matter fields localized on the brane, located at $y = 0$, and $g_{\mu \nu}$ is the 4-dimensional metric ($N$ are 5-dimensional indices whereas $\mu$, $\nu$ are 4-dimensional indices). We will be mostly interested in
the 4-dimensional vacuum energy so that we can write the 4-dimensional matter action as:

\[ S_m = - \int d^4x \sqrt{g^{(4)}} \lambda_b f^2(\phi). \] (63)

Five-dimensional Einstein equations projected on the brane, provide the following Friedmann equation:

\[ H^2 = \frac{1}{18M^6} \lambda_b^2 f^2(\phi) \left[ f^2(\phi) - 3M^2 f'^2(\phi) \right] + \frac{1}{3} V(\phi). \] (64)

The other equations, including the \( \phi \) equation of motion, ensure that this vanishes, irrespective of the precise value of \( \lambda_b \), for the following metric:

\[ ds^2 = e^{-\alpha(y)} dx^\mu dx^\nu + dy^2, \] (65)

where the derivative of the function \( \alpha(y) \) with respect to \( y \) is fixed on the brane by junction conditions (assuming a symmetry \( y \to -y \))

\[ \alpha'(0) = \frac{\lambda}{3M^5} f^2(\phi)|_{y=0}. \] (66)

In other words, the cosmological constant is, to a first order, not sensitive to the corrections to the vacuum energy coming from the Standard Model interactions.

For specific values of the potential, such a dynamics localizes the gravity around the brane. For example, with vanishing potential, the solution of the equations is obtained for

\[ f(\phi) = e^{\phi/(M\sqrt{3})}. \] (67)

One obtains a flat 4-dimensional spacetime (indeed, in this case, this is the unique solution [60]) although the vacuum energy may receive non-vanishing corrections.

The price to pay is the presence of a singularity close to the brane. It remains to be seen what is the interpretation of this singularity, how it should be treated and whether this reintroduces fine tuning [62, 63, 64]. Also a full cosmological treatment, \textit{i.e.} including time dependence, is needed.

Presumably, supersymmetry plays an important role in this game if one wants to deal with stable solutions. Supersymmetry indeed may prove to be in the end the rationale for the vanishing of the cosmological constant. The picture that would emerge would be one of a supersymmetric bulk with vanishing cosmological constant and with supersymmetry broken on the brane (remember that supersymmetry is related to translational invariance)
Models along these lines have been discussed recently by Gregory, Rubakov and Sibiryakov [66]: the four-dimensional gravity is localized on the brane due to the existence of an unstable graviton boundstate. Presumably in such models one does not recover the standard theory of gravity.

7 Conclusion

The models discussed above are many. This is not a surprise since the cosmological constant problem, although it has attracted theorists for decades, has not received yet a convincing treatment. What is new is that one expects in a not too distant future a large and diversified amount of observational data that should allow to discriminate among these models. One may mention the MAP and PLANCK satellites on the side of CMB measurements. The SNAP mission should provide, on the other hand, large numbers of type Ia supernovae which should allow a better handle on this type of measurements and a significant increase in precision. But other methods will also give complementary information: lensing, galaxy counts [67], gravitational wave detection [68, 39], ...

Acknowledgments:

I wish to thank Christophe Grojean, Michael Joyce, Reynald Pain, James Rich and Jean-Philippe Uzan for discussions and valuable comments on the manuscript. I thank the Theory Group of Lawrence Berkeley National Lab where I found ideal conditions to finish writing these lecture notes.
References


[65] H. Verlinde, [hep-th/0004003].


26
Figure 1: Best-fit coincidence regions in the $\Omega_M - \Omega_\Lambda$ plane, based on the analysis of 42 type Ia supernovae discovered by the Supernova Cosmology Project [7].
Figure 2: Best fit coincidence regions in the $\Omega_M - w_X$ plane for an additional energy density component $\Omega_X$ with equation of state $w_X = p_X/\rho_X$, assuming flat cosmology ($\Omega_M + \Omega_X = 1$); based on the same analysis [7] as in Fig.1.