\[ \int_{x_1}^{x_2} \left( x^2 - (x')^2 \right) \, dp = \int_{0}^{\infty} (x^2 - (x')^2) \, dp \]

In the aforementioned literature, it is clear that the non-analytic chi-squared hypothesis of the proton with a non-normalized field of operators in the vacuum is a critical aspect of the non-analytic behavior of the proton. Specifically, we find that a non-analytic behavior of the proton is a key feature in the field of operators in the vacuum.

In this letter, we establish the first time the non-analytic non-normalization of the proton. The non-analytic behavior of the proton is a key feature in the field of operators in the vacuum.

Abstract

Dynamical Symmetry Breaking in the Sea of the Proton

JLAB-THY-00-11
ADP-00-02/TH-04
where \( g_A \) is the axial charge of the nucleon (understood to be taken in the chiral SU(2) limit, \( m \to 0 \)), and \( \mu \) is a mass parameter. This result also generalizes to higher moments, each of which has a non-analytic component, so that the \( d - \bar{n} \) distribution itself, as a function of \( x \), has a model-independent, LNA component. The presence of non-analytic terms indicates that Goldstone bosons play a role which cannot be cancelled by any other physical process (except by chance at a particular value of \( m_\pi \)). Such insight is vital when it comes to building models and developing physical understanding of a system.

In deep-inelastic scattering the one-pion loop contribution to the \( n \)-th moment of the \( d(x) - \bar{n}(x) \) difference is given by [2,5]:

\[
(\langle \bar{T} - T \rangle^{(n)}) = \int_0^1 dx \; x^n \; (\bar{d}(x) - \bar{n}(x)) = \frac{2}{3} \; V^{(n)}_{\pi} \cdot \frac{f^{(n)}_{\pi N}}{Q^N},
\]

where \( V^{(n)}_{\pi} \) is the \( n \)-th moment of the valence pion structure function\(^1\), and \( f^{(n)}_{\pi N} \) is the \( n \)-th moment of the pion distribution function in the nucleon (or the \( N \to \pi N \) splitting function):

\[
f^{(n)}_{\pi N} = \int_0^1 dy \; y^n \; f_{\pi N}(y).
\]

The momentum dependence of the pion distribution function is given by [5,7]:

\[
f_{\pi N}(y) = \left( \frac{g_{\pi NN}^2}{16 \pi^2} \right) y \int_{t_{\text{min}}}^{\mu^2} \frac{dt}{(t + m_\pi^2)^2}.
\]

where \( t = -k_\mu k^\mu \) (\( k_\mu \) is the four-momentum of the pion), with a minimum value \( t_{\text{min}} = M^2 y^2/(1 - y) \) determined from the on-shell condition for the recoil nucleon, and \( g_{\pi NN} \) is the \( \pi NN \) coupling constant. Since the non-analytic structure of pion loops does not depend on the short-distance behavior, we have for simplicity introduced an ultra-violet cut-off, \( \mu \), to regulate the integral in Eq. (4). One could have equally well used a form factor for the \( \pi NN \) vertex, or a more elaborate regularization procedure.

It is vital to understand that this contribution to \( d - \bar{n} \) is a leading twist contribution to the structure function of the nucleon. The hard scattering involves the constituents of the pion itself, while the momentum of the pion is typical of those met in chiral models of nucleon structure, namely a few hundred MeV/c. The fact that the momentum associated with the pion is low is the reason one can discuss the LNA structure of \( d - \bar{n} \). There may, of course, be other terms which contribute to the physical difference between \( d \) and \( \bar{n} \), which cannot be expressed in the factorized form of Eq. (2), such as interactions of the spectator quark in the pion with the recoil nucleon. However, the LNA behavior of \( d - \bar{n} \) is entirely determined by the one-pion loop and cannot be altered by such contributions.

Taking the \( n \)-th moment of the distribution in Eq. (4), the LNA chiral log contribution from a pion loop is:

\(^1\)The assumption implicit in the appearance of the pion valence distribution is that the sea of the pion is flavor symmetric. The generalization to the case where this is not so is straightforward, but this contribution would be confined to very small values of Bjorken \( x \).
\[ f_{\pi N}^{(n)}_{\text{LNA}} = \left(3M^2g_A^2/(4\pi f_\pi)^2\right) \times \begin{cases} \frac{(-1)^{n/2}((n+4)/(2n+4))(m_\pi/M)^{n+2}\log(m_\pi^2/\mu^2)}{(n = 0, 2, 4, \cdots)}, \\ \frac{(-1)^{(n+1)/2}((n+5)/2)(m_\pi/M)^{n+3}\log(m_\pi^2/\mu^2)}{(n = 1, 3, 5, \cdots)}, \end{cases} \]

where the PCAC relation has been used to express the \( \pi N N \) coupling constant in terms of the axial charge \( g_A \) (both \( g_A \) and the nucleon mass, \( M \), are taken in the chiral SU(2) limit). For the \( n = 0 \) moment, conservation of baryon number requires that \( V_\pi^{(0)} = 1 \), which leads directly to Eq. (1). The LNA contributions to the \( n > 0 \) moments are suppressed in the chiral limit by additional powers of \( m_\pi^2 \). The scale dependence of \( V_\pi^{(n)} \) for \( n > 0 \) introduces a \( Q^2 \) dependence into the higher moments of \( \vec{d} - \vec{\pi} \). In particular, the observed decrease with \( Q^2 \) of the \( n > 0 \) moments of \( \vec{d} - \vec{\pi} \) arises from the QCD evolution of the momentum fraction carried by valence quarks in the pion \( \pi \rightarrow p^{(0)} \).

Another contribution known to be important for nucleon structure is that from the \( \pi \Delta \) component of the nucleon wave function [8]. For a proton initial state, the dominant Goldstone boson fluctuation is \( p \rightarrow \pi^- \Delta^{++} \), which leads to an excess of \( \pi^- \) over \( \pi^0 \). The one-pion loop contribution to the \( n \)-th moment of \( \vec{d} - \vec{\pi} \) from this process can be written in a similar form as Eq. (2):

\[ (\bar{\mathcal{D}} - \mathcal{U})^{(n)} = -\frac{1}{3} V_\pi^{(n)} \cdot f_{\pi \Delta}^{(n)}, \]

where \( f_{\pi \Delta}^{(n)} \) is the \( n \)-th moment of the \( \pi \Delta \) momentum distribution [9],

\[ f_{\pi \Delta}(y) = \left(\frac{g_{\pi N \Delta}^2}{16\pi^2}\right) y \int_{t_{\text{min}}}^{t_{\text{max}}} dt \left( t + (M_\Delta - M)^2 \right) \left( t + (M_\Delta + M)^2 \right)^2 \frac{6M_\Delta^2}{6M_\Delta^2 (t + m_\pi^2)^2}, \]

with \( t_{\text{min}} = M^2 y/(1 - y) + \Delta M^2 y/(1 - y) \), and \( \Delta M^2 = M_\Delta^2 - M^2 \) (again the masses and the coupling \( g_{\pi N \Delta} \) are implicitly those in the chiral limit). Evaluating the \( n \)-th moment of the \( \pi \Delta \) distribution explicitly, one finds the following LNA behavior:

\[ f_{\pi \Delta}^{(n)}_{\text{LNA}} = \frac{6}{25} \frac{g_A^2}{(4\pi f_\pi)^2} \frac{(M_\Delta + M)^2}{M_\Delta^2} (\Delta M)^2 \log(m_\pi^2/\mu^2), \]

where SU(6) symmetry has been used to relate \( g_{\pi N \Delta} \) to \( g_A \).

We stress that the current analysis aims only at establishing the model-independent, chiral behavior of flavor asymmetries, without necessarily trying to explain the entire asymmetries quantitatively. It is interesting, nevertheless, to observe that with a mass scale \( \mu \sim 4\pi f_\pi \sim 1 \text{ GeV} \), the magnitude of the LNA contribution (at the physical pion mass) to the \( n = 0 \) moment of \( \vec{d} - \vec{\pi} \) is quite large \( \gtrsim 0.2 \), most of which comes from the \( \pi N \) component. For comparison, we recall that the latest experimental values for the asymmetry \( (\bar{\mathcal{D}} - \mathcal{U})^{(0)} \) lie between \( \approx 0.1 - 0.15 \) [3].

In addition to \( \pi \Delta \) intermediate states, contributions from other, heavier baryons and mesons to the \( \vec{d} - \vec{\pi} \) asymmetry have been considered in meson cloud models [10]. Unlike the situation that we have explored for the (pseudo-Goldstone) pion, however, there is no direct, model independent connection with the chiral properties of QCD for mesons such as the \( \rho \) and \( \omega \).
One can generalize the preceding analysis to the flavor SU(3) sector by considering the chiral behavior of the $s$ and $\bar{s}$ components of the sea of the nucleon associated with kaon loops. One finds that the non-trivial moments of the difference between the $s$ and $\bar{s}$ distributions are non-analytic functions of $\bar{s}^{2} + m_{s}^{2}$ with $m_{s}$ the strange quark mass.

As originally proposed by Signal and Thomas [11], virtual kaon loops are one possible source of non-perturbative strangeness in the nucleon [12]. Unlike the case of SU(2) flavor asymmetry, however, where only the direct coupling to the pion plays a role, both the kaon and hyperon (for example, the $\Lambda$) carry non-zero strangeness and hence contribute to strange observables. Furthermore, the different momentum distributions of $\bar{s}$ quarks in the kaon and $s$ quarks in the hyperon lead to different $s$ and $\bar{s}$ distributions as a function of $x$, as well as to non-zero values for strange electromagnetic form factors [12].

The $n$-th moment of the $s - \bar{s}$ difference arising from a one-kaon loop can be written [11]:

$$
(S - \bar{S})^{(n)} = \int_{0}^{1} dx \ x^{n} (s(x) - \bar{s}(x)) = V_{A}^{(n)} \cdot f_{AK}^{(n)} - V_{K}^{(n)} \cdot f_{KA}^{(n)} ,
$$

where $f_{AK}^{(n)}$ is the $n$-th moment of the $N \rightarrow K\Lambda$ splitting function:

$$
f_{K\Lambda}(y) = \left( \frac{g_{K\Lambda}^{2}}{16\pi^{2}} \right) y \int_{t_{\text{min}}}^{t^{2}} \frac{dt}{t + (M_{\Lambda} - M)^{2}} ,
$$

with $t_{\text{min}} = M^{2} y^{2} / (1 - y) + \Delta M^{2} y / (1 - y)$ and $\Delta M^{2} = M_{\Lambda}^{2} - M^{2}$. The corresponding moment of the $\Lambda$ distribution, $f_{\Lambda K}^{(n)}$, can be evaluated from $f_{K\Lambda}^{(n)}$ through the symmetry relation between the splitting functions:

$$
f_{\Lambda K}^{(n)} = f_{K\Lambda}^{(n)} (1 - y) .
$$

Zero net strangeness in the nucleon implies the vanishing of the $n = 0$ moment, $(S - \bar{S})^{(0)} = 0$, which follows from Eq. (11) and strangeness number conservation, $V_{A}^{(0)} = V_{K}^{(0)} = 1$. For higher moments, however, this is no longer the case, so that in general $(S - \bar{S})^{(n)}$ will be non-zero for $n > 0$. In particular, the LNA components of the strange distributions will be given by:

$$
f_{K\Lambda}^{(n)} = \frac{2\pi}{25} \frac{M^{2} g_{A}^{3}}{(4\pi f_{x})^{2}} (M_{\Lambda} - M)^{2} (-1)^{n} \frac{m_{K}^{2n+2}}{\Delta M^{2n+2}} \log(m_{K}^{2} / \mu^{2}) ,
$$

where we have used SU(6) symmetry to relate $g_{K\Lambda}$ to $g_{A} / f_{x}$. It is especially interesting to note that while the LNA part of the $n$-th moment of $\bar{s}$ is of order $m_{s}^{2n+2} \log m_{K}^{2}$, from Eq.(11) the LNA contribution to the $n$-th moment of $s$ is of order $m_{s}^{2n+1} \log m_{K}^{2}$. As a consequence the entire $x$-dependence of $s(x) - \bar{s}(x)$ has a LNA component of order $m_{s}^{2n+1} \log m_{K}^{2}$. Since the LNA terms in the chiral expansion are model-independent, and in general not cancelled by other contributions, this result establishes the fact that the process of dynamical symmetry breaking in QCD implies that the $s$ and $\bar{s}$ distributions must have a different dependence on Bjorken $x$.

Experimental evidence for a strange–antistrange asymmetry is being sought in deep-inelastic neutrino and antineutrino scattering experiments by the CCFR Collaboration [13]. At the present level of precision it is not possible to resolve the asymmetry, which, as we
have shown, is expected on quite general grounds. Nevertheless, it should be amenable to future measurements.

A similar analysis can also be performed for spin-dependent quark distributions. Although there will be no contribution to polarized asymmetries from direct coupling to the Goldstone bosons, there will be indirect effects associated with chiral loops via the interaction with the baryon which accompanies the meson “in the air”. Such processes will renormalize the axial charge, for example, as well as give rise to polarization of strange quarks. Interestingly, Goldstone boson loops will not give rise to any flavor asymmetries for spin-dependent antiquark distributions, $\Delta \bar{d} - \Delta \bar{s}$, for which the only known source is Pauli blocking effects in the proton [14].

In summary, we have derived the leading non-analytic chiral behavior of flavor asymmetries in the proton which are associated with Goldstone boson loops. These results establish the fact that the measurement of flavor asymmetries in the nucleon sea reveals direct information on dynamical chiral symmetry breaking in QCD.

ACKNOWLEDGMENTS

We would like to thank W. Detmold and J. Goity for a careful reading of the manuscript. This work was supported by the Australian Research Council, U.S. DOE contract DE-AC05-84ER40150, and FAPESP (96/756-6, 98/2249-4).
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