Non-unitarity of CKM matrix from the vector singlet quark mixing and neutron electric dipole moment

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In the standard model (SM) the lowest order contribution to the quark electric dipole moment (EDM) occurs at the three loop level. We show that the non-unitarity of the CKM matrix in models with an extended quark sector typically gives rise to a quark EDM at the two loop level which has no GIM-like suppression factors except the external quark mass. The induced neutron EDM is of order $10^{-29}$ e cm and can be well within the reach of the next generation of experiments if it is further enhanced by long distance physics as happens in the SM.

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An important target of particle physics is the determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1], which parameterizes the charged current interactions of quarks. In the standard model (SM) with three generations of quarks the CKM matrix is a $3 \times 3$ unitary matrix, and the $CP$ violation is due to the presence of a nonzero phase in this matrix. The unitarity of the CKM matrix is essential in suppressing flavour changing neutral current (FCNC) processes [2] [3]. Using the CKM unitarity relation the $CP$ violating phase information can be elegantly displayed in terms of unitary triangles. The most interesting challenge of the $B$-factories now entering into operation, and of future collider $B$ experiments is to try to pin down the angles in the unitary triangles [4]. If the measured angles violate either of the ‘triangle conditions’, or correspond to a point $(\rho, \eta)$ [5] which is outside the allowed region, then we will have evidence for new physics.

Violation of the CKM unitarity can appear in models with an extended quark sector [6]. Various constraints on the possibility that exotic quarks mix with the ordinary SM quarks have been derived from low energy charged and neutral current phenomenology, $Z$ physics, flavor changing neutral current processes and $CP$ violation in neutral meson systems [7] [8] [9]. In this letter we propose to examine the unitarity of the CKM matrix in a different setting, namely, by investigating possible unitarity violating effects on the neutron electric dipole moment (EDM). We shall find that information from the neutron EDM is complementary to that from FCNC processes in $B$ physics and serves as a self-consistency check of the relevant theory.

In the minimal SM quark EDM’s vanish at the one loop level because the relevant amplitudes do not change the quark flavor and each CKM matrix element is accompanied by its complex conjugate so that no $T$-violating complex phase can arise. At the two loop level individual diagrams can have a complex phase, but it has been shown by Shabalin [10] that their sum vanishes strictly. The null result was confirmed afterwards by several groups [11]. It is thus thought that in the SM the lowest order contribution to quark EDM’s occurs at the three loop level. A recent calculation [12] shows they are of order $10^{-35}$ to $10^{-34}$ e cm for $u$ and $d$ quarks. The extreme smallness of the quark and neutron EDM’s in the SM makes them particularly suited for searching for new physics. The current experimental upper bound [13] [14], $|d(n)| < 6.3 \times 10^{-26}$ e cm, has put very stringent constraints on extensions of the SM, such as additional Higgs fields, right-handed currents, or supersymmetric partners [15]. We reanalyzed the problem in Refs. [16] and [17] and found that the complete two loop cancellation can be attributed to two special features in the SM: the purely left-handed structure of the charged current and the unitarity of the $3 \times 3$ CKM matrix. The cancellation introduced by the unitarity is the flavour diagonal analog of the GIM suppression in the FCNC processes, with the mere difference being that the cancellation is complete in the EDM case. Thus it is expected that in models with an extended quark sector the contributions to quark EDM’s at the two loop level are no longer cancelled thoroughly because of violation of the CKM unitarity, and that potentially large EDM’s for quarks and the neutron can then be induced. Ignoring possible logarithmic factors, they are of order, $d(q) \sim e g^2 \pi^{-4} \delta \tilde{m}_q / m_W^2$, where $g$ is the semi-weak coupling, $\delta$ is the rephasing invariant measure of $CP$ violation [18]. Note that there are no GIM-like suppression factors except the external light quark mass which is required by chirality flip. Numerically they are of order $10^{-29}$ e cm, well within the reach of the next generation of experiments [19] if they are further enhanced by long distance physics as happens in the SM.
We consider here a model with one extra singlet down-type quark in a vector-like representation of the SM gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$. In addition to the three quark generations, each consisting of the three representations $(i = 1, 2, 3)$

$$Q_L^i(3, 2)_{+1/6}, \quad u_R^i(3, 1)_{+2/3}, \quad d_R^i(3, 1)_{-1/3},$$

we have the following vector-like representation:

$$d_4(3, 1)_{-1/3} + \tilde{d}_4(3, 1)_{+1/3}$$

(2)

Such a quark representation appears, for example, in $E_6$ GUTs [20]. The model can be considered as a minimal extension of the SM in the sense that there is no other change in the gauge and scalar sectors. In particular, the charged currents remain purely left-handed and the $CP$ violation is still encoded in the CKM matrix.

After spontaneous symmetry breaking, the down-type singlet quark $(d_4)$ mixes with the ordinary three down-type quarks so that the weak and mass eigenstates are related by a $4 \times 4$ unitary matrix,

$$\begin{pmatrix}
  d' \\
  s' \\
  b' \\
  d_4'
\end{pmatrix}
  =
  \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} & V_{ud} \\
  V_{cd} & V_{cs} & V_{cb} & V_{cd} \\
  V_{td} & V_{ts} & V_{tb} & V_{td} \\
  V_{bd} & V_{bs} & V_{bb} & V_{bd}
\end{pmatrix}
  \begin{pmatrix}
  d \\
  s \\
  b \\
  d_4
\end{pmatrix}
  =
  \begin{pmatrix}
  d' \\
  s' \\
  b' \\
  d_4'
\end{pmatrix}
  L$$

(3)

In the basis where the up-type quarks are diagonalized, the submatrix consisting of the first three rows in the above matrix appears in $SU(2)_L$ $W$ and $Z$ couplings, and it is the generalized CKM matrix in this model. Note that there still exist unitarity relations among the up-type quarks although the matrix is no longer unitary. The non-unitarity of the matrix leads to FCNC $Z$ couplings amongst down-type quarks. These latter couplings are of order $O(V_{ub}V_{uj})$ so that their contributions to EDM’s are suppressed compared to the terms retained later and are thus ignored. However these couplings are important in discussions of FCNC and $CP$ violation processes [8] [9].

Let us consider the quark EDM [21]. The effective Lagrangian for the EDM interaction is defined as $\mathcal{L}_{ed} = -i d/2\bar{\psi}\gamma_5\sigma_{\mu\nu}\psi F^{\mu\nu}$, where $F^{\mu\nu}$ is the electromagnetic tensor and $d$ is the EDM of the fermion $\psi$. Again, no $T$-violating complex phase can arise at the one loop level. The contributing Feynman diagram at the two loop level is shown in Fig. 1. Following our previous works [16] [17], we use the background field [22]- [24] (or the nonlinear [25]) $R^1$ gauge with $\xi = 1$. In this gauge, there is no $W^\pm G^\mp A$ coupling and the $W^+ W^- A$ coupling is also very simple [16],[ A is the background electromagnetic field and $G^\pm$ are would-be Goldstone fields.] We use Greek and Latin letters to denote up- and down-type quarks respectively, and the external quark is denoted as $u_e$ or $d_e$. There are four groups of contributions, denoted as $WW, WG, GW$ and $GG$, where the first and second letters refer to the bosons exchanged in the outer and inner loops respectively.

**FIG. 1.** The Feynman diagram that contributes to the EDM of the up-type quark $u_e$. A background electromagnetic field is understood to be attached to internal lines in all possible ways. The dashed lines represent $W^\pm$ and $G^\pm$ bosons. The diagram for the down-type quark $d_e$ is obtained by substitutions: $\alpha \rightarrow i$ and $j, k \rightarrow \alpha, \beta$.

Consider for example the $u_e$ quark EDM. We found as in the SM [16] that the contribution from $WW$ is automatically cancelled without using any unitarity conditions while other contributions have the following separate structure:

$$V_{ek}V_{ak}V_{e\alpha}V_{e\beta}[H_{\alpha}(k) - H_{\alpha}(j)],$$

(4)

where $H_{\alpha}(k)$ is a function of masses of $u_e, d_k, u_e$ and $W^\pm$. The crucial point is that the dependence on $d_j$ and $d_k$ masses splits up. This is mainly responsible for the thorough or partial cancellation occurring in the SM and beyond. Summing up the pair of mirror-reflected diagrams (i.e., $j \leftrightarrow k$) doubles the imaginary part of the product of CKM matrix elements while removing its real part. The summation over six pairs of $(jk)$, since we have four down-type quarks $d_i, d_j, d_k$ and $d_l$, completely cancels contributions amongst themselves due to the unitarity of the $4 \times 4$ matrix $V$. For example, the $H_{\alpha}(j)$ term is

$$2i\text{Im}[V_{e\gamma}V_{e\alpha}(V_{e\gamma}V_{ak} + V_{e\alpha}V_{om} + V_{e\gamma}V_{ai})] = 2i\text{Im}[V_{e\gamma}V_{e\alpha}(\delta_{e\alpha} - V_{e\gamma}V_{e\alpha})] = 0.$$

(5)

In other words, the up-type quark EDM’s vanish strictly in the model with an extra down-type singlet quark as in the SM. However, for the down-type quark EDM’s in the model, we have one less up-type quark in virtual loops to complete the unitarity cancellation so that the cancellation is not thorough. For example, summing over the pairs of $(\alpha\beta)$ in the contribution to the $d_e$ quark EDM (see Fig. 1 for notations) and using the unitarity of $V$, the $H_i(\alpha)$ term is

$$2i\text{Im}[V_{e\gamma}V_{e\alpha}(V_{e\gamma}V_{ai} + V_{om} + V_{e\gamma}V_{ai})] = 2i\text{Im}[V_{e\gamma}V_{e\alpha}(\delta_{e\alpha} - V_{e\gamma}V_{e\alpha})],$$

(6)

which is generally non-vanishing. Here $u_\alpha, u_\beta$, and $u_\gamma$ are the three up-type quarks. Therefore, the $d_e$ quark EDM is proportional to,

$$2i \sum_{i, \alpha} \text{Im}(V_{e\gamma}V_{e\alpha}V_{\alpha i})H_i(\alpha).$$

(7)

Note that to avoid complete cancellation due to $\sum_{i}(V_{e\alpha}V_{\alpha i}) = \delta_{\alpha e} = 0$, $H_i(\alpha)$ must involve the $d_i$ mass.
Now let us evaluate analytically the $d_e$ quark EDM, $d(d_e)$. This is facilitated by the mass hierarchy in the SM, $m_t \gg m_W \gg m_q$, where $q$ stands for other five quarks, and the assumption that $m_{d_4} \gg m_t$. We should discriminate two kinds of down-type quarks with $d_4$ heavy and others light, as well as two kinds of up-type quarks with top heavy and others light. We want to retain only the terms that are least suppressed by light quark masses. Since the charged current is purely left-handed, the chirality flip needed for the EDM operator has to be made by the external quark mass. We found that $H_t(\alpha)$ is proportional to $m_{u_2}$ for all of $WG$, $GW$ and $GG$ contributions when $u_3$ is light. Therefore we only need to keep the top quark in up-type quarks. The leading terms involving the heavy $d_4$ quark come from the $WG$ and $GG$ contributions:

$$d(d_{\text{heavy}}) = em_{d_4}G_F^2m_W^2(4\pi)^{-4}\text{Im}[V_{te}V_{4e}^*V_{40}V_{0e}]$$

$$\left[Q_u \left( \frac{23}{9} - \frac{8}{9}\mu_t + \frac{4}{3}\mu_t \ln \frac{m_t}{\mu_t} - \frac{16}{3}\mu_t \ln \frac{\mu_t}{m_t} \right) + Q_d \left( -\frac{59}{9} - \frac{8}{3}\mu_t - 2\mu_t \ln \frac{m_t}{\mu_t} + 10\ln \mu_t \right) \right],$$

where $\mu_t = m_t^2/m_{u_2}^2$ and $\mu_4 = m_{d_4}^2/m_{d_4}^2$. The leading terms involving the light $d$ quarks are independent of their masses so that we may use $\sum_i V_{ai}^*V_{0i} = -V_{40}V_{04}$ to sum up their contributions and obtain:

$$d(d_{\text{light}}) = em_{d_4}G_F^2m_W^2(4\pi)^{-4}\text{Im}[V_{te}V_{4e}^*V_{40}V_{0e}]$$

$$\left[Q_u \left( -8 + \frac{16\mu_t^2}{3} - 12\ln \mu_t + 8\ln^2 \mu_t \right) + Q_d \left( -4 - \frac{8\pi^2}{3} + 4\ln \mu_t - 8\ln^2 \mu_t \right) \right],$$

which comes from the $GW$ contribution.

We note that the $d_e$ quark EDM, $d(d_e)$, occurs at order $g^4(m_{d_4}/m_W^2)$ and there is no further suppression due to GIM mechanism [2]. This is typical for models without CKM unitarity. Some terms are further enhanced by the heavy top mass, while the absence of the heaviest $m_{d_4}^2$ enhancement is consistent with general arguments based on gauge invariance and naive dimensional analysis [26].

For numerical analysis, we use the following parameters: $G_F = 1.2 \times 10^{-5}$ GeV$^{-2}$, $m_W = 80$ GeV and $m_t = 175$ GeV. Then we have for the $d$ quark,

$$d(d_{\text{heavy}}) = \text{Im}[V_{te}V_{4e}^*V_{40}V_{0e}] \cdot \frac{m_t}{10\text{ MeV}},$$

$$\begin{cases} -5.3, & 0.86, +2.3 \times 10^{-26} \text{ e cm}, \\ 200, 300, 400 \text{ GeV}, \end{cases}$$

$$d(d_{\text{light}}) = \text{Im}[V_{te}V_{4e}^*V_{40}V_{0e}] \cdot \frac{m_t}{10\text{ MeV}},$$

$$\begin{cases} -3.3, & 3 \times 10^{-26} \text{ e cm}. \end{cases}$$

Note that the same combination of CKM elements is involved in the two contributions. Since the ‘heavy’ part is generally smaller by one order of magnitude we retain below only the ‘light’ part which is independent of the $d_4$ mass. The product of CKM elements can be expressed in terms of the $3 \times 3$ submatrix elements, e.g.,

$$\text{Im}[V_{td}V_{4e}^*V_{40}V_{0d}] = \text{Im}[V_{td}V_{t0}^*Z_{sb}] - \text{Im}[V_{ts}V_{t0}^*Z_{ds}],$$

where $Z_{ij} = V_{ui}V_{uj}^* + V_{ei}V_{ej}^* + V_{ti}V_{tj}^*$, $i, j = d, s, b$ are precisely the couplings appearing in the FCNC $Z$ interactions amongst ordinary down-type quarks. The most stringent bounds on them come from the neutral meson mixing and FCNC decays. Here we adopt the bounds obtained by requiring that the new tree level FCNC effects do not exceed the experimental values. Some bounds may be relaxed if destructive interference occurs between them and other contributions, e.g., the box diagrams. We take $|Z_{ds}| \leq 3 \times 10^{-4}$, $|Z_{db}| \sim |Z_{dd}| \leq 10^{-3}$. These bounds are actually interrelated with the extraction of other elements like $V_{td}$ and $V_{ts}$. We do not attempt here a global analysis which is beyond the main interest of the present work, but simply adopt the following values for numerical estimate: $|V_{td}| \sim 0.2$, $|V_{ts}| \sim |V_{ts}| \sim 0.04$. Then, $\text{Im}[V_{td}V_{t0}^*V_{40}V_{0d}] \leq 2 \times 10^{-5}$, and

$$|d(d_{\text{light}})| \leq 6.5 \times 10^{-30} \frac{m_t}{10\text{ MeV}} \text{ e cm.}$$

Using the $SU(6)$ relation for the neutron, we obtain

$$|d(n)| \leq 0.8 \times 10^{-29} \frac{m_t}{10\text{ MeV}} \text{ e cm.}$$

A few comments are in order.

(1) The above discussion can be easily generalized to other models with exotic quarks. At the two loop level both up- and down-type quark EDM’s vanish strictly in the model with a sequential fourth generation. In the model with an extra up-type singlet quark, the down-type quark EDM’s vanish identically at two loop order. Since all down-type quarks are light in this case, the leading terms for the $u_e$ EDM must be proportional to $m_{u_2}, m_{d_4}^2$ and are thus very small compared to the case considered above.

(2) We have also studied the $P$ and $T$ violating purely gluonic operators, e.g., the dimension-6 Weinberg operator [27]. We found that they are severely suppressed by light quark masses in the current case as in the SM [28]. Their contribution to the neutron EDM can be ignored. To obtain the quark chromoelectric dipole moment (CEDM), one merely replaces $gQ_{u,d}$ by $g_v$ in Eqs. (8) and (9) with $g_v$ being the QCD coupling. It is then clear that the quark CEDM is much smaller than the quark EDM so that the latter remains to be the dominant contribution in the neutron EDM.

(3) In the static limit the fermionic part of the EDM is identical to the spin operator. Since a significant amount of the proton spin is derived from the polarized strange quark sea [29], it seems reasonable that the strange quark also contributes to the neutron EDM [30]. For the strange quark EDM, we have enhancement factors from masses and CKM elements. For the latter the dominant one is $\text{Im}[V_{td}V_{td}^*Z_{sb}]$. Suppose $\eta \sim 10\%$ of the proton spin is accounted for by the strange sea, then we have

$$|d(n)|_\text{strange} \leq 0.8 \times 10^{-29} (m_s/m_d)(|V_{ts}|/|V_{td}|)\eta \text{ e cm.}$$
Another possible enhancement for the neutron EDM originates from long distance physics [31]. Although there are still controversies concerning this, it seems reliable to get an enhancement factor of two orders of magnitude in the SM. If this persists in the case considered here, improvement of the upper bound on the neutron EDM in the near future will provide an interesting test of the unitarity of the CKM matrix which will be complementary to or even competitive with the bound from $B$ physics.

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[6] For recent review, see Y. Nir, CP Violation In and Beyond the Standard Model, hep-ph/9911321