Hyperon Beta–Decay and Axial Charges of the Lambda
in view of Strongly Distorted Baryon Wave–Functions

H. Weigel*

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics
Massachusetts Institute of Technology, Cambridge, Massachusetts 02139
hep–ph/0005115  MIT–CTP#2982

Abstract

Within $SU(3)$ chiral soliton models we suggest that flavor symmetry breaking mainly resides in the baryon wave–functions while the charge operators maintain a symmetric structure. Sizable symmetry breaking in the wave–functions is required to reproduce the observed spacing in the spectrum of the $\frac{1}{2}^+$ baryons. The matrix elements of the flavor symmetric charge operators nevertheless yield $g_A/g_V$ ratios for hyperon beta–decay which agree with the successful $F&D$ parameterization of the Cabibbo scheme. Demanding the strangeness component in the nucleon to vanish for infinitely large symmetry breaking, determines the structure of the singlet axial charge operator and yields the various quark flavor components of the axial charge of the $\Lambda$–hyperon. The suggested picture gains support from calculations in a realistic model using pion and vector meson degrees of freedom to build up the soliton.


I. INTRODUCTION

Some time ago it has been suggested [1] that the study of polarized $\Lambda$’s could be utilized to gain information about the proton spin structure, i.e. the nucleon axial vector matrix elements. For this to be a sensible program it is necessary that large polarizations of the up and down quarks, $\Delta U_{\Lambda}$ and $\Delta D_{\Lambda}$, in the iso–singlet $\Lambda$ carry over to the corresponding fragmentation functions. Once the $\Lambda$ fragmentation functions are known [2,3], the polarization of the $\Lambda$ (from $\Lambda \rightarrow p\pi$) will provide information about the polarized strange quark distribution in the nucleon [4]. Model analyses [5] for $\ell N \rightarrow \Lambda X$ also indicate that information on the quark spin distribution within the $\Lambda$ will further illuminate the proton spin

---

*Heisenberg Fellow

e-mail:weigel@ctp.mit.edu
structure. In addition its spin distribution among the quark flavors is relevant for the polarization of Λ’s being produced in e⁺–e⁻ annihilation at high energies since the strange quark that emerges from this annihilation is longitudinally polarized [6]. These are some of the issues that motivate interest in the axial current matrix elements (or quark spin structure) of the Λ.

Using results on the axial current matrix elements from deep-inelastic scattering as well as hyperon beta-decay data together with flavor covariance indeed results in sizable polarizations for the non-strange quarks, \( \Delta U_\Lambda = \Delta D_\Lambda \approx -0.20 \) together with \( \Delta S_\Lambda \approx 0.60 \) for the strange quark [1,2,6]. The use of flavor covariance is strongly motivated by the feature that the Cabibbo scheme [7] utilizing the F&D parameterization for the flavor changing axial charges works unexpectedly well [8] as the comparison in table I exemplifies. Clearly, any model that reproduces the data equally well with a minimal set of parameters can be regarded as a reasonable description of hyperon beta-decay.

### TABLE I.
The empirical values for the \( g_A/g_V \) ratios of hyperon beta-decays [9], see also [8]. For the process \( \Sigma \to \Lambda \) only \( g_A \) is given. Note that the standard definition for this decay parameter differs from that in ref [11] by a factor \( \sqrt{6}/2 \). Also the flavor symmetric predictions are presented using the values for F&D which are mentioned in section III. Analytic expressions which relate these parameters to the \( g_A/g_V \) ratios may e.g. be found in table I of [13].

<table>
<thead>
<tr>
<th></th>
<th>( n \to p )</th>
<th>( \Lambda \to p )</th>
<th>( \Sigma \to n )</th>
<th>( \Xi \to \Lambda )</th>
<th>( \Xi \to \Sigma )</th>
<th>( \Sigma \to \Lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>emp.</td>
<td>1.258</td>
<td>0.718 ± 0.015</td>
<td>0.340 ± 0.017</td>
<td>0.25 ± 0.05</td>
<td>1.287 ± 0.158</td>
<td>0.61 ± 0.02</td>
</tr>
<tr>
<td>F&amp;D</td>
<td>1.258</td>
<td>0.725 ± 0.009</td>
<td>0.339 ± 0.026</td>
<td>0.19 ± 0.02</td>
<td>1.258 = ( g_A )</td>
<td>0.65 ± 0.01</td>
</tr>
</tbody>
</table>

More recently a chiral soliton model\(^1\) motivated analysis of the axial charges of the hyperons has been performed [11]. Up to next-to-leading order in flavor symmetry breaking (linear in the strange quark mass, \( m_s \)) all operators for the respective matrix elements were collected. Their coefficients were determined from known data on hyperon beta-decay. A model result was used to relate octet and singlet currents which are not related by group theory. Then the polarization for the non-strange quarks in the \( \Lambda \) was predicted to be small, \( \Delta U_\Lambda = -0.02 \pm 0.17 \) in contrast to \( \Delta S_\Lambda = 1.21 \pm 0.54 \); with errors of the \( \Xi \) decay data penetrating through this analysis. Some of the results (for the central values) raise questions in view of the study representing a perturbation expansion in flavor symmetry breaking: The axial singlet matrix element of the \( \Lambda \), \( \Delta \Sigma_\Lambda \), turned out to be about twice as large as that of the nucleon, \( \Delta \Sigma_N \). Also, the \( \mathcal{O}(m_s) \) terms contributed almost 50% to \( \Delta S_\Lambda \). This indicates that at this order the expansion has not converged (if it does at all) or that in chiral soliton models the flavor symmetric point may not be the most suitable one to expand about.

In the present note we will therefore focus on a description with the symmetry breaking mainly residing in the baryon wave-functions, including important higher order contributions. Sizable deviations from flavor symmetric (octet) wave-functions are needed in the

\(^1\)For reviews on soliton models for three flavors see ref [10].
chiral soliton approach to account for the pattern of the baryon mass–splittings [10]. The proposed picture implies that the \textit{strange} quark component in the sea is suppressed, a scenario which has also been considered in ref [12]. On the other hand we will assume that the current operators, from which the charges are computed, are dominated by their flavor symmetric components. We will find that the proposed approach approximately reproduces the data with no (or minimal) explicit symmetry breaking in the axial charge operator. The present studies represent a refinement of some earlier calculations as we now include $1/N_C$ corrections in the axial charge operator which were omitted in ref [13]. In addition we present the results obtained from a complete calculation in a realistic vector meson soliton model. This calculation supports the suggested picture.

II. SYMMETRY BREAKING IN THE BARYON WAVE–FUNCTIONS

Here we briefly review the energy eigenvalue problem for the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons as it arises in the collective treatment of chiral soliton models. This approach was initiated in ref [14] and leads to exact eigenstates for an arbitrary strength of the flavor symmetry breaking. The collective coordinates for flavor rotations are introduced via

$$U(\vec{r}, t) = A(t)U_0(\vec{r})A^\dagger(t), \quad A(t) \in SU(3).$$

$U_0(\vec{r})$ describes the soliton field configuration embedded in the isospin subgroup of flavor $SU(3)$. An appropriate parameterization of the collective coordinates in terms of eight “Euler–angles” is given by

$$A = D_2(I) e^{-i\lambda_4} D_2(R) e^{-i(\rho/\sqrt{3})\lambda_8},$$

where $D_2$ denote $SO(3)$ rotation matrices of three Euler–angles for each, rotations in isospace ($I$) and coordinate–space ($R$). Substituting this configuration into the model Lagrangian yields upon canonical quantization the Hamiltonian for the collective coordinates $A$:

$$H = H_s + \frac{3}{4} \gamma \sin^2 \nu.$$  

The symmetric piece of this collective Hamiltonian only contains Casimir operators and may be expressed in terms of the $SU(3)$–right generators $R_a$, with $[A, R_a] = (1/2)A\lambda_a$, where $a = 1, \ldots, 8$:

$$H_s = M_{cl} + \frac{1}{2\alpha^2} \sum_{i=1}^3 R_i^2 + \frac{1}{2\beta^2} \sum_{\alpha=4}^7 R_\alpha^2.$$  

$M_{cl}$, $\alpha^2$, $\beta^2$ and $\gamma$ are functionals of the soliton, $U_0(\vec{r})$. The symmetry breaking term in the collective Hamiltonian (3) depends on only one of the eight ‘Euler–angles’ (2). This suggests the following parameterization of the baryon eigenfunctions [14],

$$\Psi(I, I_3, Y; J, J_3, Y_R) = \frac{1}{\sqrt{N}} \sum_{M_L, M_R} D_{I_3,M_L}^{(I)*} f_{M_L,M_R}^{(I,Y;J,Y_R)}(\nu) e^{iY_R \theta} D_{M_R, -J_3}^{(J)*}.$$  

3
The unit baryon number sector constrains the right hypercharge to \( Y_R = 1 \). The flavor hypercharge quantum number emerges via the constraint \( Y - Y_R = 2(M_L - M_R) \) for the intrinsic (iso-)spin projections \( M_L \) and \( M_R \).

The generators \( R_a \) can be expressed in terms of derivatives with respect to the ‘Euler-angles’. Then the eigenvalue problem \( H\Psi = \epsilon\Psi \) reduces to sets of ordinary second order differential equations for the isoscalar functions \( f^{(I;Y;J;Y_R)}_{M_L,M_R}(\nu) \). The product \( \omega^2 = \frac{\gamma^2}{\beta^2} \) appears as a continuous parameter in the eigenvalue equation. Hence the eigenfunctions (5) parametrically depend on \( \omega^2 \) which is thus interpreted as the effective strength of the flavor symmetry breaking. A value in the range \( 5 \lesssim \omega^2 \lesssim 8 \) is required to obtain reasonable agreement with the empirical mass differences for the \( \frac{1}{2}^+ \) and \( \frac{3}{2}^+ \) baryons [10]. In particular, reproducing the observed spacing \( (M_A - M_N) : (M_\Sigma - M_\Lambda) : (M_\Xi - M_\Sigma) = 1 : 0.43 : 0.69 \) demands a sizable \( \omega^2 \) since a leading order treatment of the eigenvalue equation (3) incorrectly yields \( 1 : 1 : \frac{1}{2} \). In the exact treatment we get significantly closer to the empirical values, e.g. for \( \omega^2 = 6.0 \) and \( \omega^2 = 8.0 \) we find the ratios \( 1 : 0.69 : 0.70 \) and \( 1 : 0.61 : 0.77 \), respectively\(^2\). As an example we list the admixture of the nucleon wave–functions with states carrying nucleon quantum numbers dwelling in higher dimensional \( SU(3) \) representations in table II. We compare the exact calculation outlined above with the first order approximation. We observe that in the relevant range for \( \omega^2 \) the first order approximation has only limited

<table>
<thead>
<tr>
<th>( \omega^2 )</th>
<th>( 8 )</th>
<th>( 10 )</th>
<th>( 27 )</th>
<th>( 35 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>0.977</td>
<td>0.170</td>
<td>0.128</td>
<td>0.018</td>
</tr>
<tr>
<td>6.0</td>
<td>0.955</td>
<td>0.231</td>
<td>0.184</td>
<td>0.036</td>
</tr>
<tr>
<td>8.0</td>
<td>0.927</td>
<td>0.278</td>
<td>0.233</td>
<td>0.056</td>
</tr>
<tr>
<td>10.0</td>
<td>0.904</td>
<td>0.314</td>
<td>0.276</td>
<td>0.077</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.200</td>
<td>0.130</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.300</td>
<td>0.196</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.400</td>
<td>0.261</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>1.000</td>
<td>0.500</td>
<td>0.326</td>
<td>0.000</td>
</tr>
</tbody>
</table>

validity. In particular the \( 10 \)–amplitude is overestimated by this approximation.

The feature that the effective symmetry breaking parameter also contains the moment of inertia, \( \beta^2 \) for rotations into strangeness direction allows the possibility that the symmetry breaking in the wave–functions, which is measured by \( \omega^2 \), to be large albeit the explicit symmetry breaking, measured by \( \gamma \), is not (and \textit{vice versa}). Furthermore this allows for the scenario of having strong symmetry breaking in the wave–functions without even having symmetry breaking components in the current operators since almost all symmetry breaking can eventually be included in non–derivative terms in the Lagrangian which do not contribute to currents. In the next section we will study whether such a picture can be consistent with the observations on hyperon beta–decays. These decays are well parameterized by the

\(^2\)One might want to add other symmetry breaking operators to (3) but it should be reminded that they are of lower order in \( 1/N_C \).
III. CHARGE OPERATORS

In chiral soliton models the effect of derivative type symmetry breaking terms is mainly indirect. They provide the splitting between the various decay constants and thus significantly increase \( \gamma \) because it is proportional to \( f_K^2 m_K^2 - f_\pi^2 m_\pi^2 \approx 1.5 f_\pi^2 (m_K^2 - m_\pi^2) \). Besides this indirect effect the derivative type symmetry breaking terms may be omitted. Then the octet axial charge operator

\[
\int \! d^3 x A^{(a)}_i = c_1 D_{ai} - c_2 D_{a8} R_i + c_3 \sum_{\alpha, \beta = 4}^7 d_{\alpha\beta} D_{aa} R_\beta, \quad a = 1, \ldots, 8, \quad i = 1, 2, 3. \tag{6}
\]

does not contain any symmetry breaking components. Here we have introduced the adjoint representation of the collective rotations, \( D_{ab} = \frac{1}{2} \text{tr} (\lambda_a \lambda_b A) \). In principle, the constants \( c_n, n = 1, 2, 3 \), are functionals of the soliton. The \( c_2 \)-term originates solely from the abnormal parity terms in the action, e.g. the Wess–Zumino–Witten term, while the \( c_3 \)-term additionally acquires contributions from field components which are induced by the collective rotations. Both, \( c_2 \) and \( c_3 \) are subleading in \( 1/N_C \) as the appearance of the generators, \( R_a \), suggests. A well-known problem of many chiral soliton models is the too small prediction for the axial charge of the nucleon, \( g_A \), when the constants \( c_n \) are computed using the soliton solution. In this section we will not address that problem but rather use the empirical value \( g_A = 1.258 \) as an input to determine the \( c_n \).

It turns out that for pure octet wave–functions the matrix elements of the operators multiplying the constants \( c_1 \) and \( c_3 \) have the same ratio \( F/D = 5/9 \) while the operator associated with \( c_2 \) has \( F/D = -5/3 \). This suggests to put \( c_1 + c_3/2 = -(3F + 5D)/2 \) and \( c_2 = (9F - 5D)/\sqrt{3} \) with the empirical values \( g_A = F + D = 1.258 \) and \( F/D = 0.575 \pm 0.016 \), i.e. \( c_1 + c_3/2 \approx -2.69 \) and \( c_2 \approx 0.09 \). Of course, these relations are valid only for \( \omega^2 = 0 \). To see that the notation of \( F \& D \) Clebsch–Gordan coefficients breaks down already at moderate \( \omega^2 \) we consider the ratios \( \langle B | D_{a3} | B' \rangle / \langle B | \sum_{\alpha, \beta = 4}^7 d_{\alpha\beta} D_{aa} R_\beta | B' \rangle \) in figure 1. The fact that the operators \( D_{a3} \) and \( \sum_{\alpha, \beta = 4}^7 d_{\alpha\beta} D_{aa} R_\beta \) have the same \( F/D \) ratio is reflected by all ratios assuming the same value when flavor symmetric wave–functions are used (\( \omega^2 = 0 \)). However, we see that already at moderate symmetry breaking the notion of \( F/D \) ratios becomes inadequate as these operators evolve quite differently. With these significant dependencies on the effective symmetry breaking of matrix elements of the various operators contributing to the axial charges on the effective symmetry breaking it seems difficult to imagine that the empirical results for the hyperon decays, which are well described by the symmetric formulation, can be reasonably reproduced at realistic \( \omega^2 > 5 \).

Before attempting such a fit we can get more insight into the relevance of the constants \( c_n \) from the axial singlet current. Although it is not related to the octet current (6) by group theoretical means, the fact that we can consider flavor symmetry breaking as a continuous parameter provides further information. In the limit of infinite symmetry breaking \( \omega^2 \to \infty \) the model should reduce to the two flavor formulation. In particular the strangeness contribution to the axial charge of the nucleon should vanish in that limit. Noting that
FIG. 1. The ratio of the matrix elements $\langle B|D_{a3}|B'\rangle/\langle B|\sum_{\alpha,\beta=4}^7 d_{3\alpha\beta} D_{a\alpha} R_\beta |B'\rangle$ for the relevant baryon states $B$ and $B'$ as a function of the effective symmetry breaking parameter $\omega^2$. The right panel shows the dependence of the vector matrix elements on symmetry breaking (9). They are normalized to the symmetric case.

$\langle N|D_{83}|N\rangle \rightarrow 0$ and $\langle N|\sum_{\alpha,\beta=4}^7 d_{3\alpha\beta} D_{8\alpha} R_\beta |N\rangle \rightarrow 0$ while $\langle N|D_{88}|N\rangle \rightarrow 1$ in that limit, we demand

$$\int d^3 r A_i^{(0)} = -2\sqrt{3} c_2 R_i \quad i = 1, 2, 3.$$  \hfill (7)

for the axial singlet current because it leads to the strangeness projection

$$\int d^3 r A_i^{(s)} = \frac{1}{3} \int d^3 r \left( A_i^{(0)} - 2\sqrt{3} A_i^{(8)} \right)$$

$$= -\frac{2}{\sqrt{3}} \left\{ c_1 D_{8i} + c_2 (1 - D_{88}) R_i + c_3 \sum_{\alpha,\beta=4}^7 d_{i\alpha\beta} D_{8\alpha} R_\beta \right\}. \hfill (8)$$

Actually all model calculations in the literature [16,17] are consistent with this requirement. It is simply a consequence of embedding the soliton in the isospin subgroup of flavor $SU(3)$. The analysis of the famous proton spin puzzle yielding $\Delta \Sigma_N = \langle N| \int d^3 r A_i^{(0)} |N\rangle = 0.20 \pm 0.10$ then suggests $c_2 = 0.12 \pm 0.06$ in agreement with the above estimate from the flavor symmetric description of hyperon decays.

In order to completely describe the hyperon beta–decays we also demand matrix elements of the vector charges. These are obtained from the operator

$$\int d^3 r V_{0}^{(a)} = \sum_{b=1}^{8} D_{ab} R_b = L_a,$$  \hfill (9)

which introduces the left $SU(3)$ generators $L_a$. The relevant matrix elements are protected by the Ademollo–Gatto theorem [18] stating that deviations from the $SU(3)$ relations start at second order in symmetry breaking. Consequently, symmetry breaking in the vector currents is not only ignored in the $F&D$–parameterization but also in the linear treatment of ref [11]. However, for the strongly distorted wave–functions this may still be sizable as is also shown in figure 1. Of course, the Ademollo–Gatto theorem is reproduced in this model as the slope of these curves vanishes at $\omega^2 = 0$. 

6
We now attempt to determine the constants \( c_n \) to reasonably fit the ratios \( g_A/g_V \) for the hyperon beta–decays (only \( g_A \) for \( \Sigma^+ \rightarrow \Lambda e^+\bar{\nu}_e \)). The values for \( g_A \) and \( g_V \) are obtained from the appropriate matrix elements of the operators in eqs (6) and (9), respectively. We first have to fix a value, \( \omega_{\text{fix}}^2 \) for which we want to obtain the best fit. We adopt the following strategy: we choose \( c_2 \) according the proton spin puzzle and subsequently determine \( c_1 \) and \( c_3 \) at \( \omega_{\text{fix}}^2 = 6.0 \) such that the nucleon axial charge, \( g_A \) and the \( g_A/g_V \) ratio for \( \Lambda \rightarrow p e^- \bar{\nu}_e \) are reproduced. For example, setting \( \Delta \Sigma = 0.2 \) yields \( c_1 = -1.97 \), \( c_2 = 0.12 \), and \( c_3 = -1.38 \). This is not too different from the above consideration in the symmetric case as \( c_1 + c_3/2 = -2.66 \). (However, these numbers do not necessarily reflect that \( c_3/c_1 \sim 1/N_C \).) The matrix elements for the \( n \rightarrow p \) and \( \Lambda \rightarrow p \) transitions enter this determination of the \( c_n \). The comparison with figure 1 tells us that the deviations from the symmetric limit have turned out unexpectedly small. We are now left with predictions not only for the decay parameters of the other decay processes but we can also study the variation with symmetry breaking of all relevant decays. This is shown in figure 2. Obviously the dependence on flavor symmetry breaking is very moderate, on the order of only a few percent. In view of the model being an approximation this dependence may be considered irrelevant and the results can be viewed as being in reasonable agreement with the empirical data, cf. table I. The observed independence of \( \omega^2 \) shows that these predictions are not sensitive to the choice of \( \omega_{\text{fix}}^2 \). In addition, since we observe this approximate independence of \( \omega^2 \), we essentially have a two parameter (\( c_1 \) and \( c_3 \), \( c_2 \) is fixed from \( \Delta \Sigma_N \) ) fit of the hyperon beta–decays. We remark that the two transitions, \( n \rightarrow p \) and \( \Lambda \rightarrow p \), which are not shown in figure 2, exhibit
FIG. 3. The contributions of the non–strange (left panel) and strange (right panel) degrees of freedom to the axial charge of the Λ. Again we used \( \omega^2 = 6.0 \).

a similar negligible dependence on \( \omega^2 \) and, by construction, they match the empirical data at \( \omega^2 = 6.0 \). It should be noted that the use of the exact solution to the eigenvalue problem, which leads to the non–linear behavior is important in this regard. A linearized version (in symmetry breaking) would not have necessarily yielded this result. In particular a first order description would fail for the process \( \Xi \to \Sigma \), for which \( g_A/g_V \) is a non–monotonous function of \( \omega^2 \). Comparing the results shown in figure 2 with the data in table I we see that the calculation using the strongly distorted wave–functions agree at least as good with the empirical data as the flavor symmetric \( F&D \) fit.

We also observe that the singlet current does not get modified. Hence we have the simple relation

\[ \Delta \Sigma_N = \Delta \Sigma_A \]  \( \text{(10)} \)

implying that even for strongly distorted baryon wave–functions, the operator \( \int d^3r A_i^{(0)} \) behaves like a singlet.

In figure 3 we display the flavor components of the axial charge of the Λ. We see that also the various contributions to the axial charge of the Λ only exhibit a moderate dependence on the effective symmetry breaking. The non–strange component, \( \Delta U_A = \Delta D_A \) slightly increases in magnitude. The strange quark piece, \( \Delta S_A \) then grows with symmetry breaking since we keep \( \Delta \Sigma_A \) fixed. It should be remarked that the results shown in figure 3 agree nicely with an \( SU(3) \) analysis applied to the data [1,2,6]: \( \Delta U_A = \Delta D_A \approx -0.20 \) and \( \Delta S_A \approx 0.60 \). Finally we remark that the observed independence on the symmetry breaking does not occur for all matrix elements of the axial current. An interesting counter–example is the strange quark component in the nucleon, \( \Delta S_N \). For \( \Delta \Sigma = 0.2 \), say, it is significant at zero symmetry breaking, \( \Delta S_N = -0.131 \) while it decreases (in magnitude) to \( \Delta S_N = -0.085 \) at \( \omega^2 = 6 \).

Of course, one could try to add symmetry breaking components to the currents by allowing all possible operators at the next–to–leading order in symmetry breaking to eliminate the small deviations form the empirical data. As these deviations are potentially small it might well be that this could be accomplished by a single operator of even higher order. In turn this would make the approach quite unpredictable. In addition the errors in the empirical data (\( cf. \) table I) may penetrate to the fitted coefficients \( c_n \). It seems thus more appropriate to revert to realistic models in which we can calculate the coefficients of the next–to–leading order terms and which have been tested at other instances.
IV. SPIN CONTENT OF THE $\Lambda$ IN A REALISTIC MODEL

We consider a realistic soliton model which contains pseudoscalar and vector meson fields. It has been established for two flavors in ref [15] and been extended to three flavors in ref [16] where it has been shown to fairly describe the parameters of hyperon beta-decay (cf. table 4 in ref [16]).

Starting point is a three-flavor chirally invariant theory for pseudoscalar and vector mesons. The model Lagrangian contains abnormal parity terms [19] to accommodate processes like $\omega \rightarrow 3\pi$. These terms contribute to $c_2$ and $c_3$. A minimal set of symmetry breaking terms is included [20] to account for different masses and decay constants. This effective theory contains topologically non-trivial static solutions, which are constructed by imposing ansätze in the isospin subgroup

$$\xi(\vec{r}) = \exp\left(\frac{i}{2} \vec{r} \cdot \vec{r} F(r)\right), \quad \omega_0(\vec{r}) = \omega(r) \quad \text{and} \quad \rho_{i,a}(\vec{r}) = \frac{G(r)}{r} \epsilon_{ija} \tilde{r}_j, \quad (11)$$

while all other field components vanish classically. Here $\xi = \exp(i\vec{r} \cdot \vec{r}/2f_\pi)$ refers to the non-linear realization of the pion fields. The radial functions are determined by extremizing the static energy functional subject to boundary conditions appropriate to unit baryon number. Collectively rotating this configuration induces field components which are classically absent. From this, eight real radial functions emerge. They solve inhomogeneous linear differential equations with the soliton profiles (11) acting as sources. In regard of the discussion in the preceding section it is interesting to note that despite of strong symmetry breaking in the baryon wave–functions the model predictions for the magnetic moments approximately obey the respective $SU(3)$ relations [16].

Covariant expressions for the (axial–)vector currents are obtained by introducing appropriate sources. Substituting the above described ansätze and applying the quantization rules for the collective coordinates yields the charges as linear combinations of functionals, $c_n[F,\omega,G,...]$ of the meson profile functions and operators in the space of the collective coordinates $A$. In this model the derivative type symmetry breaking terms add symmetry breaking pieces to the axial charge operator,

$$\delta A_i^{(a)} = c_4 D_{as} D_{si} + c_5 \sum_{a,\beta=4}^7 d_{i\alpha\beta} D_{aa} D_{\beta\beta} + c_6 D_{ai}(D_{8s} - 1) \quad \text{and} \quad \delta A_i^{(0)} = 2\sqrt{3} c_4 D_{si}. \quad (12)$$

The identical coefficient $c_4$ in the octet and singlet currents arises from the model calculations, it is not demanded by the above mentioned consistency condition of having vanishing strangeness contribution in the nucleon for large symmetry breaking since for $\omega^2 \rightarrow \infty$ we find $\langle N|D_{8s}D_{8s}|N\rangle \rightarrow 0$ as well as $\langle N|D_{8s}|N\rangle \rightarrow 0$.

Once the model parameters are agreed on, the coefficients $c_1,\ldots,c_6$ are uniquely determined as are the parameters in the collective Hamiltonian, which in this model is more involved than eq (3). Thus the baryon wave–functions as well as the current operators are fixed and all relevant decay parameters can be computed. Unfortunately the model parameters cannot be completely determined in the meson sector [15]. We use the remaining freedom to accommodate baryon properties in three different ways as shown in table III. The set denoted by ‘b.f.’ refers to an overall best fit to the baryon spectrum. It predicts the
axial charge somewhat on the low side, $g_A = 0.88$. The set named ‘mag.mom.’ labels a set of parameters which yields magnetic moments which are close to the respective empirical data (with $g_A = 0.98$) and finally the set labeled ‘$g_A$’ reproduces the axial charge of the nucleon and also reasonably accounts for hyperon beta–decay [16]. For all three sets the ef-

tective symmetry breaking is sizable, $\omega^2 \approx 10$. However, its effect is somewhat mitigated by additional symmetry breaking terms ($\sim \sum_{i=1}^{3} D_{8i} R_i$, $\sum_{\alpha=4}^{7} D_{8\alpha} R_\alpha$) in the collective Hamiltonian (3). We observe that in particular the predictions for the axial properties of the $\Lambda$ are quite insensitive to the model parameters. The variation of the model parameters only seems to influence the isovector part of the axial charge operator. Surprisingly the singlet matrix element of the $\Lambda$ is smaller than that of the nucleon, although this effect is tiny. As this difference emerges solely from the $c_4$ term this ordering is a reflection of $c_4$ being positive in this model. It should be noted that in other models $c_4$ is predicted to be negative [21], although small in magnitude as well; implying that $\Delta \Sigma_{\Lambda} \approx \Delta \Sigma_N$ in general.

Similar to the fit of the previous section the full model calculation predicts sizable polarizations of the up and down quarks in the $\Lambda$ which are slightly smaller in magnitude but nevertheless comparable to those obtained from the $SU(3)$ symmetric analyses.

V. CONCLUSIONS

In the collective approach to chiral solitons large deviations from flavor symmetric (octet) wave–functions are required to accommodate the observed pattern of the baryon mass–splitting. Especially, contributions which arise beyond next–to–leading order in symmetry breaking are needed for this purpose. In this report we have suggested a picture for the axial charges of the low–lying $1^+$ baryons which manages to reasonably reproduce the empirical data without introducing (significant) flavor symmetry breaking in the corresponding operators. Rather, the sizable symmetry breaking resides almost completely in the baryon wave–functions. This scenario is especially motivated by the Yabu–Ando treatment of the Skyrme model which has the major symmetry breaking components in the potential part of the action and thus no (or only minor) symmetry breaking pieces in the current operators. The empirical data for these decay parameters are as reasonably reproduced as in the Cabibbo scheme of hyperon beta–decay. Repeatedly we emphasize that the present picture is not a re–application of the Cabibbo scheme since in the present calculation the ‘octet’ baryon wave–functions have significant admixture of higher dimensional representations (cf. table II). Furthermore the individual matrix elements which enter this calculation may

<table>
<thead>
<tr>
<th></th>
<th>$\Lambda$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta U = \Delta D$</td>
<td>$\Delta S$</td>
</tr>
<tr>
<td>b.f.</td>
<td>$-0.155$</td>
<td>$0.567$</td>
</tr>
<tr>
<td>mag. mom.</td>
<td>$-0.166$</td>
<td>$0.570$</td>
</tr>
<tr>
<td>$g_A$</td>
<td>$-0.164$</td>
<td>$0.562$</td>
</tr>
</tbody>
</table>
strongly vary with the effective symmetry breaking (cf. figure 1); only when combining them to the full $g_A/g_V$ ratios the strong dependence on the strength of symmetry breaking cancels.

In the present treatment we may consider arbitrarily large symmetry breaking in the nucleon wave-function. In this limit the two flavor model must be retrieved. This consistency condition relates coefficients in the axial singlet current operator to the respective octet components, which are not otherwise related to each other by group theory. In turn we are enabled to completely disentangle the quark flavor components of the axial charge. It results in sizable up and down quark polarizations in the $\Lambda$. Again, a picture emerged which, after some cancellations, agrees with that of the flavor symmetric treatment for known data. These results were obtained utilizing a parameterization of a charge operator which did not contain any symmetry breaking component.

We have also considered a realistic model, wherein the parameters entering the charge operators are actually predicted. These operators contain non-vanishing symmetry breaking pieces, which are, however, small. Essentially this model calculation confirmed the results obtained in the parametrically treatment.

Acknowledgments

The author would like to thank R. L. Jaffe and J. Schechter for helpful conversations and useful references.

This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DF-FC02-94ER40818 and the Deutsche Forschungsgemeinschaft (DFG) under contract We 1254/3-1.
REFERENCES