SU(2) WZW D-branes and quantized worldvolume $U(1)$ flux on $S^2$

Alexander KLING*, Maximilian KREUZER† and Jian-Ge ZHOU‡

Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstraße 8–10, A-1040 Wien, AUSTRIA

Abstract

We discuss possible D-brane configurations on SU(2) group manifold in the sigma model approach. When we turn the boundary conditions of the spacetime fields into the boundary gluing conditions of chiral currents, we find that for all D-branes except the spherical D2-branes, the gluing matrices $R^a_b$ depend on the fields, so the chiral Kac-Moody symmetry is broken but conformal symmetry is maintained. To match the spherical D2-branes derived from the sigma model with those from the boundary state approach, the $U(1)$ gauge field can be determined and the resulting $U(1)$ worldvolume flux has to be quantized.

*e-mail: kling@hep.itp.tuwien.ac.at
†e-mail: kreuzer@hep.itp.tuwien.ac.at
‡e-mail: jgzhou@hep.itp.tuwien.ac.at
1 Introduction

In recent years there has been much interest in the study of D-branes on group manifolds (see for instance [1–7]). String theory on group manifolds is governed by a WZW model which has two distinct descriptions: conformal field theory (CFT) and the sigma model realization. Since the WZW model is a typical example of an exact string background, whose CFT is known explicitly, one approach to find possible D-brane configurations is to impose gluing conditions on the chiral currents $J^a(z)$ and $\bar{J}^a(\bar{z})$ in terms of which the CFT is defined. Actually the boundary state approach has been applied widely to find D-brane configurations on group manifolds [2–4,6,7]. In [4] it was found that the D-branes in WZW models associated with the gluing condition $J^a = -\bar{J}^a$ along the boundary are the configurations of ‘integer’ conjugacy classes. As the gluing conditions in the boundary state approach are defined on the chiral currents rather than on the spacetime fields there is an obvious lack of geometric interpretation of WZW boundary states and in particular of the corresponding D-brane configurations. Since the WZW model provides also the example of the string background with a sigma model description which allows a complementary study of the D-brane configurations, it is interesting to compare the D-brane configurations obtained from the sigma model realization with those from the boundary state approach (CFT) in order to see how they match with each other.

The other motivation for this work is to see the quantization of the worldvolume $U(1)$ flux on the spherical D2-brane. In [8,9] it was suggested that the $U(1)$ worldvolume flux $\int F$ rather than that of $\int [(2\pi\alpha')^{-1}B + F]$ should be quantized. In [8,9], the quantization problem was mainly discussed from the Born-Infeld theory, so it is quite interesting to see whether we can study it from the worldsheet perspective. Since the $U(1)$ gauge field appears in the action of the sigma model, we wonder if the D-brane configurations constructed from the sigma model approach can give us the answer to this question.

Motivated by the above, in this paper we study WZW D-branes on the group manifold of $SU(2)$ from the sigma model point of view. We construct D0-, D1-, D2-, and D3-branes by the sigma model approach, then compare the resulting D-branes with those from the boundary state approach. Our strategy is that we turn the boundary conditions of the spacetime fields into the gluing condition of the chiral current at the boundary, for D2-, D3-branes we try to adjust the $U(1)$ gauge field to make the gluing matrices be field independent in order to check chiral Kac-Moody symmetry. For the spherical D2-branes we find that in order to keep the infinite-dimensional symmetry of the current algebra, the $U(1)$ worldvolume gauge field strength has to take the form $F = -\frac{\kappa}{2\pi} \psi_0 \epsilon_2$ where $\kappa$ is the integer level of the associated current algebra and $\psi_0$ describes the radius of the spherical D2-branes. From the general Bohr-Sommerfeld quantization condition [4,5], the radius of the spherical D2-branes is quantized, i.e. with the values $\psi_0^{(n)} = n\pi/\kappa$. By matching the spherical D2-branes in the CFT approach to those in the complementary sigma model, the $U(1)$ worldvolume flux $\int F$ has to be quantized. For other D-branes we find it impossible to adjust the $U(1)$ gauge field to make the gluing matrices $R^a_b$ field
independent (the gluing matrix is defined by the gluing condition $J^a(z) + R^a_b J^b(\bar{z}) = 0$ at the boundary). The dependence of the gluing matrices $R^a_b$ on the spacetime fields certainly breaks the chiral Kac-Moody symmetry, but we find that conformal invariance is maintained for these D-branes.

2 Parametrization of the sigma model action and chiral currents

We start with the $SU(2)$ WZW action on a disc with the gauge field $A$ at the boundary [1]

$$ S = \int_{\Sigma} tr(g^{-1} \partial gg^{-1} \bar{\partial}g) + \int_{\Sigma} g^* B + 2\pi\alpha' \int_{\partial\Sigma} g^* A $$  \hspace{1cm} (1)$$

where $\Sigma$ is the 2D manifold with boundary $\partial\Sigma$, $B$ is a particular choice for the antisymmetric tensor field, and the overall factor in (1) has been omitted. Here we note that for the $SU(2)$ group manifold, the B-field in (1) is defined only locally\(^1\). In the case of $SU(2)$ we choose a parametrization

$$ g = \begin{pmatrix} \cos \psi - i \sin \psi \sin \theta \sin \phi \\ \sin \psi \sin \theta \cos \phi - i \sin \psi \cos \theta \\ \sin \psi \sin \theta \cos \phi + i \sin \psi \cos \theta \\ \cos \psi + i \sin \psi \sin \theta \sin \phi \end{pmatrix} $$  \hspace{1cm} (2)$$

where

$$ 0 \leq \psi \leq \pi, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi $$  \hspace{1cm} (3)$$

In these coordinates the metric and the NS three-form field are given by

$$ ds^2 = \kappa\alpha'[d\psi^2 + \sin^2 \psi (d\theta^2 + \sin^2 \phi^2)] $$  \hspace{1cm} (4)$$

$$ H = -\frac{1}{6} \kappa\alpha' tr(g^{-1} dg)^3 = 2\kappa\alpha' \sin^2 \psi \sin \theta d\psi \wedge d\theta \wedge d\phi $$  \hspace{1cm} (5)$$

where $\kappa$ is the integer level of the associated current algebra. Then the $SU(2)$ WZW action turns into

$$ S = \int \sigma d\tau \left\{ \frac{1}{2} \eta^{\alpha\beta} \kappa\alpha' \left( \partial_\alpha \psi \partial_\beta \psi + \sin^2 \psi \partial_\alpha \theta \partial_\beta \theta + \sin^2 \psi \partial_\alpha \phi \partial_\beta \phi \right) \\
+ B_{\theta\phi}(\partial_\tau \theta \partial_\sigma \phi - \partial_\sigma \theta \partial_\tau \phi) \\
+ B_{\psi\phi}(\partial_\tau \psi \partial_\sigma \theta - \partial_\sigma \psi \partial_\tau \theta) \\
+ B_{\psi\phi}(\partial_\tau \psi \partial_\sigma \phi - \partial_\sigma \psi \partial_\tau \phi) \right\} + 2\pi\alpha' \int_{\partial\Sigma} g^* A $$  \hspace{1cm} (6)$$

\(^1\)The proper global form for Wess-Zumino term is $S = \frac{2\pi}{15\pi} \int_{\Sigma+D^2} tr(g^{-1} \partial g)^3 - \frac{2\pi}{15\pi} \int_{D^2} g^*(B + 2\pi\alpha' F)$, where $\Sigma + D^2$ has no boundary, and the disc $D^2$ is mapped to D-brane submanifold [5].
where $\eta^{\alpha\beta} = \text{diag}(-1, 1)$.

The WZW model possesses chiral currents (with $\partial_{\pm} = \partial_{\tau} \pm \partial_{\sigma}$),

$$ J = -\partial_+ gg^{-1}, \quad \bar{J} = g^{-1} \partial_- g \tag{7} $$

Inserting the parametrization (2) into (7) we have

$$ J^{a} = -\bar{e}^{a}_{\mu} \partial_+ X^{\mu}, \quad \bar{J}^{a} = e^{a}_{\mu} \partial_- X^{\mu} \tag{8} $$

where

$$ e^{a}_{\mu} = \begin{pmatrix}
\cos \theta & -\sin \psi \cos \psi \sin \theta & \sin^2 \psi \sin^2 \theta \\
\sin \theta \cos \phi & \sin \psi \cos \psi \cos \theta \cos \phi & -\sin \psi \cos \psi \sin \theta \sin \phi \\
\sin \theta \sin \phi & \sin \psi \cos \psi \cos \theta \sin \phi & \sin \psi \cos \psi \sin \theta \cos \phi \\
+ \sin^2 \psi \cos \phi & -\sin^2 \psi \sin \theta \cos \theta \sin \phi
\end{pmatrix} \tag{9} $$

and

$$ \bar{e}^{a}_{\mu} = \begin{pmatrix}
-\cos \theta & \sin \psi \cos \psi \sin \theta & \sin^2 \psi \sin^2 \theta \\
-\sin \theta \cos \phi & -\sin \psi \cos \psi \cos \theta \cos \phi & \sin \psi \cos \psi \sin \theta \sin \phi \\
-\sin \theta \sin \phi & -\sin \psi \cos \psi \cos \theta \sin \phi & \sin \psi \cos \psi \sin \theta \cos \phi \\
+ \sin^2 \psi \cos \phi & -\sin^2 \psi \sin \theta \cos \theta \sin \phi
\end{pmatrix} \tag{10} $$

where $X^{1} = \psi, X^{2} = \theta, X^{3} = \phi$, and $J^{a}, \bar{J}^{a}$ are defined by

$$ J^{a} = \frac{i}{2} \text{tr}(\sigma^{a} \partial_+ gg^{-1}), \quad \bar{J}^{a} = -\frac{i}{2} \text{tr}(\sigma^{a} g^{-1} \partial_- g) \tag{11} $$

where $\sigma^{a}$ are the Pauli matrices. The vielbein matrices $e$ and $\bar{e}$ satisfy $e^T e = \bar{e}^T \bar{e} = G$, and $G$ is the metric matrix

$$ G_{\mu\nu} = \text{diag}(1, \sin^2 \psi, \sin^2 \psi \sin^2 \theta) \tag{12} $$

### 3 D-brane configurations constructed in the sigma model approach

When we vary the action (6) we get the equations of motion, in addition we also work out the boundary conditions from which we can construct possible D-brane configurations\(^2\).

\(^2\)The action (6) contains $U(1)$ gauge field $A$ only at the boundary $\partial \Sigma$, however when we vary the action (6), the boundary part has the sort of $B + 2\pi \alpha' F$ term.
We first consider the case with
\[(B + 2\pi \alpha' F)_{\theta\phi} = \kappa \alpha' (\psi - \frac{\sin 2\psi}{2} + f) \sin \theta\] (13)
and the other components are zero at the boundary \(\partial \Sigma\). Here the form of \(B_{\theta\phi}\) is smooth everywhere except at the point \(\psi = \pi\), as \(\kappa\) is integer this potential singularity is unobservable [8] and \(F_{\theta\phi} = \frac{\kappa}{2\pi} f \sin \theta\) in D-brane submanifold\(^4\).

As \(dF = 0\) we demand that on the D-brane worldvolume \(f\) should be constant. Since we have the freedom of changing \(B\) by an exact 2-form which is nothing but the \(U(1)\) gauge field strength \(F\), the quantity \(f\) is an undetermined parameter at this stage, and it can not be fixed by the leading order condition of conformal invariance at the boundary given by [3]
\[\partial_\mu [\sqrt{G} G^{\mu\nu} G^{\rho\sigma} (B + 2\pi \alpha' F)_{\nu\rho}] = 0\] (14)
where the metric is given by (12).

With the choice of \((B + 2\pi \alpha' F)_{\theta\phi}\) in (13) we can read off the boundary condition by varying the action (6) and we find
\[\left. (\delta \psi \partial_\sigma \psi) \right|_{\partial \Sigma} = 0\]
\[\left. \delta \theta \left( \sin^2 \psi \partial_\sigma \theta - (\psi - \frac{\sin 2\psi}{2} + f) \sin \theta \partial_\sigma \phi \right) \right|_{\partial \Sigma} = 0\]
\[\left. \delta \psi \left( \sin^2 \psi \sin \theta \partial_\sigma \phi + (\psi - \frac{\sin 2\psi}{2} + f) \partial_\sigma \theta \right) \right|_{\partial \Sigma} = 0\] (15)
By exploiting (15) we can construct D0-, D1-, D2- and D3-brane configurations by the combinations of the various boundary conditions.

D0-brane:
\[\psi \big|_{\partial \Sigma} = \psi_0, \quad \theta \big|_{\partial \Sigma} = \theta_0, \quad \phi \big|_{\partial \Sigma} = \phi_0\] (16)

D1-branes:
\[\psi \big|_{\partial \Sigma} = \psi_0, \quad \theta \big|_{\partial \Sigma} = \theta_0, \quad \partial_\sigma \phi \big|_{\partial \Sigma} = 0\] (17)
\[\psi \big|_{\partial \Sigma} = \psi_0, \quad \partial_\sigma \theta \big|_{\partial \Sigma} = 0, \quad \phi \big|_{\partial \Sigma} = \phi_0\] (18)
\[\partial_\sigma \psi \big|_{\partial \Sigma} = 0, \quad \theta \big|_{\partial \Sigma} = \theta_0, \quad \phi \big|_{\partial \Sigma} = \phi_0\] (19)

\(^3\)Here we choose \(B_{\theta\phi} = \kappa \alpha' (\psi - \frac{\sin 2\psi}{2}) \sin \theta\), and the other choice of \(B\) will be considered below.

\(^4\)The ends of open string are sensitive to the concrete choice of \(B\) and \(F\), the bulk of string feels only the NS 3-form field \(H\), so \(F \neq 0\) only on the D-brane submanifold.
Spherical D2-brane:\n
$$\psi\bigg|_{\partial \Sigma} = \psi_0$$

\[
\left(\sin^2 \psi \partial_\sigma \theta - \left(\psi - \frac{\sin 2\psi}{2} + f\right) \sin \theta \partial_\tau \phi\right)\bigg|_{\partial \Sigma} = 0
\]

\[
\left(\sin^2 \psi \sin \theta \partial_\sigma \phi + \left(\psi - \frac{\sin 2\psi}{2} + f\right) \partial_\tau \theta\right)\bigg|_{\partial \Sigma} = 0
\]

(20)

where $$\psi_0, \theta_0, \phi_0$$ are arbitrary constants. When we replace the Dirichlet boundary condition in (20) by a Neumann one $$\partial_\sigma \psi\big|_{\partial \Sigma} = 0$$ we have a D3-brane.\n
4 Comparison of the D-brane configurations between two approaches and quantized $$U(1)$$ worldvolume flux on $$S^2$$\n
Now we compare the D-brane configurations derived from the above sigma model with those from the boundary state approach. To do so, we construct the gluing condition $$J^a(z) + R^a_b \tilde{J}^a(\bar{z})\big|_{\partial \Sigma} = 0$$ from the boundary condition of the spacetime fields $$\psi, \theta, \phi$$ for various D-brane configurations. We try to adjust the undetermined parameter $$f$$ to see whether we can get spacetime field independent gluing matrices $$R^a_b$$ in order to check the infinite-dimensional symmetry of the current algebra.

For the following comparison, we need the explicit expression for $$J^a, \tilde{J}^a$$, from (8)-(10) we rewrite them as

\[
J^1 = \cos \theta \partial_\tau \psi + \cos \theta \partial_\sigma \psi - \sin \psi \cos \psi \sin \theta \partial_\sigma \theta + \sin^2 \psi \sin^2 \theta \partial_\tau \phi
+ \sin \theta (\sin^2 \psi \sin \theta \partial_\sigma \phi - \sin \psi \cos \psi \partial_\tau \theta)
\]

\[
J^2 = \sin \theta \cos \phi \partial_\tau \psi + \sin \theta \cos \phi \partial_\sigma \psi - \sin^2 \psi \sin \phi \partial_\tau \theta + \sin \psi \cos \psi \cos \theta \cos \phi \partial_\sigma \theta
- \sin^2 \psi \sin \theta \cos \phi \partial_\tau \theta + \sin \psi \cos \psi \sin \theta \sin \phi \partial_\sigma \phi
- \sin \phi (\sin^2 \psi \partial_\sigma \phi + \sin \psi \cos \psi \sin \theta \partial_\tau \phi)
- \cos \theta \cos \phi (\sin^2 \psi \sin \theta \partial_\sigma \phi - \sin \psi \cos \psi \partial_\tau \theta)
\]

\[
J^3 = \sin \theta \sin \phi \partial_\tau \psi + \sin \theta \sin \phi \partial_\sigma \psi + \sin^2 \psi \cos \phi \partial_\tau \theta + \sin \psi \cos \psi \cos \theta \sin \phi \partial_\sigma \theta
- \sin^2 \psi \sin \theta \cos \phi \partial_\tau \theta + \sin \psi \cos \psi \sin \theta \cos \phi \partial_\sigma \phi
+ \cos \phi (\sin^2 \psi \partial_\sigma \theta + \sin \psi \cos \psi \sin \theta \partial_\tau \phi)
- \cos \theta \sin \phi (\sin^2 \psi \sin \theta \partial_\sigma \phi - \sin \psi \cos \psi \partial_\tau \theta)
\]

(21)

5 When $$\psi_0 = 0$$ and $$\pi$$, the spherical D2-branes reduce to D0-branes, and the D0-branes described by (16) can be derived from the D0-branes of $$\psi_0 = 0$$ and $$\pi$$ by inner automorphism.

6 When we require that B-form potential is globally defined on D-brane submanifold, i.e., $$[H]_{\text{MD}} = 0$$, then D3-brane configuration on $$S^3$$ should be excluded.
\[ J^1 = - \cos \theta \partial_\tau \psi + \cos \theta \partial_\sigma \psi - \sin \psi \cos \psi \sin \theta \partial_\sigma \theta + \sin^2 \psi \sin^2 \theta \partial_\tau \phi \\
- \sin \theta (\sin^2 \psi \sin \theta \partial_\phi - \sin \psi \cos \psi \partial_\tau) \]

\[ J^2 = - \sin \theta \cos \phi \partial_\tau \psi + \sin \theta \cos \phi \partial_\sigma \psi - \sin^2 \psi \sin \phi \partial_\tau \theta + \sin \psi \cos \psi \cos \theta \cos \phi \partial_\sigma \theta \\
- \sin^2 \psi \sin \theta \cos \phi \partial_\phi \theta - \sin \psi \cos \psi \sin \theta \sin \phi \partial_\sigma \phi \\
+ \sin \phi (\sin^2 \psi \partial_\theta \theta + \sin \psi \cos \psi \sin \theta \partial_\phi) \\
+ \cos \theta \cos \phi (\sin^2 \psi \sin \theta \partial_\phi \theta - \sin \psi \cos \psi \partial_\tau \theta) \]

\[ J^3 = - \sin \theta \sin \phi \partial_\tau \psi + \sin \theta \sin \phi \partial_\sigma \psi + \sin^2 \psi \cos \phi \partial_\tau \theta + \sin \psi \cos \psi \sin \phi \partial_\sigma \theta \\
- \sin^2 \psi \sin \theta \cos \phi \sin \phi \partial_\phi \theta + \sin \psi \cos \psi \sin \phi \partial_\sigma \phi \\
- \cos \phi (\sin^2 \psi \partial_\phi \phi + \sin \psi \cos \psi \sin \phi \partial_\sigma \phi) \\
+ \cos \theta \sin \phi (\sin^2 \psi \sin \phi \partial_\sigma \phi - \sin \psi \cos \psi \partial_\tau \phi) \]

(22)

Now let us first consider the spherical D2-brane characterized by (20). In the boundary state approach the spherical D2-brane is described by the gluing condition [4]

\[ J^a = \bar{J}^a \]  

(23)

at the boundary \( \partial \Sigma \). Here we should notice that we have turned the gluing condition for the spherical D2-brane in the boundary state approach (in closed string picture) into that in open string picture\(^7\). Comparing \( J^a \) and \( \bar{J}^a \), we find that to match the spherical D2-brane described by the boundary conditions (20) in sigma model to that described by the gluing codition (23) in the boundary state approach, we have to demand\(^8\)

\[ \psi_0 - \frac{\sin 2\psi_0}{2} + f = - \sin \psi_0 \cos \psi_0 \]  

(24)

which results in

\[ f = -\psi_0 \]  

(25)

In [4] it was shown that the D-brane configurations in the WZW model associated with the gluing condition \( J^a = -\bar{J}^a \) (in closed string picture) are the conjugacy classes, and in the case of \( SU(2) \) group the D-brane configurations are spherical D2-branes, which are described by the boundary conditions (20) in the sigma model approach.

To see how the position of the spherical D2-brane is quantized, let us recall that the action (1) can be derived from WZW action. Since a closed loop on \( S^2 \) can be contracted in distinct ways which gives rise the ambiguity\(^9\)

\[ \Delta I = \frac{1}{2\pi \alpha'} \left( \int_M H - \int_{S^2} (B + 2\pi \alpha' F) \right) \]  

(26)

\(^7\)There should be a minus sign difference between open and closed string picture [6].

\(^8\)For example, let us consider \( J^1 = \bar{J}^1 \), the first line of \( J^1 \) is equal to that of \( \bar{J}^1 \) with the help of the first equation in (20), but the second line differs a minus sign. To get \( J^1 = \bar{J}^1 \) at the boundary \( \partial \Sigma \), we must demand \( \left. \sin^2 \psi \sin \theta \partial_\sigma \phi - \sin \psi \cos \psi \partial_\tau \theta \right|_{\partial \Sigma} = 0 \). When we exploit the third equation in (20), we obtain (24).

\(^9\)Since the Wess-Zumino term is \( \frac{k}{12\pi} \text{tr}(g^{-1}dg)^3 \), comparing with (5) we have the factor 1/2\(\pi \alpha' \).
and we require it to be the value of $2\pi n$ with integer $n$, where $M$ is one of the 3-balls bounded by the conjugacy class $S^2$ [4, 5]. Inserting (5), (13) and (25) into (26) we have

$$\psi_0^{(n)} = \frac{n\pi}{\kappa}$$

(27)

which indicates that the $n$-th sphere locates at $\frac{n\pi}{\kappa}$. Then the $U(1)$ worldvolume gauge field strength on the $n$-th spherical D2-brane is $F = -\frac{\kappa}{2\pi} \psi_0^{(n)} \epsilon_2 = -\frac{n}{2} \epsilon_2$. So the $U(1)$ worldvolume flux $\int F$ is quantized $(-2\pi n)^{11}$, which supports the hypothesis in [8].

What we have learnt from the above is that if we require the spherical D2-brane configuration derived from the sigma model to match that from the boundary state approach, the $U(1)$ worldvolume gauge field strength has to be fixed. Here we emphasize that because of $dF = 0$ the parameter $f$ should be constant on the D-brane worldvolume. For the spherical D2-brane we have $f = -\psi_0$ which is consistent with its boundary condition $\psi|_{\partial \Sigma} = \psi_0$.

For D0-,D1-brane configurations the gluing condition can be written

$$J^a + R^a_b J^b = 0$$

(28)

with

$$R = \bar{e} ye^{-1}$$

(29)

where the vielbein matrices $e, \bar{e}$ are defined in (9,10) and the matrix $y$ is defined by

$$\partial_+ X^\mu = y^\mu_\nu \partial_- X^\nu.$$

(30)

For the D0-brane $y = \text{diag}(1,1,1)$ and for the three types of D1-branes the matrices $y$ are \text{diag}(1,1,-1), \text{diag}(1,-1,1), \text{diag}(-1,1,1) respectively.

Since there is no place to put the magnetic field strength on D1-brane worldvolume which implies that these D1-brane configurations are unstable. For instance, the D1-brane cycle with the boundary condition (17) will shrink to a point like object which forms a nonmarginal bound state with the stable spherical D2-brane. Except in the case of spherical D2-branes for all other D-branes the gluing matrices $R^a_b$ depend on the spacetime fields, which indicates that the chiral Kac-Moody symmetry is broken. For the $SU(2)$ group manifold, the energy-momentum tensor is $T(z) = \frac{1}{\kappa+2} J^a J^a$, since $R^T R = 1$, we have $T(z) = \bar{T}(\bar{z})$ at the boundary, which shows that in the presence of the D0-,D1-branes even though the chiral Kac-Moody symmetry is broken, the theory still preserves conformal invariance.

---

10 In deriving (27), we exploit that $\int_M H = \int d\Omega_2 \int_0^{\psi_0} 2\kappa \alpha \sin^2 \psi d\psi = 4\pi \kappa \alpha (\psi_0 - \frac{\sin 2\psi_0}{2})$, and $\int_{S^2} (B + 2\pi \alpha' F) = \kappa \alpha \int d\Omega_2 (-\frac{\sin 2\psi_0}{2}) = -2\pi \kappa \alpha \sin 2\psi_0$.

11 When we replace (13) by $(B + 2\pi \alpha' F)_{\theta \phi} = \kappa \alpha' (\psi - \pi - \frac{\sin 2\psi}{2} + \tilde{f}) \sin \theta$, the singular point is transferred to $\psi = 0$. By the same procedure we get $\tilde{f} = \pi - \psi_0$, and the $U(1)$ worldvolume flux $\int F = 2(\kappa - n)\pi$.

12 Certainly, there is $\Lambda$-transformation under which $F$ is not invariant, but it does not affect $\int F$ [8].
5 Summary and discussion

In the above, we have constructed possible D-brane configurations from the sigma model. In order to see what the counterparts of these D-branes are in the boundary state approach, we turned the boundary conditions of the spacetime fields into the gluing condition of the chiral currents at the boundary. We have shown that except for spherical D2-brane configurations the gluing matrices for all other D-brane configurations depend on the spacetime fields. For the spherical D2-branes we have seen that the configurations derived from the sigma model do not match those from the boundary state approach automatically. If we demand that they coincide with each other, we have to put a strong restriction on the form of the \( U(1) \) gauge field strength. Actually, the gauge field strength can be determined by such a matching, which indicates that open strings are quite sensitive to not only the concrete choice of the 2-form potential \( B \) but also the \( U(1) \) gauge field strength \( F \) on the D-brane worldvolume. Furthermore we have found that the \( U(1) \) worldvolume flux \( \int F \) has to be quantized, which supports the hypothesis in [8].

In (13) we considered the special choice for the field \( B + 2\pi \alpha' F \) at the boundary \( \partial \Sigma \) from which we constructed spherical D2-branes and others. One may ask how about the other two choices for the NS B-field, where the nonzero \( B + 2\pi \alpha' F \) takes the following forms respectively

\[
I) \quad (B + 2\pi \alpha' F)_{\psi \theta} = 2\kappa \alpha' (\sin^2 \psi \sin \theta + f') \\
II) \quad (B + 2\pi \alpha' F)_{\psi \phi} = 2\kappa \alpha' (\sin^2 \psi \cos \theta + f'')
\]

where \( f' \) and \( f'' \) are undetermined parameters, which correspond to the \( U(1) \) gauge field on D-brane worldvolume. For the type I choice, the B-form potential in (31) has multivalue under the transformation \( \phi \rightarrow \phi + 2\pi \) which means it is unphysical choice. So we turn to type II choice. Inserting (32) into the action (6), we find besides D0- and D1-branes described by the boundary conditions (16)-(19) additional D2-branes with the boundary conditions

\[
\begin{align*}
\theta \bigg|_{\partial \Sigma} &= \theta_0 \\
(\partial_\sigma \psi - 2(\sin^2 \psi \cos \theta + f') \partial_\tau \phi) \bigg|_{\partial \Sigma} &= 0 \\
(\sin^2 \psi \sin^2 \theta \partial_\sigma \phi + 2(\sin^2 \psi \cos \theta + f') \partial_\tau \psi) \bigg|_{\partial \Sigma} &= 0
\end{align*}
\]

where (33) describes conic-like D2-branes. As we know, one of criteria to choose the B-form potential is that the dynamics disallows the end-points of strings to hit the singularity [1]. In type II case, there is the conical singularity on type II D2-brane which indicates that the choice (32) is physically unacceptable. Thus in the spherical coordinates the only physical choice for B-form potential is (13).

Eq.(20) shows that the D2-brane sphere should be a fuzzy sphere. Indeed, there have been some discussions of noncommutative geometry on the spherical D2-branes with
B-fields [10,11]. Especially in [10] the low-energy effective action on the fuzzy $S^2$ was proposed, then it is interesting to see whether there exists a similar Seiberg-Witten map [12] on the fuzzy sphere, and if so, how the nonlinear $A$-symmetry in noncommutative geometry is realized as in [13]. As we know, among all examples in AdS/CFT correspondence, the boundary theory of $AdS_2 \times S^2$ is most poorly understood, see [14] for references. In [15] it was argued that besides the fuzzy $S^2$ there is also a fuzzy $AdS_2$. It would be interesting to see whether there is a way to study the fuzzy $AdS_2$ in the context of WZW models.

**Acknowledgement**

We would like to thank A.Y. Alekseev, N. Ishibashi, V. Schomerus, and S. Stanciu for helpful discussion. This work is supported in part by the *Austrian Research Funds* FWF under grant Nr. M535-TPH.

**References**


[7] J. M. Figueroa-O’Farrill, S. Stanciu, *D-branes in AdS$_3 \times S^3 \times S^3 \times S^1$*, hep-th/0001199


