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CONSTRAINTS ON THE GRAVITATIONAL FORCE
ON PHOTONS FROM MICROLENSING

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Constraints on the gravitational force on photons from microlensing

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Abstract

Gravitational microlensing observations are used to study the radial dependence of the gravitational force acting on photons. The Newton inverse square law is shown to be valid at a few % level at a distance scale of a few AU. Constraints on possible non-Newtonian terms are derived.

I. INTRODUCTION

In the past decade, interest in experimental studies of deviations from Newton's law of gravitation, the so-called inverse square law, has been renewed by claims of detection of new macroscopic forces (see for instance [1] and references therein). The inverse square law has been tested over scales ranging from 1 cm to 10^8 km. At astronomical scales, the most stringent limits on possible deviations come from planetary and rocket motions [2].

The status of Newtonian gravity is more controversial at larger scales. Some authors have suggested to modify the inverse square law in an attempt to explain the galactic rotation curves [3], [4], [5] (the so-called “dark matter” problem). It has also been noted that the gravitational deflection of light is an important test of these models [6], [7].

The gravitational force on photons is known from studies of the deflection of light by the Sun at the scale of a few AU (where $1 \text{ AU} = 1.496 \cdot 10^8 \text{ km}$ is the Astronomical Unit) and by gravitational lensing observations on larger scales. The deflection of light by the Sun has

been shown to be consistent with the inverse square law [8].

Gravitational lensing (see reference [9] for a review) is a consequence of the gravitational deflection of light. It has evolved from a theoretical speculation to being an important tool for observational astrophysics. Lensing by structures on scales differing as much as clusters of galaxies and stars (“microlensing”) have been observed. Lensing by galaxies and clusters of galaxies have been used by Dar [10] to investigate the validity of the inverse square law at the kpc ($3 \cdot 10^{16}$ km) scale and above. This author compared the mass inferred from the size of the Einstein ring to the velocity dispersion inside the lensing system. An agreement was found at the 30 % level. However, as mentioned previously, the validity of inverse square law at the galactic scale has been questioned by several authors [3].

Microlensing is sensitive to gravitational interactions at a scale of a few to a few tens of AU. Hence it would fill a small part of the gap between the solar system scale (where Newton’s law is known to be accurate at the 10^{-9} level [1]) and the galactic scale. The purpose of this paper is to address the following question : can we learn something on the gravitational interaction of photons from microlensing experiments?

Microlensing observations have an obvious drawback: the lensing mass cannot be estimated in two independent ways. Hence, microlensing observations are not sensitive to the absolute value of the gravitational constant. On the other hand, the impact parameter of the lensed photons change with time when the lens moves in front of the source. This allows to test the radial dependence of the gravitational force acting on photons.

II. THE GRAVITATIONAL LENS EQUATION

A. Angular position of images

The simplest lensing configuration is shown on figure 1. A source star located at S is lensed by a massive lens star (mass M) at L and observed at O. The Schwarzschild radius

r_S and the Einstein radius r_E are defined by:

$$r_S = \frac{2GM}{c^2} \quad (1)$$

$$r_E = \sqrt{\frac{2r_S D_{LS} D_{OL}}{D_{OS}}} \quad (2)$$

$$D_{OS} = D_{OL} + D_{LS} \quad (3)$$

Since we want to study non-Newtonian gravity, G in the above equation is not necessarily the same as Newton's constant measured in the laboratory. The image positions r are given by the normalized lens equation:

$$r - r_o = \alpha(r) \quad (4)$$

where r_o is the angular position of the source relative to the lens, measured in unit of the Einstein radius, and α is related to the deflection angle β by:

$$\alpha = \frac{D_{LS} D_{OL}}{D_{OS}} \beta \quad (5)$$

B. The deflection angle

In the weak field approximation, the deflection angle is related to the gravitational potential Φ by

$$\beta = -2 \int_S^O \partial_{\perp} \Phi dz \quad (6)$$

where z is a coordinate along the source-observer axis. For a Schwarzschild lens, the potential is

$$\Phi \propto \frac{1}{r} \quad (7)$$

which gives [18]

$$\alpha(r) = \frac{r_E^2}{r} (1 + O(\frac{r_S}{r})) \quad (8)$$

Since deviations from the inverse square law are not expected to be large, the class of lenses for which

$$\alpha(r) = \frac{r_E^2}{r}(1 + f(r)) \quad f(r) \ll 1 \quad (9)$$

(hereafter called quasi-Schwarzschild (QS) lenses), is especially important.

C. Models of deviations from the inverse square law

Three models for a deviation from the inverse square law are considered in this paper. The first model is a power law model,

$$\Phi \propto r^{-\delta} \quad (10)$$

This model is empirical and convenient for lensing analysis, since the deflection angle is also found to follow a power law with index $-\delta$. When $\delta \sim 1$, this model is equivalent to a QS lens with

$$f(r) = (1 - \delta) \ln r \quad (11)$$

The second model is a Yukawa type model

$$\Phi \propto \frac{1}{r}(1 + \epsilon e^{-\frac{r}{\lambda}}) \quad (12)$$

where ϵ and λ are phenomenological parameters. Potentials of this type have been proposed to describe short range [12], [13], [11] or long range [4] modifications of gravity. The deflection angle of (12) can be expressed with the modified Bessel function K_1 :

$$\beta \propto \frac{1}{r}(1 + \epsilon \frac{r}{\lambda} K_1(\frac{r}{\lambda})) \quad (13)$$

When λ is large, this model is equivalent to a QS lens with

$$f(r) = \frac{\epsilon}{4} \frac{r^2}{\lambda^2} \ln \frac{r}{\lambda} \quad (14)$$

The last model is simply adding an extra $1/r^2$ term to the ordinary $1/r$ deflection.

$$\beta \propto \frac{1}{r} \left(1 + \epsilon' \frac{r_E}{r} \right) \quad (15)$$

An extra $1/r^2$ term exists also in General Relativity [19], but it is predicted to be very small. On the other hand, this term might be non-negligible for some exotic lenses such as lenses with scalar charges [14].

III. SCHWARZSCHILD LENSES

For a Schwarzschild lens, equation (4) is readily solved. Two images are found. However, these images are not detected individually in microlensing observations. Instead, the experiments take advantage of the apparent magnification of the flux of the lensed source.

The magnification factor is given by:

$$A(r_o) = \left| \frac{r_+}{r_o} \frac{dr_+}{dr_o} \right| + \left| \frac{r_-}{r_o} \frac{dr_-}{dr_o} \right| \quad (16)$$

The lensing is detected when the lens moves in front of the source with a transverse velocity v_t . For Schwarzschild lenses, the shape of the “light curve” (magnification A versus time t) is given by the well known formula [15]

$$A = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \quad (17)$$

$$u^2 = u_o^2 + \frac{v_t^2(t - t_o)^2}{r_E^2} \quad (18)$$

where v_t is the transverse velocity of the lens.

The microlensing events found by astronomical surveys such as EROS, MACHO or OGLE [16] fit well the expected shape. Since our goal is to constrain the deviations from Newtonian gravity, the expected magnification for QS lenses has to be computed and compared to the shape (17). The first step is to solve the lens equation (4).

IV. SOLUTION OF THE LENS EQUATION AT LARGE DISTANCES

It is assumed here that the lens equation (4) has only 2 solutions by analogy with Schwarzschild lenses. One of the solutions, denoted by r_+ , goes continuously to r_o when r is large compared to r_E . r_+ is easily found by application of Lagrange inversion formula [17] :

$$r_+ = r_o + \sum_{n=1}^{\infty} \frac{1}{n!} \frac{d^{n-1}}{dr_o^{n-1}} (\alpha(r_o))^n \quad (19)$$

The second solution of the lens equation is denoted by r_- and goes continuously to 0 when r is large compared to r_E . To obtain this solution, the lens equation is rewritten as

$$y = \alpha(r) \quad (20)$$

$$y = -r_o + \alpha^{-1}(y) \quad (21)$$

The Lagrange inversion formula now gives:

$$r_- = -\alpha^{-1}(r_o) + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n!} \frac{d^{n-1}}{dr_o^{n-1}} \left(\frac{d\alpha^{-1}(r_o)}{dr_o} (\alpha^{-1}(r_o))^n \right) \quad (22)$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{n!} \frac{d^{n-1}}{dr_o^{n-1}} (\alpha^{-1}(r_o))^n \quad (23)$$

To provide some insight into these formulæ , we give the explicit values of $r_{\pm}$ in the power law model. One finds:

$$r_+ = r_o + \frac{r_E^{1+\delta}}{r_o^\delta} + \sum_{n=2}^{\infty} \frac{(-1)^{n-1} ((n\delta) \dots (n\delta + n - 2))}{n!} \frac{r_E^{n(\delta+1)}}{r_o^{n(\delta+1)-1}} \quad (24)$$

$$r_- = -r_E \left(\frac{r_E}{r_o} \right)^{\frac{1}{\delta}} + \sum_{n=2}^{\infty} \frac{(-1)^{n-1} \left(\left(\frac{n}{\delta} \right) \dots \left(\frac{n}{\delta} + n - 2 \right) \right)}{n!} \left(\frac{r_E}{r_o} \right)^{\left(\frac{n}{\delta} + n - 1 \right)} \quad (25)$$

It is clear that these expressions have a restricted validity range in r_o . Using the methods of reference [17], the series for r_+ are found to converge for

$$r_o \geq \frac{\delta + 1}{\delta^{\frac{\delta}{\delta+1}}} r_E \quad (26)$$

while the series for r_- converge if the same inequality is true with δ changed into $1/\delta$. In the worst case (that of Schwarzschild lenses), the expansions converge for $r_o \geq 2r_E$.

Since the expansions are seen to converge only at large source distances r_o , only data with magnifications of less than 5 % are considered in this paper. This corresponds to $2.5 r_E$ for a Schwarzschild lens.

For a Schwarzschild lens, the expressions for $r_{\pm}$ simplify to

$$r_+ = r_o + \sum_{n=1}^{\infty} \frac{(2(n-1)!(-1)^{n-1})}{n!(n-1)!} \frac{r_E^{2n}}{r_o^{2n-1}} \quad (27)$$

$$r_- = -\left(\sum_{n=1}^{\infty} \frac{(2(n-1)!(-1)^{n-1})}{n!(n-1)!} \frac{r_E^{2n}}{r_o^{2n-1}}\right) \quad (28)$$

$$= r_o - r_+ \quad (29)$$

a well known result.

We now obtain approximate expressions for QS lenses. In these expressions, only first order terms in f are kept, since $|f(r)| \ll 1$. Furthermore, the expansion is restricted to second order terms in r_E^2/r^2 for r_+ and first order terms for r_- . The image r_+ is found by simple application of equation (19) and reads

$$r_+ = r_o + \frac{r_E^2}{r_o}(1 + f(r_o)) - \frac{r_E^4}{r_o^3}(1 + 2f(r_o) - r_o \frac{df}{dr}) \quad (30)$$

Equation (23) involves $\alpha^{-1}(r_o)$. This is found by solving

$$r_o = \alpha(r) = \frac{r_E^2}{r}(1 + f(r)) \quad (31)$$

for r . One possible way of inverting this equation is to use again Lagrange formula. Keeping only first order terms in f , this gives

$$\alpha^{-1}(r_o) = \frac{r_E^2}{r_o}(1 + f(\frac{r_E^2}{r_o})) \quad (32)$$

Finally, the result for r_- is obtained by replacing the expression for α^{-1} in formula (23)

$$r_- = -\frac{r_E^2}{r_o}(1 + f(\frac{r_E^2}{r_o})) \quad (33)$$

V. MAGNIFICATION

It is now straightforward to calculate the magnification factor for a QS lens, keeping only the lowest order terms in f and r_E^2/r_o^2 . It has a simple expression:

$$A(r_o) = 1 + \frac{r_E^2}{r_o} \frac{df}{dr} + 2 \frac{r_E^4}{r_o^4} \quad (34)$$

In the large r_o limit, the magnification is obtained by summing the magnification of a Schwarzschild lens and an additional term. In this limit, a Schwarzschild lens has a magnification which is only second order in r_E^2/r_o^2 . The non-Newtonian term of the magnification (34) is first order both in f and r_E^2/r_o^2 , and can become dominant at large enough r_o . It is basically this circumstance which allows one to look for deviations from the inverse square law.

VI. CONSTRAINTS ON THE GRAVITATIONAL FORCE FROM A MICROLENSING EVENT

The detailed study of all available microlensing data and the extraction of the best possible constraints is outside the scope of this paper and will be the subject of future work. However, it is worth seeing how formula (34) works on a practical example. For this, we use a microlensing event detected towards the Galactic Bulge by the OGLE [20] collaboration. Since we want to be sensitive to low magnifications, to have small photometric errors and a good temporal coverage, it is best to select a long duration microlensing observation with a bright source. A good candidate for our purpose is the event 1999-BUL-44. The I band light curve of the alert has been released on the Web ¹. The typical photometric error is 0.8%. Since the fits give acceptable χ^2 , no rescaling of the errors was attempted. A magnification of 1% corresponds roughly to 3 Einstein radii for a Schwarzschild lens, so the range of useful data is narrow. Furthermore, the ratio r_E^2/r_o^2 is only about 0.15 for $r_o \sim 2.5 r_E$, and this is

¹The Web address of OGLE alerts is <http://www.astrouw.edu.pl/ogle/ogle2/ews/ews.html>

not very small compared to 1. As a consequence, the magnification of a Schwarzschild lens is not very well fitted by the formula:

$$A(r_o) = 1 + 2 \frac{r_E^4}{r_o^4} \quad (35)$$

The next term of the expansion is $-8r_E^6/r_o^6$, clearly non-negligible. The formula (34) has to be improved by adding more terms to the expansion. This gives:

$$A(r_o) = 1 + \frac{r_E^2}{r_o} \frac{df}{dr} + \frac{r_E^4}{r_o^4} (2 + 2f(r_o) + 2f(\frac{r_E}{r_o}) - 2\frac{df}{dr}r_o + \frac{d^2f}{dr^2}r_o^2) - 8\frac{r_E^6}{r_o^6} \quad (36)$$

The contribution of non-Newtonian terms can be obtained directly from the residuals of the fit to the shape (17) by application of equations (34) or (36).

The analysis is done in three steps. In a first pass, the light curve is fitted to the shape (17). The fit is performed with the MINUIT program [21]. The instant of maximum t_o° and the Einstein time $\tau^\circ = r_E/v_t$ are estimated. One finds that

$$t_o^\circ = 2,451,460.19_{-0.59}^{+0.61} \text{ HJD} \quad (37)$$

$$\tau^\circ = 32.25_{-0.58}^{+0.59} \text{ days} \quad (38)$$

where t_o is given in Heliocentric Julian Days (HJD). These values agree very well with those given by the OGLE collaboration. The χ^2 value is 180 for 142 degrees of freedom (dof). This has a chi-square probability of $\sim 1.5\%$, which is acceptable, but the error bars might be underestimated by $\sim 10\%$. The data points and the overall fit are shown on figure 2. Then the data is divided in two parts: a set of 28 points located between 2.5 and 5 τ° from the time of maximum t_o° (the “usable” data points) and the complementary set. The latter is fitted again to the shape (17) to provide new values t_o^1 and $\tau^1 = r_E/v_t$. One finds

$$t_o^1 = 2,451,460.07_{-0.59}^{+0.61} \text{ HJD} \quad (39)$$

$$\tau^1 = 32.19_{-0.59}^{+0.61} \text{ days} \quad (40)$$

The χ^2 for the fit is 153 for 114 dof, which can again be “cured” by increasing the errors by 10 % . In the last step, the “usable” data points are fitted to formula (36). The unknown value of r_E/r_o is simply replaced by $\tau^1/(t - t_o^1)$, which is accurate in the large r_o limit.

The fit of the power law model (11) is illustrated on figure 3. It gives

$$\delta - 1 = -0.006 \pm 0.066 \quad (41)$$

The gravitational potential is found to be Newtonian at the 7% level.

The fit of the Yukawa type model (13) gives

$$\frac{\epsilon r_E^2}{\lambda^2} = 0.5 \cdot 10^{-5} \pm 10^{-1} \quad (42)$$

This can be converted into a 68% CL limit:

$$10 \epsilon r_E^2 \leq \lambda^2 \quad (43)$$

The fit to the parameter of model (15) gives

$$\epsilon' = -0.008 \pm 0.05 \quad (44)$$

The maximum contribution of an extra $\epsilon' r_E^2/r^2$ in the gravitational potential is thus

$$\epsilon' \leq 5 \cdot 10^{-2} (68 \% \text{ CL}) \quad (45)$$

The fits to the parameter of models (11), (13) and (15) are very good, with χ^2 values of ~ 26 for 26 dof. It is clear that a rescaling of the errors by 10 % would not spoil very much the goodness of fit.

The limits obtained above are expressed in units of the Einstein radius r_E . They have to be converted into physical units. While this is not possible on an event by event basis, an average value for r_E can be obtained. Furthermore, it is sufficient for our purpose to give an order of magnitude estimate. With typical values of

$$D_{OS} = 8 \text{ kpc} \quad (46)$$

$$D_{LS} = 3 \text{ kpc} \quad (47)$$

$$M = 0.3 M_\odot \quad (48)$$

one finds $r_E \sim 2 \cdot 10^8 \text{ km} \sim 1.4 \text{ AU}$.

The limit on model (13) is thus

$$\lambda \geq 6\sqrt{\epsilon} \cdot 10^8 \text{ km} \quad (68\% \text{ CL}) \quad (49)$$

VII. CONCLUSION

In this paper, we have studied the radial dependence of the gravitational force acting on photons with microlensing observations. Newton's inverse square law was found to be valid at a few % level at a distance scale of roughly 1 A.U. Some constraints on non-Newtonian gravity were derived. These constraints are weaker than the limits of reference [2], but they have been obtained by a completely different method. Finally, the constraints might be improved by systematically using all available “well measured” microlensing observations.

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FIGURES

FIG. 1. An observer located at O receives photons from a source star at S . The source star is lensed by a lens star at L . The distance between the observer and the lens is D_{OL} , the distance between the lens and the source is D_{LS} . The lens plane is normal to the line of sight OS .

FIG. 2. Microlensing observation OGLE 1999-BUL-44. The solid line shows the overall fit to a Schwarzschild lens. The arrows show the data points selected for the gravitational force analysis.

FIG. 3. Microlensing observations of OGLE 1999-BUL-44. The solid line shows the fit to relation (36) with $f=0$ (pure Schwarzschild lens). The dotted lines show the expected magnification when the gravitational potential is $1/r^{1\pm.066}$ (deviation by 1σ of the fitted value of model (11)).