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THE AUXILIARY FIELDS OF SUPERGRAVITY

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ABSTRACT

We solve the problem of what the minimal set of auxiliary fields of supergravity is, needed to close the gauge algebra, and present the invariant action. The result is simple: only a scalar, a pseudoscalar and an axial world vector field are needed. Our approach is closely related to superspace geometry.

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Since the beginning of supergravity, the structure of the auxiliary fields has been one of the most intriguing problems. Soon after the gauge action of supergravity was constructed ^{1),2)}, it was found that its gauge algebra only closes on-shell since there are extra terms present in the commutator of two local supersymmetry transformations proportional to the spin $\frac{3}{2}$ field equations in second order supergravity ³⁾, and to all field equations in first order supergravity. It was conjectured from the analogous situation in global supersymmetry that the algebra might close off-shell if sufficiently many auxiliary fields were added to action and transformation laws. When the problem of how to couple matter to the gauge action was solved ⁴⁾, the results strongly suggested that at least one of the auxiliary fields was an axial vector field. However, the problem of closing the gauge algebra by adding enough auxiliary fields remained unsolved since it was not clear what the starting point, the linearized limit, looked like.

Recently, an approach based on superspace geometry provided the structure of Poincaré and conformal supergravity at the linearized level ⁵⁾. It was found that in the Poincaré case there were three auxiliary fields : one scalar, one pseudoscalar and one axial vector field, while in the conformal case there was only an axial vector field in addition to the spin 2 and spin $\frac{3}{2}$ fields. These results agreed with the recently constructed complete, non-linear gauge action of conformal supergravity ⁶⁾ which contains in addition to higher derivative spin 2 and spin $\frac{3}{2}$ fields only one additional field : a propagating axial vector field. A proof of the hypothesis that one needs in conformal supergravity no more than this axial vector field was furnished by the discovery that here the gauge algebra closes off-shell ⁶⁾. From the above results one can conclude that in full non-linear Poincaré supergravity there are at least three auxiliary fields. The point of view taken in this paper is that there are no more than that.

There are, in principle, several ways to proceed from here. One might use a superspace approach ⁷⁾. This approach has led recently to the construction of an invariant Lagrangian in superspace ⁸⁾ which is the determinant of the achtbein field and whose field equations are equivalent to those of conventional supergravity. However, the elimination of the redundant fields by choosing suitable gauge conditions is far from trivial algebraically and has not yet been executed. A closely related approach starts with a Lie algebra valued vector multiplet. In fact, an action as well as a prescription for the elimination of the many (126) auxiliary field components in this approach has been given some time ago ⁹⁾. However, in a recent publication, this programme was worked out and more auxiliary

fields than the scalar, pseudoscalar and axial vector mentioned above were found ¹⁰⁾. In particular, a vector field, a second axial vector field and two spin $\frac{1}{2}$ fields were found, while the resulting action turned out to be complicated.

Our approach starts at the other end, by using only what seems strictly necessary in ordinary space-time and by keeping action and transformation rules as simple as possible. This approach has led to a very simple solution which we now first present, while a discussion of its derivation is given afterwards.

The action is given by

$$\mathcal{L} = \mathcal{L}(\text{SG}) + \frac{1}{3}e(S^2 + P^2 + A_\mu A^\mu) \quad (1)$$

where $\mathcal{L}(\text{SG})$ is the second order action of supergravity ¹⁾ and where S , P and A_μ denote the scalar, pseudoscalar and axial vector auxiliary fields. The local supersymmetry transformation rules under which this action is invariant and under which the gauge algebra closes, are given by

$$\begin{aligned} \delta_S e_\mu^a &= \kappa \bar{\epsilon} \gamma^a \psi_\mu \\ \delta_S \psi_\mu &= 2\kappa^{-1} D_\mu \epsilon - \frac{1}{3} \gamma_\mu (iS + \gamma_5 P) \epsilon \\ &\quad + i \left(A_\mu - \frac{1}{3} \gamma_\mu A \right) \gamma_5 \epsilon \\ \delta_S S &= \frac{i}{2} \bar{\epsilon} \gamma \cdot R - \frac{\kappa}{2} \bar{\epsilon} \gamma \cdot \psi S - \frac{i\kappa}{2} \bar{\epsilon} \gamma_5 \gamma \cdot \psi P - \frac{\kappa}{2} \bar{\epsilon} \gamma_5 A \cdot \psi \\ \delta_S P &= \frac{1}{2} \bar{\epsilon} \gamma_5 \gamma \cdot R - \frac{\kappa}{2} \bar{\epsilon} \gamma \cdot \psi P + \frac{i\kappa}{2} \bar{\epsilon} \gamma_5 \gamma \cdot \psi S + \frac{i\kappa}{2} \bar{\epsilon} A \cdot \psi \quad (2) \\ \delta_S A_\mu &= \frac{3i}{2} \bar{\epsilon} \gamma_5 (R_\mu - \frac{1}{3} \gamma_\mu \gamma \cdot R) - \frac{i\kappa}{2} \bar{\epsilon} \psi_\mu P + \frac{\kappa}{2} \bar{\epsilon} \gamma_5 \psi_\mu S \\ &\quad - \frac{\kappa}{2} \bar{\epsilon} \gamma \cdot \psi A_\mu + \kappa \bar{\epsilon} A \psi_\mu - \frac{\kappa}{4} \epsilon_{\mu\nu\rho\sigma} A^\nu \bar{\epsilon} \gamma_5 \gamma^\rho \psi^\sigma \end{aligned}$$

We note - en passant - that as far as the superconformal fields e , ψ and A are concerned, their transformation laws are a linear combination of the Q supersymmetry laws of conformal supergravity ⁶⁾ together with a field dependent S supersymmetry transformation ⁶⁾.

In this equation eR^μ is the spin $\frac{3}{2}$ field equation with $e = \det e^a_\mu$

$$eR^\mu = \epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma_\nu \left(\partial_\rho + \frac{1}{2} \omega_\rho^{ab} \tau_{ab} \right) \psi_\sigma$$

$$\omega_\rho^{ab} = \omega_\rho^{ab}(e) + \frac{\kappa^2}{4} \left(2 \bar{\psi}_\rho \gamma^a \psi^b + \bar{\psi}^a \gamma_\rho \psi^b \right) \quad (3)$$

These are our final results.

These results are obtained as follows. The action is the simplest choice possible and reduces to the linearized limit of Ref. 5). For the transformation rules we took the usual rules for $\delta_S e^a_\mu$ and $\delta_S \psi_\mu$, and we added the R terms, obtained by covariantizing the results of Ref. 5). We then completed these rules by adding the most general expressions, linear (for ψ_μ) or quadratic (for S, P and A_μ) in all fields, but always linear in the auxiliary fields. No extra terms were added to $\delta_S e^a_\mu$ since in second order formalism the gauge algebra already closes on e^a_μ 3). We then fixed the coefficients of all these terms as follows. The requirement that the action be invariant and that the linearized theory satisfies the algebra of global supersymmetry left only the coefficients of the last two terms in $\delta_S S$ and $\delta_S P$ and of the last term in $\delta_S A_\mu$ undetermined. From $[\delta_S(\epsilon_1), \delta_S(\epsilon_2)] e^a_\mu$ the uniform structure of this commutator was determined

$$[\delta_S(\epsilon_1), \delta_S(\epsilon_2)] = \delta_S [-\bar{\epsilon}_2 \gamma^\alpha \epsilon_1 \psi_\alpha] + \delta_G [2 \bar{\epsilon}_2 \gamma^\alpha \epsilon_1]$$

$$+ \delta_L \left[2 \bar{\epsilon}_2 \gamma^\alpha \epsilon_1 \omega_\alpha^{ab} - \frac{4\kappa i}{3} \bar{\epsilon}_2 \tau^{ab} \epsilon_1 S - \frac{4\kappa}{3} \bar{\epsilon}_2 \tau^{ab} \gamma_5 \epsilon_1 P \right.$$

$$\left. - \frac{2i}{3} (\bar{\epsilon}_2 \gamma_c \epsilon_1) A_d \epsilon^{abcd} \right] \quad (4)$$

i.e., it consists of the usual field-dependent local supersymmetry (S) and general co-ordinate (G) transformations, but contains extra auxiliary field dependent local Lorentz rotations (L). By requiring that the $\bar{\epsilon}_2 \gamma^\alpha \epsilon_1 \psi_S$ terms in $[\delta_S(\epsilon_1), \delta_S(\epsilon_2)] \psi_\mu$ are of the form $\delta_S(-\bar{\epsilon}_2 \gamma^\alpha \epsilon_1 \psi_\alpha)$ the remaining coefficients in $\delta_S S$ and $\delta_S P$ were fixed. Finally the coefficient of the last term in $\delta_S A_\mu$ followed by requiring that in the same commutator the

$\bar{\epsilon}_2 \gamma^\alpha \epsilon_1 A \psi_\alpha$ terms reproduce exactly the last two terms in $\delta_s \psi_\mu$ with again $\epsilon = -\bar{\epsilon}_2 \gamma^\alpha \epsilon_1 \psi_\alpha$. At this stage action and transformation laws were fixed and to our relief all subsequent checks on closure of the gauge algebra were satisfied.

In the process of checking the closure relations we used a recent expression for the variation of the full spin $\frac{3}{2}$ field equation

$$\delta_s (e R^\mu) = \frac{1}{2} (\bar{\epsilon} \sigma^{ab} R^\rho) (\gamma_a \psi_b g_{\rho\mu} - \gamma_a \psi_\rho e_{b\mu} + \gamma_\rho \psi_b e_{a\mu}) - (\gamma^a \epsilon) (e G_a^\mu - \frac{1}{2} \epsilon^{\lambda\mu\rho\sigma} \bar{\psi}_\lambda \gamma_5 \gamma_a D_\rho \psi_\sigma) \quad (5)$$

It follows from the observation that if a gauge field ϕ^k transforms as $\delta \phi^k = \xi^\alpha \phi_\alpha^k$, then the ϕ^k field equation $I_{,k}^{(6)}$ transforms as $\delta I_{,k} = -I_{,j} \xi^\alpha R_{\alpha,k}^j$. Other useful formulae used are

$$R_{\mu\nu} - R_{\nu\mu} = -\frac{\kappa}{2} \bar{\psi}_\nu \gamma^\alpha \psi_{\mu\alpha} + \frac{\kappa}{2} \bar{\psi}_\mu \gamma^\alpha \psi_{\nu\alpha} - \frac{\kappa}{2} \bar{\psi} \cdot \gamma \psi_{\nu\mu} \quad (6)$$

where $\psi_{\mu\nu} = D_\mu \psi_\nu - D_\nu \psi_\mu$, as well as the identities ³⁾

$$\begin{aligned} \gamma^\alpha \psi_{\alpha\lambda} &= R_\lambda - \frac{1}{2} \gamma_\lambda \gamma \cdot R \\ \gamma_\alpha \psi_{\beta\lambda} + \gamma_\beta \psi_{\lambda\alpha} + \gamma_\lambda \psi_{\alpha\beta} &= -e \epsilon_{\alpha\beta\lambda\tau} \gamma_5 R^\tau \end{aligned} \quad (7)$$

These formulae were needed for the $\psi A (\bar{\epsilon}_2 \sigma^{ab} \epsilon_1)$ terms and the $\psi \delta \bar{\epsilon}_2 \sigma^{ab} \epsilon_1$ terms.

Finally, we note that a very helpful trick is that in second order supergravity one need not vary $\omega_{\mu ab}$ ["1.5 order formalism" ¹¹⁾].

We now discuss our results. Through the introduction of these auxiliary fields one can start writing the transformation rules of the gauge fields in matter-coupled theories as independent of the matter fields. This probably will simplify future matter couplings considerably. Also, our results might indicate which auxiliary fields in the superspace approach still can be eliminated. In (2) we find local Lorentz rotations with tensorial parameters $\bar{\epsilon}_2 \sigma^{ab} \epsilon_1$. It is conceivable that the requirement made in Ref. 9) of closure without such parameters leads to more auxiliary fields. The non-closure of

the gauge algebra in supergravity without auxiliary fields has led to new quantization rules, in particular to a new four-ghost coupling ^{12)-15),10)}. In our case, where the gauge algebra closes, four-ghost couplings are produced by elimination of the auxiliary fields after Faddeev-Popov quantization, as conjectured in Ref. 15) and shown for the model of Ref. 9) in Ref. 10). However, it is interesting to note that the action in (1) is invariant under three separate subalgebras of (2) :

- i) the usual terms for e^a_μ and ψ_μ together with the diagonal terms in the auxiliary field sector;
- ii) the result in (2) itself, and
- iii) the off-diagonal elements of the auxiliary field sector.

In order to reproduce the four-ghost coupling of Ref. 12)-15), one might instead consider i), ii)-i) and iii) as a basis in gauge space.

We close with a comment on the matter coupling. The Maxwell-Einstein system ⁴⁾, being superconformally invariant, couples only to A_μ with a minimal coupling with respect to chiral invariance

$$D_\mu \lambda \rightarrow D_\mu \lambda + \frac{i}{2} A_\mu \gamma_5 \lambda \quad (8)$$

The transformation law in Ref. 4)

$$\delta \psi_\mu = \frac{2}{\kappa} D_\mu \epsilon + \frac{\kappa}{4} (\not{x} \gamma_5 \gamma^\rho \lambda) (g_{\rho\mu} + \sigma_{\rho\mu}) \gamma_5 \epsilon \quad (9)$$

is precisely (2), after elimination of the auxiliary fields, while also the λ^4 terms in the action are thus reproduced. Further work on matter coupling is in preparation.

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R E F E R E N C E S

- 1) D.Z. Freedman, P. van Nieuwenhuizen and S. Ferrara, Phys.Rev.D13, 3214 (1976).
- 2) S. Deser and B. Zumino, Phys.Letters 62B, 335 (1976).
- 3) D.Z. Freedman and P. van Nieuwenhuizen, Phys.Rev.D14, 912 (1976).
- 4) S. Ferrara, J. Scherk and P. van Nieuwenhuizen, Phys.Rev.Letters 37, 1035 (1976).
- 5) S. Ferrara and B. Zumino, CERN preprint TH.2418 (Nov. 1977) - to be published in Nuclear Physics B.
- 6) M. Kaku, P.K. Townsend and P. van Nieuwenhuizen, Phys.Rev.Letters 39, 1109 (1977) - to appear.
- 7) P. Nath and R. Arnowitt, Phys.Letters 56B, 177 (1975);
J. Wess and B. Zumino, Phys.Letters 66B, 361 (1977);
R. Grimm, J. Wess and B. Zumino, Karlsruhe preprint, to be published in Phys.Letters B;
L. Brink, M. Gell-Mann, P. Ramond and J. Schwarz - to be published;
Y. Ne'eman and T. Regge, Princeton preprint (Dec. 1977);
W. Siegel, Harvard University preprint (Dec. 1977).
- 8) J. Wess and B. Zumino, CERN preprint TH.2453 (Jan. 1978).
- 9) P. Breitenlohner, Phys.Letters 67B, 49 (1977); Nuclear Phys.B124, 500 (1977).
- 10) B.Q.P.J. De Wit and M.T. Grisaru, CERN preprint TH.2451 (Jan. 1978).
- 11) P.K. Townsend and P. van Nieuwenhuizen, Phys.Letters B67, 439 (1977).
- 12) E.S. Fradkin and M.A. Vassiliev, Phys.Letters 72B, 70 (1977).
- 13) A. Batalin and G.A. Vilkovisky, Phys.Letters 69B, 309 (1977).
- 14) R. Kallosh, Zh.ETF Pis'ma v Red. 26, 575 (1977).
- 15) G. Sterman, P.K. Townsend and P. van Nieuwenhuizen, Stony Brook preprint, to be published in Phys.Rev.D.

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