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SPECULATIONS ON QUARK OBSERVATION

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ABSTRACT

Results of non-perturbative calculations in several models of relativistic quantum field theory lead to the concept of a stable particle whose mass is not well-defined. Assuming that quarks are such particles, we propose several possible methods of quark observation.

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Quarks, as proposed by Gell-Mann ¹⁾, are very useful in classifying baryons and mesons. Furthermore, in connection with J/ψ ²⁾ in the charmonium scheme ³⁾, quarks are treated not only as mathematical constructs but also as physical entities satisfying Schroedinger's equation. Yet none of the efforts to search for free quarks in flight have produced any positive results ⁴⁾. The word confinement ⁵⁾ is used to describe this paradoxical situation.

Recent calculations on two-dimensional Yang-Mills theory ⁶⁾ in the 't Hooft limit ⁷⁾ show that the quark propagator in the light-cone gauge has no pole (and hence no asymptotic states, as expected for confinement due to a linearly rising Coulomb potential) but has a branch cut instead ⁸⁾. Such branch cuts have previously been seen in the boson propagator in the relativistic quantum field theory based on the two-dimensional Ising model ⁹⁾⁻¹¹⁾ and in the related time-dependent X-Y model ¹²⁾. Through unitarity such branch cuts have direct effects on the S matrix, and hence must represent physical objects. The salient feature of these objects is that they have no well-defined mass and hence, for conciseness, shall be called indeterminate-mass particles (imp).

It is the purpose of this letter to take these two-dimensional results seriously and investigate the possibility that quarks are imps ¹³⁾.

Since the branch cut in the two-dimensional Yang-Mills theory has some bothersome features ⁸⁾, and since the properties of the X-Y model are not sufficiently well developed for our purpose, the properties of imps are best inferred from the two-dimensional Ising model [which is formally related to the ϕ^4 field theory in the limit of infinite unrenormalized coupling constant ¹⁴⁾]. There the imp arises when the symmetry is spontaneously broken. Moreover, if a symmetry-breaking interaction (i.e. an external magnetic field) is introduced, the imp breaks up into poles ¹⁵⁾. This is dramatically different from cuts due to multiparticle states or resonances.

Although their detailed behaviour, and in fact their very existence, depends on dynamics and hence is difficult to ascertain, we speculate that the quark-imps have the following general properties:

- i) Since an imp can be described by a collection of poles in the limit when the spacing and residue go to zero, it seems likely that, in the present picture, quarks are rather large objects. On the other hand, it is difficult to believe that, in a high-energy production process, quarks are born large. The most natural picture is that the size of a quark increases indefinitely with time, consistent with the absence of asymptotic states.

ii) In the limited experience of exact model calculations ^{11),15)}, the mass distribution $\rho(m)$ is zero if m is less than some m_0 but is non-zero for all $m > m_0$. However, there is strong coupling of u and d quarks ¹⁾ to pions. Therefore it is reasonable to expect that, for u and d , $\rho(m)$ is cut off when $\Delta m = m - m_0 \gtrsim m_\pi$, where $m_0 > m_p/3$. For the other quarks ^{1),2),16)} both m_0 and Δm are larger.

iii) We take the quarks to have normal electromagnetic properties. Thus, for example, they are deflected in the usual way in a magnetic field. Since we envisage the quark, in analogy to a proton, as being a smooth extended object with many degrees of freedom ¹⁷⁾, a quark of radius r is unlikely to couple efficiently to photons of wavelength much less than r . Accordingly a quark is able to ionize or excite an atom only for a limited period of time after its birth. After this limited period of time, quarks cannot be detected by ionization devices such as bubble chambers, cloud chambers, nuclear emulsions, scintillation counters, spark chambers, Charpak chambers and drift chambers. In the present picture, this is the explanation for the failure of in-flight quark searches, which are all carried out away from the production vertex.

Assuming that these unconventional properties are not just an artifact of the lower dimensionality of the models, we can distinguish two general classes of methods of detection. It is not clear to us whether the search is better carried out at a proton or an electron accelerator.

1. QUARK DETECTION NEAR THE PRODUCTION VERTEX

For a short time after its production the quark behaves like a normal particle. In order to get some semiquantitative insight, consider for definiteness quark production by a 400 GeV proton and assume that the incident proton is fragmented into three quarks with mass of order $m_p/3$. Such an event is probably quite rare. Since the excitation energy for a silver bromide crystal in emulsion ¹⁸⁾ is 2.5 eV, corresponding to a photon wavelength of 5000 Å, a very rough estimate of the track length is a fraction of a millimeter.

If this estimate were correct, it would be difficult to observe quarks in either bubble chambers or wire chambers. However, such a track is surely long enough to be seen in nuclear emulsions. These tracks in nuclear emulsions have two characteristics:

- a) The grain density corresponds to a charge $\frac{1}{3}e$ or $\frac{2}{3}e$ as the case may be.
- b) The terminus of the track will not show increasing ionization but will just fizzle out.

If a large number of such tracks is actually observed, nuclear emulsions can give information about the strong interactions of quarks. When the radii of the quarks are of the order of fermis, they must interact strongly among themselves and with hadrons. However, it is less clear whether they still interact so strongly after they reach atomic sizes or larger. If large quarks do have strong interactions, then some tracks may show a star before the terminus is reached.

2. QUARK DETECTION AWAY FROM THE PRODUCTION VERTEX

When the sizes of the quarks grow larger, no ionization device can be used efficiently in their detection. So far as these detectors are concerned, the charged quarks masquerade as neutral particles. We discuss three other possible methods based on electromagnetic, strong, and weak interactions.

a) Electromagnetic

If a gas Čerenkov counter is used just above the threshold so that the radiation is nearly in the forward direction, then under the conditions of 1.) above it would remain effective up to perhaps one metre downstream from the target. If large quarks have reduced strong interactions, then a shield may be placed in front of the Čerenkov counter to diminish background. Our rough estimate indicates that it would be difficult to have detectors based on liquid or solid scintillators ¹⁹⁾ used as Čerenkov counters. It is probably also difficult to use transition radiation because its frequency is too high.

b) Strong

By a suitable combination of bending magnets and shields, quarks may be transported to regions of space inaccessible to neutral particles. Possible strong interactions of large quarks can then be observed with, for example, hadron calorimeters.

c) Weak

With the same arrangement of bending magnets and shields, weak decay can also be used as a signal for the possible presence of quarks. Decays where only charged particles are produced are easiest to detect. An example of such a decay is

$$s \longrightarrow d + \pi^+ + \pi^-.$$

The angular distribution of the produced pions gives information about the quark form factor. On the other hand, if the rate for weak decay decreases sufficiently rapidly with increasing size, it is even conceivable that more than one quark are stable.

There is a large number of open questions concerning indeterminate-mass particles. We mention only a few.

A) In the above discussion we have tacitly assumed a linear rate of growth for the imp radii. In practice there will be a distribution of growth rates related to the time dependence of the internal structure, and this will in turn lead to, for example, a distribution of track lengths in nuclear emulsions. Such distributions depend on detailed dynamics, but there exists the possibility that the distributions may extend to sufficiently long track lengths so that the range of usefulness of the detection devices discussed above would be significantly enhanced. It would be most interesting to determine experimentally whether such long tracks exist.

B) With the growth rate in 1.) above, it is unlikely that quarks can be observed in a magnetic levitometer experiment ²⁰⁾. However, if in nuclei or macroscopic matter the growth of quarks can be arrested and/or the quarks can be bound, then levitometer experiments can succeed.

C) From the present point of view, all quarks are imps but it is not necessary that all imps are quarks. There may be imps which are bosons and/or have integer charge. In particular, the question may be raised whether gluons are also imps.

D) Finally, we list a few other questions which readily come to mind. Can detectors operating at frequencies much below the optical be devised to observe large quarks? Can synchrotron radiation from imps be measured and does it have astrophysical significance?

As noted above, our knowledge of indeterminate-mass particles comes mainly from the two-dimensional Ising model. Therefore, the remainder of this paper is devoted to a presentation of those Ising-model properties which motivated our study of imps. Of course, unlike the quark imp, the Ising-model imp is a neutral boson.

In the absence of a magnetic field the thermodynamics of the two-dimensional Ising model was first given by Onsager ²¹⁾ and there is a phase transition at a temperature T_c . For $T < T_c$, as obtained exactly by Yang ²²⁾,

there is a spontaneous magnetization, i.e., spontaneous symmetry breaking. As $T \rightarrow T_c^+$ or $T \rightarrow T_c^-$, a Euclidean, and hence relativistic, quantum field theory emerges ¹¹⁾. In both cases there is no massless particle and hence no long range force. Otherwise these two field theories are rather different. The field theory as $T \rightarrow T_c^+$ shows no peculiar feature. Here we concentrate on the case $T \rightarrow T_c^-$.

$\alpha)$ Two-point function

In momentum space the two-point Green's function (i.e., the propagator) has the spectral representation

$$\tilde{G}_2^c(k) = \int_0^\infty dm \frac{\rho(m)}{k^2 + m^2}, \quad (1)$$

where ²³⁾

$$\rho(m) = \begin{cases} 0 & \text{for } 0 \leq m \leq m_0, \\ \text{const. } (m^2 - m_0^2)^{1/2} / m^2 & \text{for } m_0 \leq m \leq 2m_0, \end{cases} \quad (2)$$

and, for $m \geq 2m_0$, $\rho(m)$ has threshold singularities whenever m/m_0 is an integer. This weight function $\rho(m)$ does not contain any delta functions.

$\beta)$ String behaviour of the connected three-point function

The asymptotic behaviour of the three-point Green's function in coordinate space for large separations is qualitatively different depending on whether the three points are nearly collinear or not. Near the collinearity with the point 2 between the points 1 and 3, we define the variable

$$\tau^2 = R_{12} + R_{23} - R_{13} \geq 0, \quad (3)$$

where $R_{ij} = |\vec{r}_i - \vec{r}_j|$. Then ¹⁵⁾ we have for $R_{13} \rightarrow \infty$

$$G_3^c(1, 2, 3) \sim \text{const. } R_{13}^{-2} e^{-2R_{13}} F(\tau), \quad (4)$$

where

$$F(\tau) = (2\pi^2)^{-1} e^{-2\tau^2} + \frac{1}{2} \pi^{-3/2} \tau e^{-\tau^2} \text{Erf } \tau + (4\pi)^{-1} [1 - (\text{Erf } \tau)^2], \quad (5)$$

with

$$\text{Erf } \tau = 2\pi^{-1/2} \int_0^\tau dx e^{-x^2}. \quad (6)$$

(Here we have taken $m_0=2$). We plot $F(\tau)$ in Fig. 1.

The variable τ and the shape of the curve in Fig. 1 strongly suggest that we are dealing with a string. These Green's function strings are to be contrasted with the strings described by the Lagrangian of Nambu²⁴⁾, Nielsen²⁵⁾, and Susskind²⁶⁾. It is tempting to believe that L-strings and G-strings have much in common, but their precise relation needs to be investigated further.

v) Break-up of the imp in an external symmetry breaking field

The string structure is also present in the higher N point functions. When $\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N$ are all nearly collinear with $\vec{r}_1$ and $\vec{r}_N$ at the extremities, then the generalization of (4) for large $R=R_{1N}$ is

$$G_N^c \sim \text{const. } R^{-2} e^{-2R} F_{N-2}(\tau_2, \tau_3, \dots, \tau_{N-1}), \quad (7)$$

where τ_1 is the deviation of $\vec{r}_1$ from collinearity, divided by $R^{1/2}$.

From this form we already may infer an important property of the two-point Green's function in a magnetic field h . Without loss of generality, let h be positive. We may write $G_2^c(R;h)$ for large R as an expansion in terms of the N -point Green's functions at $h=0$

$$G_2^c(R;h) \sim R^{-2} e^{-2R} \sum_{N=0}^{\infty} \frac{1}{N!} c_N (h R^{3/2})^N, \quad (8)$$

where

$$c_N = \int_{-\infty}^{\infty} d\tau_1 d\tau_2 \dots d\tau_N F_N(\tau_1, \tau_2, \dots, \tau_N). \quad (9)$$

If the imp breaks up into a sum of poles, we must have, for large R ,

$$G_2^c(R; h) \sim R^{-1/2} e^{-2R} \sum_{\ell=0}^{\infty} a_{\ell}(h) e^{-\kappa_{\ell}(h)R}. \quad (10)$$

A comparison of (8) and (10) shows

$$\kappa_{\ell}(h) = \kappa_{\ell} h^{2/3}, \quad (11)$$

and

$$a_{\ell}(h) = a_{\ell} h. \quad (12)$$

Explicit calculation¹⁵⁾ with the exact F_N shows that this break-up does indeed occur. Furthermore, we find that a_{ℓ} is independent of ℓ and

$$\kappa_{\ell} = (3\lambda_{\ell})^{2/3}, \quad (13)$$

where λ_{ℓ} are the roots of

$$J_{1/3}(\lambda_{\ell}) + J_{-1/3}(\lambda_{\ell}) = 0, \quad (14)$$

and J denotes a Bessel function.

Therefore the part of the cut near m_0 in the two-point Green's function does break up into a series of poles. This is the reason why the imp is in no way related to cuts due to multiparticle states or resonances. In addition it should be particularly pointed out that the $h^{2/3}$ behaviour of $\kappa_\rho(h)$ guarantees that the magnetization of the two-dimensional Ising model for $T < T_c$ cannot be analytically continued from $h > 0$ into a "meta-stable" region for $h < 0$.

In summary we have seen that the two-dimensional relativistic quantum field theory obtained from the Ising model as $T \rightarrow T_c^-$ shows the four properties of spontaneous symmetry breaking, confinement, strings, and indeterminate-mass particles. Since all four features are absent for the corresponding theory obtained from $T \rightarrow T_c^+$, it is tempting to believe that they are four different aspects of the same phenomenon. The appearance of several approximate symmetries in nature can be most easily understood in terms of spontaneous symmetry breaking; the lack of observation of quarks in flight points towards confinement; the linearly rising Regge trajectories for hadron resonances may be related to strings²⁷⁾. It is therefore perhaps not entirely unreasonable to hope that the fourth aspect, indeterminate-mass particles, may also appear in nature.

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FIGURE CAPTION

Fig. 1: A plot of the string profile function $F(\tau)$ defined by Eq. (5).