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STRUCTURE OF HADRON IN MIGDAL'S REGULARIZATION SCHEME FOR QCD

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A B S T R A C T

Migdal's approach to the spectrum of QCD is generalized to three-point (hadron) amplitudes. This procedure is shown to be unique once the two-point functions are known. In particular, meson form factors and structure functions are discussed including hard gluon and Regge contributions. Quark mass effects turn out to be very important.

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Quantum chromodynamic (QCD), the gauge theory of coloured quarks and gluons, may be *the* theory of strong interactions. This is supported by the short-distance properties of the theory which can be studied in perturbation theory enriched by renormalization group techniques. Other dynamical characteristics, like confinement of quarks and gluons and the spectrum of hadrons, are sensitive to the large-distance behaviour. In order to study this, one has to develop appropriate techniques going beyond ordinary (low-order) perturbation theory.

Confinement has not yet been proved. Therefore, it might be a promising strategy to assume it, perform concrete calculations and check internal consistency and consistency with experiments. Effective (singular) potentials or bag models incorporate confinement and may be very appropriate for a phenomenological description of hadron physics. Unfortunately, in the usual formulations, the relation to the original QCD becomes loose. It would be an -- at least conceptual -- advantage if one could return to QCD in some limit.

Recently, Migdal¹⁾ proposed such a method^{*)}. In his approach a constant R with the dimension of a length appears which plays the role of an inverse "bag pressure". It is, however, not fixed, as in the original formulation of the MIT bag model, but renormalized by the gluon interaction. It is suggested to be pushed to infinity in higher order perturbation theory.

Migdal studies the set of hadronic two-point functions

$$\Gamma_{ij}^{(2)}(p^2) = \int d^4x e^{ipx} \langle \Omega | T(O_i(x) O_j(0)) | \Omega \rangle \quad (1)$$

where the gauge-invariant local operators $O_i(x)$ are those appearing, for example, in the operator product expansion of QCD [compare Eqs. (7), (8)]. Asymptotic freedom allows one to calculate $\Gamma_{ij}^{(2)}(p)$ in the deep Euclidean region via perturbation theory. These expressions show quark and gluon cuts. In the case of confinement these cuts should not appear in the final result. Imposing a $1/N_c$ expansion²⁾ one expects real poles in the leading order in $1/N_c$ corresponding to stable hadrons. Therefore a new set of meromorphic two-point functions $\tilde{\Gamma}_{ij}^{(2)}(p)$ is constructed which gives an optimized approximation to the original $\Gamma_{ij}^{(2)}$ in the deep Euclidean region. This is achieved via Padé approximation.

For massless quarks and up to order g^4 conformal invariance can be used to find an explicit solution for $\tilde{\Gamma}_{ij}^{(2)}$. In this approximation it is diagonal in the

*) Lattice gauge theory is also of that type, the limit being lattice constant $a \rightarrow 0$.

rank of the operators (included in the indices i, j). Hence particle poles correspond to operators with definite rank. The (spin s) partial wave contribution to $r_{ii}^{(2)}(p^2)$ is found¹⁾ to be

$$\tilde{f}_{ii}^s(p^2) = f_{ii}^s(p^2) - \frac{1}{Q_i(p^2)} \frac{1}{2\pi i} \int_{\mathbb{C}} \frac{dt' Q_i(t') f_{ii}^s(t')}{t' - p^2} \quad (2)$$

where $f_{ii}^s(p^2)$ is the perturbation theory result containing the quark-gluon cuts. $Q_i(p^2)$ is given by

$$Q_i(t) = (R\sqrt{t})^{-v_i} J_{v_i}(2R\sqrt{t}) \quad (3)$$

with $v_i = n_i + \tau_i + g^2 \bar{\gamma}_i(n_i)$, where n_i , τ_i , $g^2 \bar{\gamma}_i$ are rank, twist, and anomalous dimension (to order g^2) of $O_i(x)$. J_v is the Bessel function^{*)}.

The cut of $f_{ii}^s(p^2)$ is cancelled by the second term on the r.h.s. of Eq. (2). Only the poles due to the zeros of $Q_i(p^2)$ remain. For $p^2 < 0$ the quantity $1/Q_i(p^2)$ decreases exponentially. $\tilde{f}_{ii}^s(p^2)$ approaches $f_{ii}^s(p^2)$ very rapidly there.

R , the only dimensional parameter, is related to the subtraction point $p^2 = -\mu^2$ where the Padé expansion is performed. Equation (2) is obtained in the limit $M, N, \mu^2 \rightarrow \infty$, $R^2 = MN/\mu^2$ fixed (M, N are the Padé indices, i.e. the degree of numerator and denominator, respectively). Taking $R \rightarrow \infty$ in Eq. (2) the set of poles condenses to the original cut. However, in Migdal's approach¹⁾ R is kept fixed in finite order of perturbation theory. Like g^2 it is fitted to the spectrum and a value $R = 2.5 \text{ GeV}^{-1}$ is obtained¹⁾ in Eq. (3). In higher orders in g^2 the matrix $\tilde{\Gamma}_{ij}^{(2)}$ is no longer diagonal and Eq. (2) becomes a matrix equation. The renormalization program can still be carried through¹⁾ in terms of R and the corresponding coupling g^2 . Including higher order terms in $f_{ij}^s(p^2)$ a fit to R is expected to yield larger and larger values, and finally $R^2 \rightarrow \infty$. This justifies the above remark about R as an inverse bag pressure.

In the following, the approach of Ref. 1 is generalized to three-point functions.

$$\Gamma_{\bar{q}jk}^{(3)}(x_1, x_2, x_3) = \langle \Omega | T(O_i(x_1) O_j(x_2) O_k(x_3)) | \Omega \rangle \quad (4)$$

*) In the geometrical approach of Preparata³⁾ a structure similar to Migdal's zeroth order result is found.

In particular, we discuss the "hadronization" of form factors and deep inelastic structure functions.

After a tensor decomposition in momentum space, the scalar structure coefficients $F_{ijk}(p_1^2, p_2^2, p_3^2)$ corresponding to Eq. (4) can be calculated in perturbation theory. Again, these quantities have quark-antiquark cuts in the external mass variables p_i^2 , and we look for a meromorphic approximation $\tilde{F}_{ijk}(p_1^2, p_2^2, p_3^2)$ which fits the original perturbation theory result in the deep Euclidean region. For consistency, the spectrum has to be the same as that of the two-point functions. There is a unique solution to this problem with a structure analogous to Eq. (2).

$$\begin{aligned}
 \tilde{F}_{ijk}(p_1^2, p_2^2, p_3^2) = & F_{ijk}(p_1^2, p_2^2, p_3^2) - \frac{1}{Q_i(p_1^2)} \frac{1}{2\pi i} \int_{\infty} \frac{dt_1 Q_i(t_1) F_{ijk}(t_1, p_2^2, p_3^2)}{(t_1 - p_1^2)} \\
 & - \frac{1}{Q_j(p_2^2)} \dots - \frac{1}{Q_k(p_3^2)} \dots \\
 & + \frac{1}{Q_i(p_1^2) Q_j(p_2^2)} \frac{1}{(2\pi i)^2} \int_{\infty} \frac{dt_1 dt_2 Q_i(t_1) Q_j(t_2) F_{ijk}(t_1, t_2, p_3^2)}{(t_1 - p_1^2)(t_2 - p_2^2)} \\
 & + \frac{1}{Q_i(p_1^2) Q_k(p_3^2)} \dots + \frac{1}{Q_j(p_2^2) Q_k(p_3^2)} \dots \\
 & - \frac{1}{Q_i(p_1^2) Q_j(p_2^2) Q_k(p_3^2)} \frac{1}{(2\pi i)^3} \int_{\infty} \frac{dt_1 dt_2 dt_3 Q_i(t_1) Q_j(t_2) Q_k(t_3) F_{ijk}(t_1, t_2, t_3)}{(t_1 - p_1^2)(t_2 - p_2^2)(t_3 - p_3^2)}
 \end{aligned}
 \tag{5}$$

The denominator functions $Q_i(t)$ are no longer determined by an optimization but are those [Eq. (3)] found in connection with the two-point functions.

The electromagnetic form factors and deep inelastic structure functions of a hadron h are determined by the matrix elements

$$R_{\mu_1 \dots \mu_n}^{\alpha, n}(p', p) = \langle h, p' | O_{\mu_1 \dots \mu_n}^{\alpha, n} | h, p \rangle \quad (6)$$

where in leading order the singlet and octet fermion operators

$$O_{\mu_1 \dots \mu_n}^{\psi, n} = S \bar{\psi} \gamma_{\mu_1} \partial_{\mu_2} \dots \partial_{\mu_n} \frac{\lambda^b}{2} \psi, \quad b = 0, 1, \dots, 8 \quad (7)$$

and the gluon operators

$$O_{\mu_1 \dots \mu_n}^{A, n} = S \text{Tr} F_{\mu_1}^\alpha \partial_{\mu_2} \dots \partial_{\mu_{n-1}} F_{\alpha \mu_n} \quad (8)$$

contribute. The meson form factor is directly given by Eqs. (6) and (7), with $n = 1$, $b = 3$, whereas the structure functions are determined via their moments

$$\int_0^1 dx x^{n-2} F_2(x, q^2) = \sum_a c^{\alpha, n}(q^2) M_n^a \quad (9)$$

The coefficient functions $c^{\alpha, n}(q^2)$ can be calculated by renormalization group methods [see, for example, Gross and Wilczek⁴⁾]. The q^2 -independent constants M_n^a defined as

$$R_{\mu_1 \dots \mu_n}^{\alpha, n}(p, p) = p_{\mu_1} \dots p_{\mu_n} M_n^a + O(p^2) \quad (10)$$

are usually not accessible, but can be calculated in our approach.

The matrix elements (6) which contain all the desired information can be extracted from the three-point functions $\tilde{\Gamma}_{ijk}^{(3)}$ [compare Eq. (5)] as the residues of the corresponding hadron poles. In order to illuminate the main features of the procedure we shall concentrate on the technically simplest case, that of scalar mesons. Detailed calculations and the extension to other mesons and especially baryons will be given elsewhere.

Using $O_i = O_k = \bar{\psi}\psi$ as interpolating meson operators we have to calculate [compare Eq. (6)]

$$R_{\mu_1 \dots \mu_n}^{\alpha, n}(p', p) = \frac{\lim_{t, t' \rightarrow M^2} (t - M^2)(t' - M^2) \tilde{\Gamma}_{ijk}^{(3)}(t, q^2, t')}{\lim_{t \rightarrow M^2} (t - M^2) \tilde{f}_{ij}^{s=0}(t)} \quad (11)$$

where $q = p' - p$, and M^2 is the first zero of $Q_1(t)$ corresponding to the mass of the scalar meson.

We start with the zeroth order contribution (Fig. 1a). The calculation is most conveniently done in light-cone coordinates in a frame with $q_- = 0$, i.e. $q^2 = -q_\perp^2$. The graph in Fig. 1a has no triple spectral function. After meromorphization in the two hadron-channels, we find

$$R_{\{p_i=-\}}^{a,n}(p', p) = (2p_-)^n \frac{N_a}{(2\pi)^3} \int_0^1 \frac{dx x^{n-1}}{(1-x)} \int d^2 k_\perp \frac{k_\perp^2 - \frac{1}{4}(1-x)q_\perp^2 + m^2(1-x)(1-x)}{1-x} \\ \times \left\{ \frac{M^2(1-x) - (\vec{k}_\perp - \frac{1}{2}(1-x)\vec{q}_\perp)^2 - m^2}{1-x} \right\}^{-1} \left\{ \frac{M^2(1-x)x - (\vec{k}_\perp + \frac{1}{2}(1-x)\vec{q}_\perp)^2 - m^2}{1-x} \right\}^{-1} \\ \times Q_i \left(\frac{(\vec{k}_\perp - \frac{1}{2}(1-x)\vec{q}_\perp)^2 + m^2}{x(1-x)} \right) \times Q_i \left(\frac{(\vec{k}_\perp + \frac{1}{2}(1-x)\vec{q}_\perp)^2 + m^2}{x(1-x)} \right) \quad (12)$$

N_a describes the normalization according to Eq. (11). m is an effective quark mass. Its influence will be discussed below.

After the $k_\perp$ loop integration and meromorphization in the hadron masses [cf. Eq. (5)] the quark lines emerging from the bound states are on mass shell and give rise to quark constituent cuts in sidewise dispersion relations in a covariant framework.

Equation (12) makes the physical meaning of $Q_1(p^2)$ (whose zeros determine the hadron spectrum) more transparent. It just plays the role of Weinberg's infinite momentum frame vertex function [see, for example, Blankenbecler⁵⁾ and references therein]. The zeros of Q_1 cancel the quark poles. This can also be explicitly seen in two-dimensional QCD⁶⁾.

Unfortunately, $Q_1(p^2)$ is explicitly known only for massless quarks and low-order perturbation theory. The introduction of quark masses, however, changes the behaviour of, for example, the structure function in a qualitative way. In order to demonstrate this, let us neglect the q^2 dependence of the structure function for a while [i.e. we use the free field behaviour $c_n(q^2) = 1$]. For massless quarks we find^{*)}

$$F_2(x) \xrightarrow{x \rightarrow 1} \text{const.} \quad (13)$$

*) In the case of baryons it behaves like $(1-x)$. This corresponds to a naïve quark-parton picture with constant independent probabilities.

In the massive case the $x \rightarrow 1$ behaviour is very sensitive to the behaviour of $Q_i(p^2)$ at large argument. Assuming Eq. (3)

$$Q_i(p^2) \underset{p^2 \text{ large}}{\sim} (R\sqrt{p^2})^{-(\nu+1/4)} \sin(2R\sqrt{p^2} + \beta)$$

we obtain

$$F_2(x) \xrightarrow{x \rightarrow 1} (1-x)^{\nu+1/2} \quad (14)$$

with $\nu = 1$ in the case considered here. The reason for this non-continuous behaviour in the quark mass lies in a corresponding behaviour of the quark cut, which gives rise to the hadron poles by meromorphization. Its branch point at $m^2/x(1-x)$ recedes far behind the hadron pole if $m \neq 0$, $x \rightarrow 1$, but stays at zero in the massless case.

Some quark mass dependence also remains away from $x = 1$, as shown in the numerical calculation of Fig. 2 [again Eq. (3) has been used for $Q_i(p^2)$]. However, quite large quark masses of the order of a few hundred MeV are necessary to obtain a significant deviation from a flat quark distribution.

Similar quark mass effects are seen also in the meson form factor. From Eq. (12) we obtain

$$F(q^2) \sim 1/\sqrt{-q^2}, \quad -q^2 \rightarrow \infty \quad (15)$$

in the massless case and

$$F(q^2) \lesssim (1/\sqrt{-q^2})^{1/2 + 5/4} \quad (16)$$

for massive quarks^{*)}. The behaviour in the lower q^2 régime is shown in Fig. 3. For comparison the available data on the pion form factor are given. With $R = 2.5 \text{ GeV}^{-1}$ taken from the discussion of the hadron spectrum¹⁾, and a quark mass of 300 MeV we obtain an almost perfect agreement^{**)}.

$F(q^2)$ is very sensitive to the value of R as shown in Fig. 3b. Hence it is indeed related to the spatial extension of the hadron, and a space-time interpretation of Migdal's approach in the sense of a bag model is possible.

*) Because of the complicated overlap of oscillations in Eq. (12) we give only an upper limit.

**) This result should not be overestimated since only the zeroth order contribution is included.

The zeroth order results discussed so far are not very realistic for the following reason. In this approximation a hadron consists of the valence quarks only, which carry the whole momentum. Qualitatively new effects appear in next order [$O(g^2)$ gluon corrections]. The graphs in Fig. 1b provide a triple spectral function giving rise [via Eq. (5)] to a vector-dominance contribution to the form factor and also to a Regge (including Pomeron) contribution to the structure function. The corresponding behaviour of the structure function near $x = 0$ is $x^{1-\alpha(0)}$, apart from renormalization group effects. The Pomeron contribution represents the usual quark-gluon sea term. There is no special problem with the Pomeron (apart from the numerical value of the intercept) in this approach. It is found as the quark-gluon singlet mixing term in the spectrum¹⁾, and appears in the current channel of the three-point function through a mixing of Fig. 1b₂ with the singlet part of Fig. 1b₁.

Other $O(g^2)$ contributions reduce the energy carried by the valence quarks further. A new phenomenon appears in connection with the hard gluon graphs Fig. 1c (for the form factor) and the $O(g^4)$ contribution (Fig. 1d) to the structure function. These are the lowest order contributions whose behaviour at large q^2 and $x \rightarrow 1$, respectively, is not sensitive to the behaviour of $Q_1(p^2)$ at large argument, and which give rise to dimensional counting⁷⁾. From these graphs we obtain

$$F_2(x) \sim (1-x)^2 \quad (17)$$

at large x and

$$F(q^2) \sim 1/q^2 \quad (18)$$

for large q^2 *). Whether these terms are really dominant at large q^2 (or $x \rightarrow 1$) depends on the details of the damping effect of $Q_1(t)$ which determines the other contributions. Absolute and relative normalizations are fixed in the approach studied here.

Numerical work is in progress on these Regge and hard gluon contributions.

In conclusion, we have shown that there is a unique parameter-free solution to the three-point functions in Migdal's approach, once the two-point function (i.e. the spectrum) is known. Practical problems arise since the wave function $Q_1(p^2)$ is explicitly known only in the case of zero quark mass. A preliminary phenomenological analysis of meson form factor and structure function, however, leads to a quark mass of a few hundred MeV, i.e. a typical constituent quark mass.

*) It is interesting to note that the coefficient in Eq. (18) is proportional to the squared quark mass due to a cancellation in the traces.

The extension to n -point amplitudes ($n \geq \phi$) is obvious, in principle, although more complicated in practice because a complete set of operators has to represent intermediate states.

The over-all physical picture emerging from this approach is quite appealing and shows that Migdal's starting point is sensible. Already a few low-order contributions give the whole range of expected physical phenomena.

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Figure captions

- Fig. 1 : Several low-order contributions to the three-point function as quoted in the text.
- Fig. 2 : Zeroth order quark density $q(x) \sim (1/x) F_2(x, q_0^2)$ for several quark masses ($m_q = 0, 0.15, 0.3, 0.45 \text{ GeV}$) and $R = 2.5 \text{ GeV}^{-1}$.
- Fig. 3 : Meson form factor for several values of quark mass m_q and R
a) $R = 2.5 \text{ GeV}^{-1}$, $m_q = 0, 0.15, 0.3, 0.45 \text{ GeV}/c^2$;
b) $m_q = 0.3 \text{ GeV}/c^2$, $R = 1.5, 2.5, 3.5 \text{ GeV}^{-1}$.
Data on pion form factor taken from Gourdin⁸).

Fig. 1

Fig. 2

Fig. 3