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CHIRALITY SELECTION RULES AND THE $U(1)$ PROBLEM ^{*)}

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A B S T R A C T

The conventional theory for the lack of $U(1)$ chirality invariance in the presence of topological charge ν fails to resolve the $U(1)$ problem : it involves selection rules which forbid chirality changes ± 2 needed to generate the charged-pion decay constant F_π . Possible remedies are considered ; if they work, F_π^2 is proportional to the average value of ν^2 per unit Euclidean volume.

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't Hooft ^{1),2)} has observed that non-zero values of ν , the topological number of Belavin et al. ³⁾, permit non-conservation of $U(1)$ chirality ^{1),2),4)}

$$\Delta \chi = 2N\nu \quad (1)$$

in N flavour quantum chromodynamics (QCD) with massless quarks. It was suggested that this mechanism be used to resolve the $U(1)$ problem ⁵⁾⁻¹¹⁾ which has long afflicted quark theories.

However, the main point of this note is that if ν is an integer, Eq. (1) and its generalization

$$\Delta \chi_L = 2L\nu, \quad (\text{exact } SU(L) \times SU(L)) \quad (2)$$

for chiralities χ_L corresponding to gauge-invariant $U(1)$ currents ^{*)}

$$J_{\mu 5} = \sum_{i=1}^L \bar{q}_i \gamma_\mu \gamma_5 q_i, \quad (L = 2, 3, \dots, N) \quad (3)$$

amount to selection rules against resolving the $U(1)$ problem. These rules forbid transitions in which the chirality changes by only two units, so $\langle \bar{q}_i q_i \rangle_\theta$ vanishes in the $SU(L) \times SU(L)$ limit for each value of $i = 1, 2, \dots, L$. (Here $\langle \rangle_\theta$ denotes the θ vacuum ^{4),12)} expectation value.) This means that the conventional scheme (WKB method applied to multi-instanton configurations) is of no use because it respects the selection rules with integer ν . To avoid the disaster and hence show that unwanted small-mass bosons are absent, we need new field configurations or methods of calculation. Specific proposals are not yet available, but the following general conclusions emerge :

*) Quark fields q_i have a flavour index $i = 1, 2, 3, 4, \dots, N$ distinguishing up, down, strange, charmed, ..., quarks. Summation over colour indices is understood. Conventions for Minkowski space are $(g_{\mu\nu}) = (1, -1, -1, -1)$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\gamma_5^2 = 1$, $\epsilon_{0123} = -\epsilon^{0123} = +1$, and for Euclidean space, $\epsilon_{1234} = +1$.

- (i) the problem is not resolved by retaining the selection rules and assuming the existence of fractional values ^{*}) of ν , even though Eq. (2) gives $\Delta\chi_L = \pm 2$ for $|\nu| = 1/L$;
- (ii) it may be possible to violate the selection rules without introducing observable zero-mass bosons or abandoning QCD. Then there is a relation for the pion decay constant F_π in terms of $\nu \neq 0$ sectors of the Euclidean vacuum functional ;
- (iii) the spontaneous breaking of $U(1)$ symmetry due to $\nu \neq 0$ effects cannot be simulated by using an effective Lagrangian which explicitly breaks the symmetry.

The first step is to show that the Ward identities ^{5),10)} for the $U(1)$ problem are modified because of $\nu \neq 0$ effects.

Since chiral $SU(N) \times SU(N)$ currents do not have anomalous divergences, their lack of conservation in the real world cannot be attributed to instanton effects. Therefore mass parameters M_i have to be included in the QCD Lagrangian density

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} F^2 + \sum_{i=1}^N \bar{q}_i (i\not{D} - M_i) q_i \\ & + \{ \text{gauge-fixing and ghost terms} \} \end{aligned} \quad (4)$$

where $F_{\mu\nu}^a$ is the field strength tensor for the colour gauge group, and D_μ is the gauge-covariant derivative for the quark field q_i . It is assumed that the chiral $SU(L) \times SU(L)$ limit ($M_1, \dots, M_L \rightarrow 0$, with $M_{L+1}, \dots, M_N \neq 0$) generates a multiplet of $(L^2 - 1)$ Nambu-Goldstone bosons. This multiplet acquires an average mass squared μ_L^2 when the $SU(L) \times SU(L)$ asymmetric term in $\mathcal{L}$,

$$\Delta_L = - \sum_{i=1}^L M_i \bar{q}_i q_i = O(\mu_L^2) \quad (5)$$

is turned on.

^{*}) Here ν is the total charge, so configurations ¹³⁾ of separated lumps with local charge $\pm \frac{1}{2}$ are not relevant : the lumps always occur in pairs with charge $\pm \frac{1}{2} \pm \frac{1}{2} = \text{integer}$.

For each value of L , ($2 \leq L \leq N$), there is a $U(1)$ problem [the " $U_L(1)$ problem"] associated with the $U_L(1)$ current $J_{\mu 5}^L$. This current has an anomalous divergence ^{8),14)}

$$\partial^\mu J_{\mu 5}^L = (g^2 L / 16 \pi^2) F \cdot F^* + D_L \quad (6)$$

$$D_L = 2i \sum_{i=1}^L M_i \bar{q}_i \gamma_5 q_i = O(\mu_L^2) \quad (7)$$

where g is the gluon coupling constant and $F_{\alpha\beta}^*$ denotes the dual tensor $\frac{1}{2}\epsilon_{\alpha\beta\gamma\delta} F^{\gamma\delta}$. Operators O_n may be classified according to their $U_L(1)$ chirality χ_L ,

$$[O_n, Q_5^L] = \chi_L(n) O_n \quad (8)$$

where Q_5^L is the partially conserved $U_L(1)$ generator given by Bardeen's prescription ⁸⁾. [Thus a right-handed quark $(1+\gamma_5)q_i$ has $\chi_L = +1$ for $1 \leq i \leq L$ and $\chi_L = 0$ for $L+1 \leq i \leq N$.]

Only the cases $L=2,3$ matter phenomenologically. They should be carefully distinguished.

- (i) For $L=2$, there are three axial-vector currents $\mathcal{F}_{\mu 5}^A$, ($A=1,2,3$) with pions as Nambu-Goldstone bosons :

$$\langle \pi(q) | \mathcal{F}_{\mu 5} | 0 \rangle = -i q_\mu F_\pi, \quad (F_\pi \approx 94 \text{ MeV})$$

The extra $U_2(1)$ current is a fourth (not ninth) axial-vector current which becomes conserved (apart from the anomalous term). The question is whether a fourth massless boson is needed in the $SU(2) \times SU(2)$ limit. The lowest mass state with the right quantum numbers [ignoring $SU(3)$ and higher symmetries] is $\eta(549)$. Breaking of $SU(2) \times SU(2)$ invariance is measured by the parameters

$$\begin{aligned} \mu_2 &= m_\pi = \text{pion mass} \\ M &= M_1 = M_2 = O(m_\pi^2) \end{aligned} \quad (9)$$

The $O(m_\pi^2)$ dependence of M follows from the fact ¹⁵⁾ that the measurable ratio M/M_3 is $O(m_\pi^2/m_K^2)$.

- (ii) The case $L=3$ refers to chiral $SU(3) \times SU(3)$ symmetry, so $\eta(549)$ becomes a member of the (π, K, η) octet of Nambu-Goldstone bosons. Properties of the ninth current $J_{\mu 5}^3$ and possibly a ninth boson ($\eta'(958)?$) have to be considered in this limit.

Consider the most important case, $L=2$. The $SU(2) \times SU(2)$ Ward identity ^{*)}

$$\begin{aligned} \langle \Delta_2 \rangle &= i \int d^4x \, T \langle \partial^\mu \mathcal{F}_{\mu 5}^3(x) \partial^\nu \mathcal{F}_{\nu 5}^3(0) \rangle \\ &= m_\pi^2 F_\pi^2 + O(m_\pi^4) \end{aligned} \quad (10)$$

is well known ¹⁷⁾; it corresponds to the equal-time commutator

$$\begin{aligned} i \Delta_2 &= -i M \sum_{i=1}^2 \bar{q}_i q_i \\ &= \left[\int d^3x \, \mathcal{F}_{05}^3(x), \partial^\nu \mathcal{F}_{\nu 5}^3 \right] \end{aligned} \quad (11)$$

There is also a $U_2(1)$ equal-time commutator

$$i \Delta_2 = \frac{1}{4} [Q_5^2, D_2] \quad (12)$$

The current $J_{\mu 5}^2$ is observable, so zero-mass particles do not couple to it :

$$\int d^4x \, \partial^\mu T \langle J_{\mu 5}^2(x) D_2(0) \rangle = 0 \quad (13)$$

Equations (6) (for $L=2$), (12) and (13) imply the identity

$$-4i \langle \Delta_2 \rangle = (g^2/8\pi^2) \int d^4x \, T \langle F.F^*(x) D_2(0) \rangle + \int d^4x \, T \langle D_2(x) D_2(0) \rangle \quad (14)$$

The term $\langle F.F^*(x) D_2 \rangle$ can be eliminated by using Eq. (6) again :

^{*)} The notation $\langle \prod_n O_n \rangle$ refers to the $\theta=0$ vacuum expectation value of an operator product $\prod_n O_n$. Parity conservation in strong interactions dictates the choice $\prod_n \theta=0$. Terms of the form $O(m_\pi^4)$ are understood to include the $\ln m_\pi$ dependence found by Li and Pagels ¹⁶⁾.

$$\int d^4x \, T \langle F.F^*(x) D_2(0) \rangle = - (g^2/8\pi^2) \int d^4x \, T \langle F.F^*(x) F.F^*(0) \rangle \quad (15)$$

Let us introduce the notation

$$\langle\langle v^2 \rangle\rangle = -i (g^2/32\pi^2)^2 T \langle \int d^4x \, F.F^*(x) F.F^*(0) \rangle \quad (16)$$

which is suggested by the definition ³⁾

$$v = (g^2/32\pi^2) \int d^4x \, F.F^*(x) \quad , \quad (\text{Euclidean space}) \quad (17)$$

An alternative representation is

$$\langle\langle v^2 \rangle\rangle = \lim_{V \rightarrow \infty} (V Z)^{-1} \sum_{v=-\infty}^{\infty} v^2 Z_v \quad (18)$$

where Z_v is the contribution of the v^{th} topological sector to the Euclidean vacuum functional $Z = \sum_v Z_v$, and V is the Euclidean four-volume.

Then Eqs. (14)-(16) yield the result

$$\langle \Delta_2 \rangle = \frac{i}{4} \int d^4x \, T \langle D_2(x) D_2(0) \rangle + 4 \langle\langle v^2 \rangle\rangle \quad (19)$$

Now we impose the requirement that there should be no isoscalar 0^- particle with mass $O(m_\pi)$:

$$\int d^4x \, T \langle D_2(x) D_2(0) \rangle = O(m_\pi^4) \quad (20)$$

Equations (10), (19) and (20) imply a relation for the pion-decay constant :

$$m_\pi^2 F_\pi^2 = 4 \langle\langle v^2 \rangle\rangle + O(m_\pi^4) \quad (21)$$

Thus F_π is generated by $v \neq 0$ contributions to the vacuum functional.

Note the absence of a new current-algebraic relation for the mass of an η like particle^{*)}. This is a consequence of Eq. (20) : if all η like particles have mass $O(1)$ [i.e., not $O(m_\pi)$], they cannot be made soft. Of course, a model-dependent calculation could well yield estimates of masses. However, $v \neq 0$ effects are irrelevant for the $SU(N) \times SU(N)$ Nambu-Goldstone bosons $\pi, K, \eta, \dots$; their masses can be chosen at will by varying the free parameters $\{M_i\}$. In particular, the lack of conservation of the $SU(3)$ currents, and hence the η - π mass difference, is entirely determined by $(M_3 - M)$; $SU(3)$ currents do not have anomalous divergences.

The WKB approximation yields a positive value for $\langle\langle v^2 \rangle\rangle$, but there is a problem with its magnitude. Contributions from the topological sector v are proportional to the fermionic determinant

$$\prod_{i=1}^N [\det(\not{D} + M_i)]_v = O\left[\prod_{i=1}^N M_i^{|v|}\right] \quad (22)$$

The right-hand side of (22) is a consequence of the index theorem¹⁸⁾⁻²⁰⁾, which places a lower bound on the number of zero eigenvalues of $\not{D}$ (for all Euclidean field configurations A_μ which can be smoothly mapped onto the hypersphere in five dimensions). Note that it is the total charge v which matters : for example, if A_μ looks like a superposition of n_+ instantons and n_- anti-instantons then, for each flavour, $\not{D}$ has at least $|v| = |n_+ - n_-|$ zero eigenvalues. According to Eqs. (9) and (22), the v^{th} sector contributes a term $O[m_\pi^4 |v|]$, so the result for the complete amplitude is

$$\langle\langle v^2 \rangle\rangle_{\text{WKB}} = O(m_\pi^4) \quad (23)$$

This estimate of $\langle\langle v^2 \rangle\rangle$ is not satisfactory : Eq. (21) shows that the $U_2(1)$ problem can be solved only if $O(m_\pi^2)$ contributions to $\langle\langle v^2 \rangle\rangle$ can be found. Similarly, $\langle \bar{q}_i q_i \rangle_{\text{WKB}}$ is $O(m_\pi^2)$ for $i=1,2$. These results are not accidents : they exhibit the difficulty with the selection rules (2) (for $L=2$) noted at the beginning of this paper.

*) Compare Weinberg's analysis¹⁰⁾ - he finds that the upper bound $\sqrt{3}m_\pi$ on the mass of the lightest isoscalar 0^- meson disappears when Kogut-Susskind poles⁹⁾ (i.e., $v \neq 0$ effects) are introduced in an effective chiral Lagrangian ; there is no constraint on m_η or other η like masses, apart from the Gell-Mann - Okubo formula.

Note that the difficulty can be seen directly in the limit of exact $SU(2) \times SU(2)$ ($M \rightarrow 0$; $M_3, \dots, M_N > 0$) without using Eq. (9). Let H_8 be the $SU(3)$ breaking part of the energy density. The (π, K, η) octet remains split in the $SU(2) \times SU(2)$ limit, so $\langle \pi | H_8 | \pi \rangle$ does not vanish ($\simeq -2m_K^2/3$). Contract both pions and use the $(3, \bar{3}) + (\bar{3}, 3)$ commutation relations :

$$3F_\pi^2 \langle \pi | H_8 | \pi \rangle = M_3 \left\langle \sum_{i=1}^2 \bar{q}_i q_i \right\rangle \quad (24)$$

The selection rule is $\Delta X_2 = 4v$, but on the right-hand side of (24), we have $\Delta X_2 = \pm 2$. To prevent F_π from vanishing in the $SU(2) \times SU(2)$ limit, we need $v = \pm \frac{1}{2}$, or a violation of the selection rule, or both.

The problem is to find a sensible method of calculation which does not respect the usual rules and hence permits an acceptable estimate of $\langle v^2 \rangle$ to be obtained. Unforeseen non-perturbative effects could arise from infinite-action configurations, but it is hard to find practical analytic methods to test proposals of this type. However, there may be an alternative : it is not clear that all finite-action configurations obey the usual rules, because the relevant theorems have been proved only for the $O(5)$ hypersphere, not for non-compact Euclidean space. An explicit violation of the index theorem in Euclidean QCD₄ has yet to be found, but Hawking²⁰⁾ has found a Euclidean gravitational metric with asymptotically negligible curvature (the Taub-NUT solution) for which the conventional index theorem is not valid. If the index theorem fails, it is possible that v is not necessarily an integer : in general, it is the index theorem which restricts topological invariants such as v to integer values on compact manifolds. Although Belavin et al.³⁾ consider non-compact Euclidean space, their analysis seems to be restricted to asymptotically smooth configurations, i.e., those which remain regular when mapped onto the $O(5)$ hypersphere. Not all smooth finite-action configurations obey this requirement, but so far, I have not found an example with $v \neq \text{integer}$. (For infinite-action paths, v can take any real value.) Non-integer values of v would imply a vacuum structure more complicated than that^{4),12)} previously considered.

There are strong restrictions on allowed violations of the selection rules. Consider the decomposition of an amplitude into v sectors :

$$T \langle \pi O_n \rangle_\Theta = \sum_\nu e^{-i\nu\Theta} T \langle \pi O_n \rangle_\nu \quad (25)$$

$$(g^2/32\pi^2) T \langle \int d^4x F.F^*(x) \pi O_n \rangle_\nu = \nu T \langle \pi O_n \rangle_\nu \quad (26)$$

[Equations (10)-(24) involve the $\Theta=0$ vacuum, but here it is convenient to permit arbitrary Θ .] Equation (6) is valid for arbitrary Θ , so it must be true for individual ν sectors :

$$\begin{aligned} & \int d^4x \partial^\mu T \langle J_{\mu 5}^L(x) \pi O_n \rangle_\nu \\ &= [2L\nu - \sum_m \chi_L(m)] T \langle \pi O_n \rangle_\nu + \int d^4x T \langle D_L(x) \pi O_n \rangle_\nu \end{aligned} \quad (27)$$

Equation (2) is recovered in the $SU(L) \times SU(L)$ limit $D_L \rightarrow 0$ only if the left-hand side of (27) vanishes for individual ν sectors. Suppose it does not vanish ^{*}) : i.e.,

$$\int d^4x e^{iq \cdot x} T \langle J_{\mu 5}^L(x) \pi O_n \rangle_\nu \sim a_\nu q_\mu / q^2, \quad (q \rightarrow 0) \quad (28)$$

[These are not Kogut-Susskind poles ⁹⁾, because $J_{\mu 5}^L$ is the gauge-invariant observable current. Kogut-Susskind poles couple to the gauge-dependent, partially conserved "symmetry" current of Bardeen ⁸⁾, and their "existence" amounts to nothing more than the existence of non-zero matrix elements of $\int d^4x F.F^*(x)$.] The $\Theta=0$ amplitudes $T \langle J_{\mu 5}^L \{ \int F.F^* \}^p \pi O_n \rangle$ are observable, so the conditions

$$\sum_\nu \nu^p a_\nu = 0 \quad (29)$$

^{*}) This is the analogue of a boundary correction to the index theorem ¹⁸⁾⁻²⁰⁾ in a non-compact space. Jackiw and Rebbi ¹⁹⁾ suggest another correction (failure of "term-by-term integration") but this seems to correspond to a change in short-distance behaviour and hence trouble with Eq. (8). The latter problem may arise only for infinite-action fields, as in their examples.

must be satisfied for all integers $p \geq 0$; otherwise, the zero-mass poles are observable. There are infinitely many sequences $\{a_\nu\}$ which satisfy (29) : they correspond to all smooth functions

$$f(\theta) = \sum_{\nu} a_{\nu} e^{-i\nu\theta}$$

with an infinite-order zero at $\theta=0$ and periodicity P (i.e., $P=2\pi$ for integer ν). Since $f(\theta)$ is the amplitude for the $q^2=0$ pole to appear in $T < \int_{\mu_5}^L \Pi O_n >_{\theta}$, strongly interacting zero-mass particles are present in some $\theta \neq 0$ worlds. However, these worlds do not communicate ⁴⁾ with our $\theta=0$ world, so there is no obvious reason to exclude this possibility.

The case $a_{\nu} \equiv 0$ with fractional ν does not lead to a resolution of the $U(1)$ problem. Equation (27) reduces to the formula

$$\xi_L(\nu) T \langle \Pi O_n \rangle_{\nu} = T \langle \int d^4x D_L(x) \Pi O_n \rangle_{\nu} \quad (30)$$

where $\xi_L(\nu)$ denotes the factor $(\sum_n \chi_L(n) - 2L\nu)$. Since D_L is a sum $D_L^+ + D_L^-$ of operators $D_L^{\pm}$ with $\chi_L = \pm 2$, Eq. (30) can be iterated $(K-1)$ times :

$$T \langle \Pi O_n \rangle_{\nu} = T \langle \prod_{k=1}^K \sum_{\epsilon_k = \pm 1} (\xi_L(\nu) + 2 \sum_{i=2}^k \epsilon_i)^{-1} \int d^4x_k D_L^{\epsilon_k}(x_k) \Pi O_n \rangle_{\nu} \quad (31)$$

(where $\sum_{i=2}^k \epsilon_i$ vanishes for $k=1$ and D_L^{ϵ} denotes $D_L^{\pm}$ for $\epsilon = \pm 1$). The iteration terminates at $K = \frac{1}{2} |\xi_L(\nu)|$ if νL is an integer ; otherwise it may be continued indefinitely. Each $D_L^{\pm}$ in (31) contributes $O(\mu_L^2)$, so the result is

$$T \langle \Pi O_n \rangle_{\nu} = O \left[\mu_L^{|\xi_L(\nu)|} \right] T \langle \Pi O_n \rangle \quad , \quad (\nu L = \text{integer}) \quad (32)$$

$$\lim_{\mu_L \rightarrow 0} \mu_L^{-2K} T \langle \Pi O_n \rangle_{\nu} = 0 \quad , \quad (\text{all } K \geq 0, \nu L \neq \text{integer}) \quad (33)$$

The problem is that $\langle\langle v^2 \rangle\rangle$ cannot be $O(\mu_L^2)$ for both $L=2$ and $L=3$. If $\prod_n O_n$ is the unit operator, the power $|\xi_L(v)|$ becomes $^{*)} 2L|v|$. The only $O(m_\pi^2)$ contribution is from $|v| = \frac{1}{2}$, but Eq. (33) requires this term to vanish faster than any power of μ_3^2 for small μ_3 . The same conclusion follows if $\langle \bar{q}_i q_i \rangle$ is considered.

Even if some a_v are not zero, it is clear that it will be necessary to go beyond the $O(\hbar)$ fermionic determinant in Eq. (22). The trouble with that determinant is that it is factorized into flavour sectors ; if a zero eigenvalue of $\not{D}$ occurs for one flavour, it occurs for all of them simultaneously.

The last point concerns the use of an effective Lagrangian ^{1),2)} to simulate $U(1)$ symmetry breaking by instantons, i.e.,

$$\mathcal{L} \longrightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \mathcal{L}_+ + \mathcal{L}_- \quad (34)$$

where $\mathcal{L}_0$ accounts for QCD without instantons and $\mathcal{L}_\pm(x)$ describes the effect of an instanton (anti-instanton) concentrated at x . If the $SU(L) \times SU(L)$ limit is being considered, $\mathcal{L}_\pm$ has $\chi_L = \pm 2L$ and the $F.F^*$ term in (6) is replaced by $i[Q_5^L, \mathcal{L}_+ + \mathcal{L}_-]$:

$$(g^2/32\pi^2) \int d^4x F.F^*(x) \longrightarrow -i \int d^4x \{ \mathcal{L}_+(x) - \mathcal{L}_-(x) \} \quad (35)$$

The problem with this substitution is that the left-hand side of (35) commutes with Q_5^L but the right-hand side does not. In other words, vertices $\mathcal{L}_\pm[q, \bar{q}]$ with the properties

$$\begin{aligned} [Q_5^L, \mathcal{L}_+ + \mathcal{L}_-] &= \text{non-trivial} \\ [Q_5^L, [\mathcal{L}_+ + \mathcal{L}_-]] &= 0 \end{aligned} \quad (36)$$

do not exist. An effective vertex can be obtained for a particular Green's function ^{1),2)}, but the point is that, in general, different Green's functions G produce different effective vertices $\mathcal{L}_\pm[q, \bar{q}; G]$. There is no universal vertex.

*) Thus Eq. (22) is a special case of (32).

It cannot be argued that the failure of $[Q_5, [Q_5, \mathcal{L}_+ + \mathcal{L}_-]]$ to vanish is unimportant. Consider the application of Eq. (6) sector-by-sector. To obtain $\langle \prod_n O_n \rangle_v$, the exponential in

$$T \langle \prod_n O_n \rangle \longrightarrow T \langle in | \prod_n O_n \exp i \int d^4x \mathcal{L}_{\text{eff}}(x) | in \rangle$$

is expanded by using the identity

$$T \exp(A + B) = T \{F(AB)\} + \sum_{v=1}^{\infty} T \{(A^v + B^v) F^{[v]}(AB)\} \quad (37)$$

where $F(z)$ is the entire function

$$F(z) = \sum_{n=0}^{\infty} z^n / (n!)^2 = I_0(2z^{1/2})$$

with v^{th} derivative $F^{[v]}(z)$, and A and B are set equal to $i \int d^4x \mathcal{L}_+(x)$ and $i \int d^4x \mathcal{L}_-(x)$, respectively. The first term in (37) gives the $v=0$ sector, while the terms $A^v F^{[v]}$ and $B^v F^{[v]}$ correspond to charge $+v$ and $-v$: this is easily checked by comparing Eqs. (26) and (35). Now consider Eq. (27) with all terms multiplied by v . Clearly, it is consistent to apply Eq. (26), because no new equal-time commutators are generated. However, there is a contradiction if we attempt to apply Eq. (35), because an additional equal-time commutator appears on the right-hand side:

$$(i/2L) T \langle \int d^4x [Q_5^L, [Q_5^L, \mathcal{L}_+(x) + \mathcal{L}_-(x)]] \prod_n O_n \rangle_v$$

This means that Eq. (6) is valid sector-by-sector in QCD but not in the $\mathcal{L}_{\text{eff}}$ formalism. A similar problem arises if we try to reproduce the $U(1)$ Ward identities involving $\langle\langle v^2 \rangle\rangle$.

The conclusion is that the substitution of $\mathcal{L}_{\text{eff}}$ for $\mathcal{L}$ completely changes the theory. The result is not QCD, but something which looks like a σ model with explicit $U(1)$ symmetry breaking and no Θ vacua. An effective Lagrangian for $v \neq 0$ effects in QCD has to be of the Weinberg type ¹⁰⁾, with $U(1)$ invariance broken spontaneously by including Kogut-Susskind poles.

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