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SCATTERING THEORY IN THE PRESENCE
OF A SOLITON ASYMPTOTIC THEORY

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A B S T R A C T

We write down LSZ reduction formulae for meson scattering in the presence of a soliton. We compute the S matrix elements between coherent states of mesons.

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1. INTRODUCTION

During the past three years many papers have been concerned with the quantization of solitons ¹⁾⁻³⁾. Most of the quantitative results have been obtained in the weak coupling limit, in an approximation where the soliton is quantized only in the first quantization. We then learn how to compute Green functions in a given superselection sector ²⁾. Green functions are computed using a perturbative expansion around a generalized free Hamiltonian $H_0(\phi)$ which is the quadratic part of the Hamiltonian $H(\phi+\phi_{cl})$ where the field ϕ has been translated with respect to the classical solution ϕ_{cl} corresponding to the studied soliton. H_0 , which is non-local, does not appear as a good candidate to be used as an asymptotic wave operator for the mesons.

In this paper we show that we have to use the Klein-Gordon operator for this purpose; this allows us to write down LSZ reduction formulae and to compute the S matrix elements between coherent states of mesons in the presence of a soliton [i.e., the generating functional of the S matrix elements ⁴⁾]. The crucial fact which leads to this asymptotic interpretation is that the classical solution connects different vacua in which the meson appears with the same mass. Actually, the mass renormalization is guaranteed in the translated theory as soon as the mass renormalization has been carried out properly in the vacuum sector.

Our analysis clearly does not rely on any particular model, neither on the space-time dimension. However, we shall illustrate some of our developments with the well-known Sine-Gordon model, to make our analysis more explicit.

2. QUANTIZATION IN A SUPERSELECTION SECTOR

Several different methods ²⁾ have been used to compute Green functions of mesons in a superselection sector, i.e., in the presence of a soliton treated like a firstly quantized object.

Let us denote by

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_\mu \phi)^2 - U(\phi) \quad (1)$$

the classical Lagrangian, $\phi_{cl}((x-x_0-vt)/\sqrt{1-v^2})$, a classical solution of the equations of motion. Quantizing the theory around this classical solution, one is led to introduce the operator

$$T = \square + U''(\phi_{cl}) \quad (2)$$

This T operator is used more or less technically as a wave operator.

Its spectrum plays a fundamental rôle in the analysis of the "translated" theory. It consists of a continuum

$$\omega_k = \pm \sqrt{k^2 + m^2}, \quad k \in]-\infty, +\infty[\quad (3)$$

and a discrete part containing at least one zero mode. This zero mode is interpreted in terms of the usual quantum q and p variables of the soliton. On the other hand, the mass m appearing in the continuum is the meson mass which comes in the theory developed around a vacuum. This fact is due to the property of ϕ_{cl} of connecting different vacua corresponding to the same meson mass. This is the reason why this continuous spectrum is interpreted as related to the mesons. Furthermore, it can be noticed that the purely mesonic contribution is obtained from the renormalized theory in the vacuum just by translation; the whole non-trivial part of this translation is related to the zero mode problem and to the soliton contribution. It then appears that also in higher orders the meson mass in the superselection sector is obtained from the meson mass in the vacuum sector and, if this mass has been renormalized to be m^2 in the vacuum sector, the same is true in the one-soliton superselection sector : in other words, for infinite time the meson is infinitely far from the soliton; then ϕ_{cl} tends exponentially to one of the vacua, $U''(\phi_{cl})$ tends exponentially to m^2 and the T operator to the Klein-Gordon operator with the mass of the meson. Asymptotically, the meson does not see the soliton and the theory is the same as in the vacuum sector - of course the in and out states are not related as in the vacuum sector.

3. GOING TO THE S MATRIX

Let us be more specific in what follows, and use as an example the Sine-Gordon model in which :

$$U(\phi) = \frac{m^2}{g^2} (1 - \cos g \phi)$$

$$\Phi_q\left(\frac{x-x_0-vt}{\sqrt{1-v^2}}\right) = \frac{4}{g} t g^{-1} \exp m \frac{x-x_0-vt}{\sqrt{1-v^2}} \quad (4)$$

$$T = \square + m^2 \cos 4 t g^{-1} \exp m \frac{x-x_0-vt}{\sqrt{1-v^2}} = \square + m^2 \left(1 - \frac{2}{\cosh^2 m \frac{x-x_0-vt}{\sqrt{1-v^2}}}\right)$$

the spectral decomposition of F , solution of $TF=0$, is of the form :

$$\begin{aligned} F(t, x) = & \int_{-\infty}^{+\infty} \frac{dk}{2\sqrt{k^2+m^2}} \left\{ e^{-i\sqrt{k^2+m^2}t + ikx} \frac{ik - m \tanh mx}{ik - m} f_k^- \right. \\ & + e^{i\sqrt{k^2+m^2}t - ikx} \frac{ik + m \tanh mx}{ik + m} f_k^+ \left. \right\} \\ & - \frac{\sqrt{m}}{2} \frac{1}{\cosh mx} (f_A + f_B t) \end{aligned} \quad (5)$$

We suppose that we have defined in and out states and computed Green functions of mesons between soliton states $|p\rangle$ and $|p'\rangle$,

$$G_{pp'}(p_1, \dots, p_n) = \langle p' | T \hat{\Phi}(p_1) \dots \hat{\Phi}(p_n) | p \rangle ; \sum_i p_i + p - p' = 0 \quad (6)$$

using some of the recently developed methods ²⁾.

We are now in a position to define coherent states, using the functions $f(x)$, solutions of the Klein-Gordon equation with mass m^2 :

$$\begin{aligned} |f_p^-\rangle_{in} &= \left\{ \exp -i \int dt dx f^-(t, x) \overleftrightarrow{\partial}_t \Phi_{in}(t, x) \right\} |p\rangle_{in} \\ \langle f_{p'}^+|_{out} &= \langle p' | \left\{ \exp i \int dt dx f^+(t, x) \overleftrightarrow{\partial}_t \Phi_{out}(t, x) \right\} \quad (7) \\ (\square + m^2) f^\pm(t, x) &= 0 \end{aligned}$$

The states $|f_p^-\rangle_{in}$ and $\langle f_{p'}^+|_{out}$ are asymptotically time independent.

Before going to LSZ reduction formulae, we must remember that the S matrix so defined will correspond to the Hamiltonian $H_{\text{int}} = H - H_0$, where H_0 is not a true free Hamiltonian; the residual interaction due to the difference between T and the free Klein-Gordon operator introduces a phase shift which can be computed exactly for a given model. More precisely :

$$\langle f_{p'}^+ | f_p^- \rangle_{\text{in}} = \frac{\langle f_{p'}^+ | S | f_p^- \rangle_{\text{Tout}}}{\langle f_{p'}^+ | f_p^- \rangle_{\text{Tout}}} \quad (8)$$

in the Sine-Gordon example, one sees from the explicit form of the eigenfunctions of T, Eq. (5), that if the spectrum of the T_{out} mesons is given by :

$$f^-(t, x) = \int \frac{dk}{2\sqrt{k^2 + m^2}} e^{-i\sqrt{k^2 + m^2}t + ikx} f_k^- \quad (9)$$

the observed spectrum will correspond to

$$g^-(t, x) = \int \frac{dk}{2\sqrt{k^2 + m^2}} e^{-i\sqrt{k^2 + m^2}t + ikx} f_k^- \frac{ik + m}{ik - m} \quad (10)$$

Using this parametrization, we write :

$$\langle f_{p'}^+ | f_p^- \rangle_{\text{Tout}} = \langle f_{p'}^+ | g_p^- \rangle_{\text{out}} \quad (11)$$

$$\langle f_{p'}^+ | S | f_p^- \rangle_{\text{Tout}} = \langle f_{p'}^+ | S | g_p^- \rangle_{\text{out}}$$

We can write down the reduction formulae :

$$\langle f_{p'}^+ | f_p^- \rangle_{\text{in}} = \langle p' | T \exp i \int dt dx f(t, x) (\square + m^2) \phi(t, x) | p \rangle \quad (12)$$

$$f(t, x) = f^+(t, x) + f^-(t, x)$$

Denoting by $\Gamma(\phi)$ the generating functional of the one-particle irreducible Green functions, computed, as we have indicated, using one of the standard methods referred above, one can get a compact formula for

$$_{out} \langle f_{p'}^+ | f_p^- \rangle_{in} ;$$

actually (12) can be written under the form :

$$_{out} \langle f_{p'}^+ | f_p^- \rangle_{in} = \exp i(\Gamma(\phi) + j\phi) \Big|_{\frac{\delta \Gamma}{\delta \phi} + j = 0, j = fK} \quad (13)$$

$$K = \square + m^2$$

The following calculation has been carried out by R. Stora. We can decompose $\Gamma(\phi)$ in its asymptotic part and the so-called interaction part :

$$\Gamma(\phi) = \Gamma_{as}(\phi) + \Gamma_{int}(\phi) = -\frac{1}{2} \phi K \phi + \Gamma_{int}(\phi) \quad (14)$$

so that ϕ , solution of $\delta \Gamma / \delta \phi + j = 0$, has to be computed through the iterative integral equation :

$$\phi = \Delta_F * \left(\frac{\delta \Gamma_{int}}{\delta \phi}(\phi) + j \right) = \tilde{\phi}(j) + \Delta_F * j \quad (15)$$

In terms of (15) we can write :

$$\Gamma(\phi) + j\phi = -\frac{1}{2} \tilde{\phi} K \tilde{\phi} + \frac{1}{2} j(\Delta_F * j) + \Gamma_{int}(\tilde{\phi} + \Delta_F * j) \quad (16)$$

Now we use the condition $j = fK$ which means that $\tilde{\phi}$ is defined through

$$\phi = f + \Delta_F * \frac{\delta \Gamma_{int}}{\delta \phi}(\phi) = f + \tilde{\phi} \quad (17)$$

So that

$$\begin{aligned} \langle f_{p'}^+ | f_p^- \rangle_{in} &= \exp i \left(\Gamma(\phi) + j\phi \right) \left| \frac{\delta \Gamma}{\delta \phi} + j = 0, j = f_K \right. \\ &= -\frac{1}{2} \tilde{\phi} K \tilde{\phi} + \Gamma_{int}(f + \tilde{\phi}) \end{aligned} \quad (18)$$

which concludes the calculation.

4. CONCLUDING REMARKS

Concerning formula (18), let us emphasize that, as far as the ultra-violet divergence structure is concerned, the generating functional $\Gamma(\phi)$ is identical with the corresponding meson vertex functional $\Gamma_{vac}(\phi)$ in the vacuum sector. Indeed, in this respect, $\Gamma(\phi)$ behaves like the translated $\Gamma_{vac}(\phi + \phi_{cl})$ and the normalization conditions which fix the ambiguities in Γ_{vac} must fix also $\Gamma(\phi)$.

Since the additional infra-red divergences coming from the zero mode problem have been solved in a very general way ^{6),7)}, one may consider the quantum theory in the soliton sector as being "renormalizable" in the conventional sense.

This conclusion can be readily generalized to the multisoliton case using the appropriate classical solutions. This fact will be crucial in computing non-trivial classical solutions in four space-time dimensions; it explains the otherwise miraculous cancellations ⁸⁾⁻¹⁰⁾. A more detailed proof as well as examples of computation will be given in a future publication.

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R E F E R E N C E S

We give here only some basic references where complete references can be found.

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