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GEOMETRODYNAMICS FOR QUARKS AND HADRONS: THE BARYON STATES

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ABSTRACT

In the geometrodynamical approach, baryons turn out to exist only in configurations where the quark co-ordinates are spatially aligned. As a consequence, the spectrum of $[SU(6), L^P]$ multiplets follows the pattern: $(56, 0^+)$, $(70, 1^-)$; $(56, 2^+)$, $(70, 2^+)$; $(70, 3^-)$; ... with their radial recurrences. This appears to solve an old quark-model puzzle.

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In this letter we intend to present the first results of the geometrodynamical approach^{1,2)} to the baryon states. The principles on which this approach is based have been discussed in a series of papers by one of us, and in particular in Ref. 2) they have been applied to describe with remarkable success several aspects of meson dynamics.

When such principles are employed to discuss the baryon (three spin- $\frac{1}{2}$ quarks) wave function

$$\Psi_{a\alpha, b\beta, c\gamma}(p; x_1, x_2, x_3) = e^{ipX} \Psi_{a\alpha, b\beta, c\gamma}(p; x, y) \quad (1)$$

where a, b and c are internal (SU_3, SU_4) indices, α, β and γ Dirac indices, and

$$\begin{aligned} X &= \frac{1}{3} (x_1 + x_2 + x_3) \\ x &= \frac{1}{\sqrt{2}} (x_2 - x_3) \\ y &= \sqrt{\frac{2}{3}} \left(x_1 - \frac{x_2 + x_3}{2} \right), \end{aligned} \quad (2)$$

they take the following form:

i) Confinement

$$\Psi_{a\alpha, b\beta, c\gamma}(p; x, y) = 0 \quad \text{for } x, y \notin R^8(p; x, y), \quad (3)$$

where $R^8(p; x, y)$ is a compact eight-dimensional space-time region with boundary $B^8(p; x, y)$.

ii) Continuity

For $x, y \in B^8(p; x, y)$ $\Psi(p; x, y)$ must be continuous, i.e.

$$\Psi_{a\alpha, b\beta, c\gamma}(p; x, y) = 0 \quad \text{for } x, y \in B^8(p; x, y). \quad (4)^3)$$

iii) Wave equation

For $x, y \in R^8(p; x, y)$ Ψ obeys the simple differential equation

$$D_1 D_2 D_3 \Psi = 0 \quad (5)$$

where $D_i = (-i \not{\partial}_i + m_i)$ is the Dirac operator acting on the i^{th} co-ordinate.

iv) Approximate freedom

The "distance" of Ψ from the "free solution" $\Psi_{a\alpha, b\beta, c\gamma}^{(0)}(p; x, y)$, determined by the free equations

$$D_i \Psi^{(0)} = 0 \quad i=1,2,3. \quad (6)$$

is minimum⁴⁾.

In order to find solutions to the conditions i)-iv) we shall proceed in the same way as in Ref. 2): we find $\Psi^{(0)}$ first and then construct a solution of (5) which minimizes the distance from $\Psi^{(0)}$.

In solving the problem for $\Psi^{(0)}$ we are faced with a great surprise. For introducing the variables q_μ and k_μ conjugate to x_μ and y_μ respectively, and separating in the rest frame $[P_\mu \equiv (M, \vec{0})]$ the time- from the space-variables as required by rotational invariance and the bag structure⁵⁾, we obtain the following spatial equations:

$$(\vec{q}^2 + \vec{k}^2) \Psi^{(0)} = \vec{Q}^2 \Psi^{(0)}, \quad (7a)$$

$$2\vec{q} \cdot \vec{k} \Psi^{(0)} = A \Psi^{(0)} \quad (7b)$$

$$(\vec{q}^2 - \vec{k}^2) \Psi^{(0)} = B \Psi^{(0)}. \quad (7c)$$

Whilst (7a) can be easily solved, and its solutions can be given in a $O(6)$ basis, which naturally embodies the necessary quark permutational symmetry, (7b) and (7c) cannot be simultaneously solved in a bounded region of space due to the quantum "spread" of the momenta $\vec{q}$ and $\vec{k}$. We find that the only way out of this difficulty is to suppress three degrees of freedom by setting $(\phi_0 \text{ constant})$

$$\begin{aligned}\vec{x} &= \vec{R} \cos \frac{\phi_0}{2} \\ \vec{y} &= \vec{R} \sin \frac{\phi_0}{2},\end{aligned}\tag{8}$$

and quantizing only the vector $\vec{R}$. The constraint (8) corresponds to a spatial configuration where the three quarks are aligned along the direction of the vector $\vec{R}$. This illustrates the profound difference between the geometrical and the field theoretical approaches to the quark degree of freedom, being very difficult in the latter to understand how a similar constrained motion could arise.

Thus, solving Eqs. (7) amounts to finding the $O(6)$ solutions of (7a) and to restricting them to the subspace (8). We get in this way

$$\varphi_{\mu, LM, n}(\vec{R}, \phi_0) = e^{i\mu\phi_0} j_L(QR) Y_L^M(\Omega_{\vec{R}}),\tag{9}$$

where the total angular momentum L is given by

$$L = 2n + |\mu| \quad (n \text{ integer}),$$

μ takes three distinct values 0, 1, 2 according to the symmetry character of the wave function: i.e. $\mu = 0$ corresponds to a completely symmetric wave function, while $\mu = 1, 2$ is associated with a mixed symmetry configuration; and finally the bag condition $\varphi(|\vec{R}| = R_s) = 0$, quantizes as usual the momentum Q according to the equation

$$Q = \frac{\beta_{nL}}{R_s},\tag{10}$$

β_{nL} being the n^{th} positive zero of j_L .

Depending on the value of the angle ϕ_0 we have different spatial configurations for the three quarks. $\phi_0 = 0$ corresponds to the well-known quark-diquark configuration⁶⁾ depicted in Fig. 1, which seems to be favoured by present experimental information.

Once we have formed $\Psi^{(0)}$ it is very easy to obtain the solution of Eq. (5) which is as close as possible to $\Psi^{(0)}$. A full account of the derivation shall be given elsewhere⁷⁾; here we shall only present the results:

a) For a completely symmetric baryonic wave function the physical states can be organized in $[SU(6), L^P]$ multiplets following the pattern in Fig. 2. Note that the representations $(20, L^P)$ and $(70, 0^+)$ are absent, and that the parity of the states is always given by $(-1)^L$.

These are all consequences of the suppression of three degrees of freedom dictated by our theory, and seem to have experimental support⁸⁾.

b) The mass of a physical baryon state is given by the equation

$$M = \sum_{\substack{abc \\ ijk}} |\Psi_{ai,bj,ck}^{(SU_8)}|^2 (E_{1a} + E_{2b} + E_{3c}) - \omega \quad (11)$$

where $\Psi^{(SU_8)}$ ($a, b, c, = 1, \dots, 4$; $i, j, k = 1, 2$) are the normalized SU_8 wave functions; (m_a are the quark masses)

$$E_{1a} = \sqrt{\vec{Q}^2 + m_a^2}, \quad E_{2b} = E_{3b} = \sqrt{\vec{Q}^2 + m_b^2} \quad (12)$$

$$|\vec{Q}| = \frac{\beta_{nL}}{R_{s,nL}} \quad (13)$$

$$\omega = \frac{\pi}{R_{t,nL}} \quad (14)$$

Following exactly the same line of reasoning as in Ref. 2), we can set

$$R_{s,nL}^2 = R_0^2 \beta_{nL} \quad (15)$$

$$R_{t,nL}^2 = R_0^2 \beta_{nL} - C, \quad (16)$$

where R_0 and C are parameters to be determined. In the Table we give the mean values of the masses of the various SU_6 multiplets, setting in (15) and (16) $R_0^2 = 4 \text{ GeV}^{-2}$, and $C = 0$.

The mean masses in GeV of the $[SU(6), L^P]$ multiplets, according to (11), ... (16), setting $C = 0$, $R_0^2 = 4 \text{ GeV}^{-2}$ and $m_q^2 = 0.10 \text{ GeV}^2$.

$[SU(6) L^P]^n$	1	2	3	4	5
$(56, 0^+)$	1.14	2.07	2.71	3.24	3.69
$(70, 1^-)$	1.60	2.39	2.97	3.46	3.89
$(56, 2^+)$ $(70, 2^+)$	1.94	2.65	3.20	3.66	4.07
$(70, 3^-)$	2.23	2.89	3.41	3.85	4.24
$(56, 4^+)$ $(70, 4^+)$	2.48	3.10	3.60	4.03	4.41
$(70, 5^-)$	2.70	3.30	3.78	4.19	4.56
$(56, 6^+)$ $(70, 6^+)$	2.91	3.48	3.94	4.34	4.71
$(70, 7^-)$	3.10	3.65	4.10	4.49	
$(56, 8^+)$ $(70, 8^+)$	3.27	3.81	4.25	4.64	

An extensive analysis of the spectrum in the presence of SU_3 and SU_4 symmetry breaking shall be carried out elsewhere⁷⁾, we only mention that according to (11) Λ and Σ^0 are degenerate and that using $m_p = 100$ MeV, $m_\lambda = 460$ MeV and $m_c = 1860$ MeV⁹⁾ we get

$$\frac{1}{2} (m_\Lambda + m_\Sigma) - m_p \simeq 170 \text{ MeV}$$

and for the charmed baryon $C^+ \equiv (pcn)$

$$m_{C^+} - m_p \simeq 1360 \text{ MeV}.$$

These results are quite satisfactory, however a more complete understanding of the masses of the baryonic states must await the completion of the program of computations of unitarity corrections, as discussed in Ref. 2).

We conclude by stressing that the first applications of the geometrodynamical approach to the study of the baryon systems have been remarkably successful. The emerging dynamics is that of a constrained three-body quantum mechanical system, which explains the failure to observe experimentally the many states predicted by the quark model^{8,10)}. If this trend is confirmed it will pose some really serious problems to the quantum chromodynamical framework¹¹⁾ and will further corroborate our approach.

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REFERENCES AND FOOTNOTES

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- 2) G. Preparata, CERN preprint TH.2271 (1977).
- 3) More precisely, continuity is to be required for suitable scalar functions
 appearing in a Lorentz-covariant decomposition of the wave function.
- 4) The distance is calculated by an appropriate definition of the norm of the
 wave functions - see Ref. 2).
- 5) In the rest frame $B^0(p; x, y)$ must necessarily decompose into a close con-
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 six-dimensional space $(\vec{x}, \vec{y})$.
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- 8) The most recent review is in K. Lanius, Proceedings of the XVIII International
 Conference on High Energy Physics, Tbilisi (1976).
- 9) These values were found to reproduce accurately the meson spectrum in Ref. 2)
- 10) See, for example, R. Horgan and R.H. Dalitz, Nuclear Phys. 66B, 135 (1973).
- 11) The baryonic spectrum in QCD appears to be, as a matter of fact, even richer
 than the naïve quark model. See G. Rossi and G. Veneziano, CERN preprint
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FIGURE CAPTIONS

Fig. 1 : The three inequivalent quark-diquark configurations.

Fig. 2 : $[SU(6), L^P]$ pattern of physical baryon states.

Fig. 1

Fig. 2