A HIGH-PRECISION METHOD FOR CALIBRATION
OF DIGITAL TIME RECORDERS

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ABSTRACT

A method, the "Synchronous Random Phase Test", for calibrating time-to-digital converters (TDCs) is described. The method is applicable to TDCs that can record several successive time intervals and has a high intrinsic precision. Application of the method to the "digitrons" of the latest CERN muon g-2 experiment shows that relevant time intervals are measured with errors that are small fractions of a ppm. Use of the method to give fast linearity checks of Drift Chamber time recorders is also considered.

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1. INTRODUCTION

A method is presented for calibrating time-to-digital converters (TDCs) which have the capability of recording several successive time intervals. Examples are the "digitrons"¹⁻³⁾ used in the muon g-2 experiments^{4, 5)}, and digital time recorders used in association with drift chamber detectors. The method has a very high intrinsic accuracy. In an application of the method to the digitrons of the latest CERN g-2 experiment⁵⁾ (described in Section 4 below), time intervals of ~ 300 μ sec are calibrated to a small fraction of a ppm. The test itself, called the "Synchronous Random Phase Test" (SRPT) is described in the following section. Section 3 is devoted to a brief comparison with other tests described in the literature, while in Section 5 application to drift chamber time recorders, which have more modest requirements for calibration and linearity than have the g-2 experiment digitrons, is considered.

2. METHOD: THE SYNCHRONOUS RANDOM PHASE TEST

The TDC is assumed to have a least count of τ (i.e. a clock frequency $1/\tau$) and to be able to record several successive time intervals. In general there will be a "Start" signal to enable the TDC, and, for each time interval recorded, a "Stop" signal. The maximum time interval that can be recorded, $t_{\max}$, is referred to as the "time window" of the TDC. In the proposed test the Stop signals are generated by a free-running oscillator (referred to as the "reference oscillator") and the Start signal is generated randomly in time by, say, counting noise or a radioactive source. In this way the phase of the pulse train generating the Stop signals is random with respect to a given time bin in the TDC. The principle of the method can be understood by referring to Figs. 1a and 1b. The probability that a stop appears in the time interval Δt between the i^{th} and j^{th} time bins of the TDC [$\Delta t = (j - i + 1)\tau$] depends on the ratio of Δt to the period T of the reference oscillator. In Fig. 1a, $\Delta t/T = 0.5$, so there is a 50% probability, for each random start, that a stop will occur in Δt . In Fig. 1b, where $\Delta t/T = 0.9$, this probability is 90%. For n_s starts, the number of stops n recorded in Δt will have a r.m.s. deviation σ_n , which has its maximum value for the situation of Fig. 1a and will be smaller for that of Fig. 1b. In fact in the limit $\Delta t/T \rightarrow 1$, $\sigma_n \rightarrow 0$ as there will always be one stop recorded in Δt . In this simple example then, if $\Delta t/T$ is very close to unity there will always be n_s stops in Δt for n_s starts, and any significant deviations from this number must be attributed to measurement error in the TDC. As any difference between the expected (n_e) and observed (n_o) number of stops must be at least one count, it is required that $\sigma_n \ll 1$ for good sensitivity. However, σ_n cannot be too small. In the case where $\Delta t/T = 1$ exactly, the TDC clock and the "Stop" pulse train are locked in phase and the statistical character of the test is lost. The allowed lower limit to σ_n is discussed quantitatively at the end of this section.

We now consider an arbitrary time interval $\Delta t = (i - j + 1)\tau$ (i, j label time bins of the TDC, $i > j$) and oscillator period T . For each random start there will either be $m = \text{INT}(\Delta t/T)$ *) or $m + 1$ stops in the interval Δt with respective probabilities q, p where

$$\begin{aligned} p &= \Delta t/T - \text{INT}(\Delta t/T) \\ q &= 1 - p. \end{aligned} \quad (1)$$

For n_s starts, the expected number of stops is

$$n_e = n_s \Delta t/T \quad (2)$$

and, since the number of stops is distributed according to the binomial distribution⁶⁾, the r.m.s. error on n_e , σ_{n_e} is given by

$$\sigma_{n_e} = \sqrt{n_s p q}. \quad (3)$$

If now T is chosen so that

$$\Delta t/T = k + \epsilon, \quad (4)$$

where k is an integer and $0 < \epsilon \ll 1$, then, from Eqs. (1) and (3), $\epsilon = p$, σ_{n_e} is small, and the observed number of stops n_o can be substituted for n_e in Eq. (2) to give the time interval Δt in terms of T , n_o , and n_s :

$$\Delta t = \frac{n_o T}{n_s}. \quad (5)$$

The general condition for σ_{n_e} to be small is

$$\Delta t/T = (i - j + 1)\tau/T = k \pm \epsilon,$$

so the requirement on the ratio τ/T is

$$\tau/T \sim k/(i - j + 1), \quad (6)$$

i.e. it should be close to a fractional real number. It is convenient to specialize formula (6) to the case:

$$\tau/T \sim 1/L \quad (L = \text{integer}). \quad (7)$$

All time intervals i, j satisfying the condition

*) INT here stands for "integral part of".

$$i - j + 1 = kL \quad (k = 1, 2, 3, \dots) \quad (8)$$

will then have σ_{n_e} small and can be calibrated via the number of stops n_o observed in the interval for n_s starts [Eq. (5)]. The error on the time interval is given by Eqs. (2), (3), and (4) as:

$$\sigma_{\Delta t} / \Delta t = \frac{1}{k} \sqrt{\frac{pq}{n_s}}. \quad (9)$$

The number of time intervals defined by Eq. (8) is given in terms of the number of time bins r in the time window of the TDC:

$$r = t_{\max} / \tau,$$

by the relation

$$N_{\text{INT}} = rk_{\max} - \frac{Lk_{\max}(k_{\max} + 1)}{2}, \quad (10)$$

where $k_{\max}$ is the largest value of k for which $kL < r$. For the g-2 digitron tests described in Section 4 below ($r = 65535$, $L = 512$, $k_{\max} = 127$), N_{INT} is very large, $> 4 \times 10^6$! However, the correctness of operation of the TDC can be established by examining only a small fraction of these intervals.

We now consider the lower limit on the probability p to ensure that random sampling of the phase of the reference oscillator is made in the interval Δt . Figure 2 shows a time interval AB of length Δt as measured in the TDC at an instant when one pulse from the reference oscillator is coincident with A. As time advances, the train of k oscillator pulses will "beat" relative to a fixed TDC time bin (say A) with a frequency ν_{beat} of:

$$\nu_{\text{beat}} = \frac{\Delta t - kT}{kT} \frac{1}{kT} = \frac{p}{k^2 T}, \quad (11)$$

where Eqs. (1) and (4) have been used. The phase of the reference oscillator will be randomly sampled if, during the time t_m of the measurement, a large number of pulses of the reference oscillator advance past the point A in Fig. 2. As this "advance time" t_a is $1/k \times$ the "beat" period $= 1/k\nu_{\text{beat}}$, the condition becomes

$$\frac{t_m}{t_a} = \frac{t_m p}{kT} \gg 1.$$

So the requirement on p is:

$$p \gg \frac{kT}{t_m}. \quad (12)$$

As $kT \approx \Delta t$ the condition may also be written:

$$p \gg \Delta t/t_m. \quad (13)$$

3. COMPARISON WITH OTHER TEST METHODS

It is of some interest to compare the method described above with two other tests which have appeared in the literature:

- i) the "random test"^{1,7)};
- ii) the "crystal oscillator test"⁴⁾.

In (i) a random pulse generator is used to start the TDC, which is stopped by a free-running oscillator of period T_R slightly less than the time window t_w of the TDC. Only one stop is then recorded for each start. The distribution of stops should be uniform in the range $0 < t < T_R$ if the TDC has linear response. An arbitrary time interval $\Delta t = t_2 - t_1$ is calibrated in terms of the reference oscillator period by the relation:

$$\Delta t = \frac{nT_R}{n_s}, \quad (14)$$

where n_s is the number of starts and n the number of counts recorded in the interval Δt . The test (i) has the advantage of checking the whole time window of the TDC in an unbiased manner, but is lacking in statistical precision. The fractional error with which the time interval Δt is checked by Eq. (14) is

$$\sigma_{\Delta t}/\Delta t = 1/\sqrt{n}. \quad (15)$$

For the muon $g-2$ experiment where $\sigma_{\Delta t}/\Delta t \sim 2 \times 10^{-6}$ is needed, 2.5×10^{11} counts are required using method (i). This is hardly practicable.

The test (ii), which was used in the previous CERN $g-2$ experiment⁴⁾, derives both the start and a series of stop pulses from the same free-running oscillator, which has arbitrary frequency relative to the digitron clock. Owing to the action of the "derandomizer"⁴⁾ of the digitron, the time interval between the start and any one of the series of stop pulses will be recorded in two adjacent digitron time bins. The distribution of counts in these bins is given by the binomial distribution, and the mathematical analysis of the error expected in the time interval for n_s starts is closely parallel to that given above for the Synchronous

Random Phase Test (SRPT). Again the statistical error becomes small if the reference oscillator is almost synchronous with a sub-multiple of the digitron clock frequency. However, the test (ii) has the disadvantage that only fixed time intervals are checked, and so the time window of the digitron can be uniformly sampled only by repeating the measurement with different oscillator frequencies. The SRPT combines in one measurement the high precision attainable if the test (ii) is modified so that the reference oscillator is made synchronous with the digitron clock, together with the uniform sensitivity to the whole time window of the "random test" (i).

4. APPLICATION TO THE CERN g-2 EXPERIMENT

A block diagram of the experimental set-up used in the tests is shown in Fig. 3.

A 3 Hz oscillator gives an interrupt to the computer and fires a 10 msec one-shot which opens the gate G. The first subsequent count from the β source provides a "start" to the digitron. A series of Stop signals at intervals of 5.12 μ sec are provided by a Hewlett-Packard 8660B frequency synthesizer running at ~ 50 MHz and divided in frequency by 2^8 using a prescaler. The clock of the digitron has a frequency of ~ 100 MHz, and the frequency of both it and the HP synthesizer are compared at intervals during the measurement with a broadcast 1 MHz standard (atomic clock). The time window of the digitron is 655.35 μ sec. The time intervals recorded by the digitron are read out via CAMAC to the PDP 11/20 computer using the normal g-2 experiment data acquisition program and written on to magnetic tape for subsequent analysis on the CERN CDC 7600 computer.

Table 1 shows the mean digitron clock and oscillator frequencies during the tests of the seven digitrons (six of these are used during the normal running of the g-2 experiment).

The errors quoted on the frequencies are $\pm 1/2$ the frequency drift observed during each test, which lasted ~ 3 hours and comprised $\sim 27K$ random starts. The fractional drift varied between 4×10^{-9} and 5×10^{-8} for the digitron clock, and between 0.0×10^{-9} and 8×10^{-9} for the frequency synthesizer.

The linearity of response and calibration of the digitron relative to the reference oscillator were checked by defining 92 time intervals Δt_i , $i = 1, \dots, 92$, of length 307.2 μ sec (30720 digitron time bins) with starting times varying in 3.1 μ sec steps from 3.1 to 285.2 μ sec. In this way the range 3.1-592.4 μ sec of the time window was checked for linearity. Almost exactly 60 oscillator pulses per start were recorded in each Δt_i . Table 2 gives the values of the probability p defined in Eq. (1) for each digitron as well as the total number of starts and the expected number of stops and error, calculated using Eqs. (2) and (3), in each

time interval. The errors quoted on p are derived from the drifts observed in $\bar{f}_{\text{dig}}, \bar{f}_{\text{osc}}$ (see Table 1). The typical value of p is $\sim 5 \times 10^{-7}$, while the ratio $\Delta t/t_m$ is $307.2 \mu\text{sec}/3 \text{ hours} = 9 \times 10^{-8}$, so the inequality for random phase sampling [Eq. (13)] is satisfied. Other evidence for the randomness of the test is presented below. Table 2 also gives the mean numbers of counts $\bar{n}_0$ observed in the intervals Δt_i and the r.m.s. deviations, for each digitron. Finally, the fractional difference between $\bar{n}_0$ and n_e is given in ppm. The r.m.s. deviations of n_0 are significantly larger than σ_{n_e} , indicating the presence of some time jitter. For the time interval chosen, this is typically at the level of 3×10^{-7} . The more important time-slewing effect shown in the last row of Table 2 is at a much lower level -- a few hundredths of a ppm with different signs for the different digitrons. Within the level of sensitivity of this test there is no evidence of any time slewing or mean calibration error.

The linearity of response of the digitron -- the lack of bias in the Δt_i measurement for different starting times t_1 -- is shown in Figs. 4a to 4g for Digitrons Nos. 1 to 7; $10^6(n_0 - n_e)/n_e$ is plotted against the starting time t_1 . Statistical errors are indicated in each case. As one count in n_0 varies from 4 to 30 statistical standard deviations, n_0 should always be equal to the integer part of n_e in the absence of time jitter. However, the latter is found to be at a level far below that needed in the g-2 experiment. The maximum error in Δt_i , which is seen in Digitron No. 6 for a starting time of $46.5 \mu\text{sec}$, is still less than 2 ppm. Furthermore, it is evident from Figs. 4a to 4g that there is no significant variation or sign bias of the time jitter with starting time of the interval.

As a check of the correctness of the statistical assumptions made, Fig. 4h shows the result of the test applied to Digitron No. 1 choosing a time interval of $307.19 \mu\text{sec}$ (i.e. 30719 digitron time bins, one less than in Figs. 4a to 4g). The value of p is now 2×10^{-3} , giving a statistical error σ_{n_e}/n_e of 4.4 ppm. The $(n_0 - n_e)/n_e$ distribution is now dominated by the statistical error, and can be seen to have the correct width with $\sim 30\%$ of the points lying outside 1 standard deviation.

In conclusion it may be stated that both the absolute calibration and linearity for all seven of the digitrons used in the CERN muon g-2 experiment are such that the systematic errors associated with the operation of the digitrons are negligible compared with the statistical error of ~ 10 ppm of the experiment.

5. APPLICATION TO DRIFT CHAMBER TDCs

The high intrinsic precision of the SRPT can be used to perform extremely rapid checks of TDCs with more modest requirements as to linearity and absolute precision than those used in the CERN g-2 experiment.

As an example of this, consider a linearity test on a Drift Time Digitizer*) with the following characteristics:

$$t_{\max} = 2048 \text{ nsec}$$

$$\tau = 4 \text{ nsec}$$

$$\text{maximum number of stops/start} = 16 .$$

τ will normally correspond to the intrinsic spatial resolution of the chamber. A minimal requirement on the linearity would seem to be that the departure from linearity at $t = t_{\max}$ should be at most τ . In the "random test", described in Section 3 above, this could be detected by a departure from flatness of slope $\tau/t_{\max} = 1/512$ in the stop distribution. To detect such a departure at the 1σ level would require $(512)^2 = 262K$ random starts. Using the SRPT with the reference oscillator frequency chosen so that the statistical error on n_0 is much less than one count, only 512 stops are needed in the full time range to check the linearity to the accuracy quoted above. This requires only $512/16 = 32$ random starts. The optimum period for the reference oscillator is then $T \sim 128 \text{ nsec}$, corresponding to $k = 32$ in Eq. (4). If p is chosen so that 1 count is 4 standard deviations, i.e. $\sigma_{\Delta t}/\Delta t = 1/2048$ then from Eq. (9), $p = 7.8 \times 10^{-3}$. Allowing a factor of 10 in the inequality (13), 2.62 msec are needed to carry out the test. For comparison, assuming 1 μsec read-out time for each recorded time interval, the equivalent "random test" requires at least 262 msec, so in this application the SRPT is ≥ 100 times faster.

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*) These figures are taken from a talk on Some possible drift time digitizer designs, given at CERN by H. Verweij to the European Muon Collaboration (11 December 1975).

REFERENCES

- 1) R.A. Swanson, Rev. Sci. Instrum. 31 (1960) 149.
- 2) H.I. Pizer, Nuclear Instrum. Methods 123 (1975) 461.
- 3) I. Pizer, L. Van Koningsveld and H. Verweij, CERN 76-17 (1976).
- 4) J. Bailey, W. Bartl, G. von Bochmann, R.C.A. Brown, F.J.M. Farley, M. Giesch, H. Jöstlein, S. van der Meer, E. Picasso and R.W. Williams, Nuovo Cimento 9A (1972) 369.
- 5) J. Bailey, K. Borer, F. Combley, H. Drumm, C. Eck, F.J.M. Farley, J.H. Field, W. Flegel, P.M. Hattersley, F. Krienen, F. Lange, G. Petrucci, E. Picasso, H.I. Pizer, O. Runolfsson, R.W. Williams and S. Wojcicki, Phys. Letters 55B (1975) 420.
- 6) See, for example, Sir Ronald A. Fisher, Statistical methods for research workers, 14th edition (Oliver and Boyd, Edinburgh, 1970), p. 63.
- 7) W. Weber, C.W. Johnstone and L. Cranberg, Rev. Sci. Instrum. 27 (1956) 166.

Table 1
Digitron and reference oscillator frequencies

Digitron No.	1	2	3	4	5	6	7
$\bar{f}_{\text{dig}} \pm 1/2 \text{ drift}$ -10^8 Hz	630.8 $\pm$ 0.2	633.0 $\pm$ 2.4	635.6 $\pm$ 0.2	636.6 $\pm$ 1.0	636.0 $\pm$ 0.4	639.1 $\pm$ 0.5	640.5 $\pm$ 0.3
$\bar{f}_{\text{osc}} \pm 1/2 \text{ drift}$ $-5 \times 10^7 \text{ Hz}$	316.0 $\pm$ 0.2	318.4 $\pm$ 0.0	318.4 $\pm$ 0.1	318.3 $\pm$ 0.0	317.9 $\pm$ 0.5	320.4 $\pm$ 0.1	320.4 $\pm$ 0.1

Table 2

Digitron test results

Digitron No.	Number of starts n_s	p ($\times 10^{-7}$)	$n_e \pm \sigma_{n_e}$	$\bar{n}_o \pm \text{r.m.s. dev.}$ (mean of 92 intervals)	$\frac{10^6(n_o - n_e)}{n_e}$
1	27460	6.3 $\pm$ 0.9	1647600.02 $\pm$ 0.13	1647599.96 $\pm$ 0.51	-0.04
2	27062	0.23 $\pm$ 0.14	1623720.06 $\pm$ 0.25	1623720.02 $\pm$ 0.39	-0.025
3	27759	6.9 $\pm$ 1.5	1665540.02 $\pm$ 0.14	1665540.05 $\pm$ 0.52	0.018
4	27897	0.3 $\pm$ 5.7	1673820.00 $\pm$ 0.03	1673819.95 $\pm$ 0.50	-0.03
5	26260	1.9 $\pm$ 5.3	1575600.00 $\pm$ 0.07	1575599.98 $\pm$ 0.42	-0.013
6	26328	4.2 $\pm$ 3.0	1579680.01 $\pm$ 0.11	1579680.01 $\pm$ 0.60	0.00
7	26422	2.4 $\pm$ 2.4	1585320.01 $\pm$ 0.08	1585319.95 $\pm$ 0.40	-0.038

Figure captions

- Fig. 1 : The principle of the Synchronous Random Phase Test. The train of "Stop" pulses at intervals T has a random phase relative to the time interval Δt . a) $\Delta t/T = 0.5$; b) $\Delta t/T = 0.9$.
- Fig. 2 : Relation between the time bins of the TDC and the reference oscillator pulse train, $k = 2$, $L = 6$ [Eqs. (4) and (7)] is shown. The group of k oscillator pulses moves relative to the time interval AB with frequency ν_{beat} [Eq. (11)].
- Fig. 3 : Block diagram for test of g-2 experiment digitrons.
- Fig. 4 : a) to g) Plots of the observed minus expected counts divided by the expected counts in 92 time intervals of 307.2 μsec (30720 time bins) versus the starting time of the interval t_1 for Digitrons Nos. 1 to 7; h) As (a) except time interval is 307.19 μsec (30719 time bins). The $\pm\sigma$ lines indicate the expected statistical error.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

b)

c)

Fig. 4

Fig. 4

Fig. 4

Fig. 4