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QUARK CONFINEMENT AND INSTANTON-LIKE
EUCLIDEAN FIELD CONFIGURATIONS

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A B S T R A C T

We introduce the concept of "c instanton" configuration and show that the QED_2 confinement may be understood as a saturation of the functional integrals in terms of such new configuration.

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In two previous publications ^{1),2)} we have shown how the instanton idea ³⁾⁻⁶⁾ can be modified in order to incorporate the two-dimensional situation as the Schwinger model QED₂. The massless Schwinger model admits no "bare" instantons, i.e., non-trivial "winding number" solution of the classical field equation, however the fermion part of the functional integration induces a kind of intrinsic Higgs picture ⁷⁾ which in turn yields to an instanton interpretation of the chiral vacuum structure. In fact the typical feature of the model is that the Euclidean configuration of the composite $\bar{\psi}((1 \pm \gamma_5)/2)\psi$ vacuum-expectation values is "saturated" by a finite number of (anti)instantons, their number depending on the number of composite chiral changing fields. For the chiral vacuum structure only the asymptotic saturation in terms of a finite number of "induced" (long distance) (anti)instantons would be necessary and this is indeed what we expect to happen ²⁾ in four-dimensional quark theories of the QCD₄ type.

It was pointed out by Rothe and Swieca ⁷⁾ that in massless QED₂ the vacuum has a still finer structure related to confinement which in the relativistic formulation corresponds to non-trivial global gauge transformation with a "half-integer" winding number ⁷⁾. So the question arises whether in the functional integral, after the fermion integration, there are induced (i.e., non-classical) field configurations which can account for this finer structure ⁸⁾. In the following we will demonstrate that this indeed is the case. The new Euclidean configurations will be called "c instantons" (where c stands either for "covering" similar to the use of this word in "covering" groups, or c may be read as "confining" anticipating the physical consequences).

Consider for simplicity the gauge invariant two-point function

$$\begin{aligned}
 & \langle \psi_i(x) e^{-ie \int_x^y A_\mu(z) dz^\mu} \psi_i^+(y) \rangle \\
 &= Z^{-1} \int [dA] \exp \left\{ -S[A] - \Gamma^{[L]}[A] \right. \\
 &\quad + e \varepsilon_{\lambda\mu} \int d^2z A_\mu(z) \partial_\lambda (D(x-z) - D(y-z)) \\
 &\quad + ie \int d^2z A_\mu(z) \partial_\mu (D(x-z) - D(y-z)) \\
 &\quad \left. - ie \int_y^x A_\mu(z) dz^\mu \right\} \langle \psi_{i0}(x) \psi_{i0}^+(y) \rangle \quad (1)
 \end{aligned}$$

The "induced" action has a similar appearance to the one appearing in the two-point function of the composite $J_{\pm}^2$). The first term is the free photon part of the interaction, the second the contribution from the fermion determinant, the third and fourth represent the external field dependence of the classical Dirac equation Green's function. In addition to the new fifth contribution which is the Euclidean counter-part of the line integral, the notable difference compared to the old functional integral is the appearance of the factor $\frac{1}{2}$ in front of the second term ⁹⁾. This effective Lagrangian associated to the gauge invariant two-point function (in a more appropriate fashion : "one string correlation function") yields the following equation of motion :

$$\begin{aligned} & [(-\partial^2 + m^2) A_{\mu} + \partial_{\mu} \partial_{\nu} A_{\nu}](z) + \text{gauge terms} \\ & = e \varepsilon_{\lambda\mu} \partial_{\lambda} (\mathcal{D}(x-z) - \mathcal{D}(y-z)) \\ & \quad + i e \partial_{\mu} (\mathcal{D}(x-z) - \mathcal{D}(y-z)) \\ & \quad - i e (x-y)_{\mu} \int_0^1 dt \delta(xt + y(1-t) - z) \end{aligned} \quad (2)$$

The solution of this (effective) inhomogeneous field equation is :

$$\begin{aligned} A_{\mu}^c(z) & = \frac{\pi}{e} \varepsilon_{\lambda\mu} \partial_{\lambda} (\mathcal{D}(x-z) - \Delta(x-z; m^2) - \mathcal{D}(y-z) + \Delta(y-z; m^2)) \\ & \quad + \frac{i\pi}{e} \partial_{\mu} (\mathcal{D}(x-z) - \Delta(x-z; m^2) - \mathcal{D}(y-z) + \Delta(y-z; m^2)) \\ & \quad - i e (x-y)_{\mu} \int_0^1 \Delta(xt + y(1-t) - z; m^2) dt \end{aligned} \quad (3)$$

It has vanishing divergence ; thus the gauge terms in the effective action do not contribute to the value of the action of this solution.

We now show that this new instanton-like configuration which, apart from a string piece and a gauge contribution, consists of half of the old "angular-like" instanton function and therefore is called c ("covering") instanton, saturates the functional integral and hence is fully responsible for the confining nature of the correlation functions.

The fastest way to prove this is to compute the functional integral (1)

$$\begin{aligned} & \int [dA] \exp \left\{ -S[A] - \Gamma^{(1)}[A] + e \int d^2z A_\mu(z) J_\mu(z; x, y) \right\} \\ &= Z \exp \left\{ \frac{e^2}{2} \int d^2z d^2z' J_\mu(z; x, y) J_\nu(z'; x, y) D_{\mu\nu}(z-z') \right\} \end{aligned} \quad (4)$$

Since

$$\begin{aligned} J_\mu(z; x, y) &= \varepsilon_{\lambda\mu} \partial_\lambda (D(x-z) - D(y-z)) \\ &\quad + i \partial_\mu (D(x-z) - D(y-z)) \\ &\quad - i(x-y)_\mu \int_0^1 dt \delta(xt + y(1-t) - z) \end{aligned}$$

has a vanishing divergence, we may also write for the above exponential :

$$\exp \frac{e^2}{2} \int d^2z d^2z' J_\mu(z; x, y) J_\mu(z'; x, y) \Delta(z-z'; m^2) \quad (5)$$

By means of

$$(-\partial^2 + m^2) A_\mu^a = e J_\mu \quad (6)$$

this may also be written as

$$\begin{aligned} & \exp \frac{1}{2} \int d^2z A_\mu^a(z) (-\partial^2 + m^2) A_\mu^a(z) \\ &= \exp \left\{ -\frac{1}{2} \int d^2z A_\mu^a(z) (-\partial^2 + m^2) A_\mu^a(z) + e \int d^2z J_\mu(z) A_\mu^a(z) \right\} \end{aligned} \quad (7)$$

The last expression is precisely the value of the effective action in (1) on the c instanton solution (3).

In a situation where several bilocal strings are present, the expression for J_μ becomes

$$J_\mu(z; x_1, \dots, x_n, y_1, \dots, y_n) = \sum_i \left\{ \gamma_{5x_i} \varepsilon_{\lambda\mu} \partial_\lambda D(x_i - z) + \gamma_{5y_i} \varepsilon_{\lambda\mu} \partial_\lambda D(y_i - z) \right\} + i \sum_i \partial_\mu (D(x_i - z) - D(y_i - z)) + i \sum_i (x_i - y_i)_\mu \int_0^1 dt \delta(x_i + y_i(1-t) - z) \quad (8)$$

which again is conserved. Formula (8) also contains a generalization with respect to arbitrary spinor components, a fact which is expressed by the occurrence of γ_5 matrices. The resulting solution for A_μ^{cl} again saturates the functional integrals; the remaining fluctuation terms are, as previously, x_1 and y_1 independent and will be absorbed in the factor Z . The reader can also check easily that the local limiting process, which leads from the gauge invariant bilocals to the previously ^{1),2)} discussed operators $J_\pm(x)$, will be accompanied by the disappearance of the string pieces and the doubling of the "angular" terms $\varepsilon_{\mu\lambda} \delta_\lambda(\dots)$ in the induced action.

The set of correlation functions of products of gauge invariant bilocals and $F_{\mu\nu}$'s characterize the model completely. In the previous treatment we indicated the computation of these functions in the description based on a degenerate vacuum. The functional description may be directly adapted to the chiral irreducible vacuum, i.e., the case when the operator U introduced in Ref. 1) is a multiple of the identity. If we insist in parity invariance, the new correlation function is given by integrating the functional integrals representing the chiral changing correlation function over that winding class which corresponds to the chiral change. It is, however, known ⁷⁾ that the model has yet another degeneracy index for the vacuum related to the condensation of colour into a spurion ¹⁾. In the Rothe-Swieca treatment this corresponds to the non-triviality of the global (Minkowski-space) "half-integer" gauge transformation $T(A_{\frac{1}{2}})$. If it would be possible to see this degeneracy directly in the functional integral instead of an "operator fit" ⁸⁾ of the physical correlation functions, one would have to modify the functional fermion integration in a very radical way. We will not pursue this matter here. However, we would like to stress that this colour-degeneracy of the vacuum in a relativistic gauge is no accident of the rather trivial nature of the (solvable) massless model. The (non-soluble)

Schwinger model with massive fermions shares this property. In that model the chiral spurion ¹⁾ U does not exist, i.e., U is proportional to the identity. In the language of Ref. 7) $T(\Lambda_1)$ is trivial whereas the half-integer gauge transformation remains non-trivial on gauge invariant states. That model also shows colour confinement if one defines colour confinement to mean that gauge invariant bilocal "dipoles" :

$$D(x,y) = \psi(x) e^{-ie \int_x^y A_\mu(\xi) d\xi^\mu} \psi^\dagger(y) \quad (9)$$

do not yield charged fields by a limiting procedure (in the case that non-trivial charge exists, by putting one charge "behind the moon"). This is the mathematical formulation ⁹⁾ of the absence of charge (colour) states in the sense that ψ cannot be linked with particles appearing in the physical spectrum. One may convert the non-existence of such a limit into simpler criteria ⁸⁾ among which the absence of the end-point stability of charges and the too big size of energy fluctuation in a dipole state for large separation are the most prominent ones. It is easy to discuss this model in the Landau gauge using known methods ^{10),11)}. One starts from the ansatz

$$\psi(x) = : \exp i \sqrt{\pi} \int^x (\tilde{\eta}(\xi) + \tilde{\Sigma}(\xi)) : \chi(x) \quad (10)$$

The gauge invariant current is

$$j_\mu = j_{\mu,\chi} - \frac{1}{\sqrt{\pi}} \epsilon_{\mu\nu} \partial^\nu (\tilde{\eta} + \tilde{\Sigma}) \quad (11)$$

As expected from the zero mass model we have instead of the Maxwell equation in the indefinite metric space ($\tilde{\eta}$ carries as in the zero mass model negative metric) :

$$\square A_\mu = -e (j_\mu - j_{\mu,\chi}) \quad (12)$$

with

$$A_\mu = -\frac{\sqrt{\pi}}{e} \epsilon_{\mu\nu} \partial^\nu \tilde{\Sigma} \quad (13)$$

and

$$\dot{j}_{\mu,\epsilon} = \dot{j}_{\mu,\lambda} - \frac{1}{\sqrt{\pi}} \epsilon_{\mu\nu} \partial^\nu \tilde{\eta} \quad (14)$$

These fields $\tilde{\Sigma}$ and $\tilde{\eta}$ together with the field ϕ from the Mandelstam representation of the spinor ¹²⁾

$$\chi = \left(\frac{t}{2\pi}\right)^{\frac{1}{2}} e^{-i\sqrt{t}\gamma^5} : e^{i\sqrt{t}\gamma^5 \phi + i\sqrt{t}\int_x^\infty \dot{\phi}(x') dx'} : \quad (15)$$

fulfil the system of Sine-Gordon-like equations

$$\square \phi = -\frac{2}{\sqrt{\pi}} M_\mu \sin 2\sqrt{\pi} (\tilde{\Sigma} + \tilde{\eta} + \phi) \quad (16a)$$

$$-\square \tilde{\eta} = -\frac{2}{\sqrt{\pi}} M_\mu \sin 2\sqrt{\pi} (\tilde{\Sigma} + \tilde{\eta} + \phi) \quad (16b)$$

$$(\square + m^2) \tilde{\Sigma} = -\frac{2}{\sqrt{\pi}} M_\mu \sin 2\sqrt{\pi} (\tilde{\Sigma} + \tilde{\eta} + \phi) \quad (16c)$$

The sum

$$\gamma = \phi + \tilde{\eta} \quad (16d)$$

is, as expected, a free zero mass field

$$\square \gamma = 0 \quad \text{with} \quad \langle \varphi(x) \varphi(y) \rangle = 0 \quad (17)$$

which characterizes the definition of the physical subspace. Differently from the zero mass situation, the exponential of the positive definite field ϕ does not carry a charge quantum number ⁷⁾. Physically this expresses the explicit breaking of chiral symmetry by the mass ; mathematically it is related to the fact that ϕ does not need any restriction for the test functions it can be smeared with. However, the line integral of $\dot{\phi}$

$$\int_x^\infty \dot{\phi}(t, x') dx' \quad (18)$$

both in the massless, as well as in the massive case, require a restriction in their smearing functions, a fact which is known ¹³⁾ to be responsible for the charge carrying property of

$$\exp i \sqrt{\pi} \int_x^{\infty} \dot{\phi} dx'$$

The indefinite Sine-Gordon-like field $\tilde{\eta}$, on the other hand, does not lead to any charge carrying structure and hence the operator

$$\exp i \sqrt{\pi} \int_x^{\infty} (\dot{\phi} + \dot{\tilde{\eta}}) dx' \quad (19)$$

which, according to (17), is a spurion operator between physical states, nevertheless carries the colour charge quantum number. This explains why the gauge transformation ⁷⁾ $T(A_1)$ is trivial, whereas the half-integer winding class transformation $T(A_{\frac{1}{2}})$ remains non-trivial since it carries the spurionic colour charge. An understanding of this phenomenon directly in terms of functional integration would be very desirable since by the non-triviality of the model one would be forced to look for structural arguments instead of relying on explicit functional integrations as in the zero mass case. This will be even more so in the model of QCD₄. The various necessary steps for the realistic case are in our opinion clearly suggested by the success of our methods in the Schwinger model.

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REFERENCES AND FOOTNOTES

- 1) N.K. Nielsen and B. Schroer - CERN Preprint TH 2250 (1976).
- 2) N.K. Nielsen and B. Schroer - CERN Preprint TH 2270 (1976).
- 3) A.A. Belavin, A.M. Polyakov, A.S. Schwartz and Yu.S. Tyupkin - Phys.Letters 59B, 85 (1975).
- 4) G. 't Hooft - Phys.Rev.Letters 37, 8 (1976).
- 5) C.G. Callan, Jr., R.F. Dashen and D.J. Gross - Phys.Rev.Letters 37, 8 (1976).
- 6) R. Jackiw and C. Rebbi - Phys.Rev.Letters 37, 172 (1976).
- 7) K.D. Rothe and J.A. Swieca - "Gauge Transformations and Vacuum Structure in the Schwinger Model", PUC Rio de Janeiro Preprint (October 1976).
- 8) Part of our results, in particular the relation to the half-integer winding of global gauge transformations in Ref. 7) have been reported on at the "Instanton Workshop" in Copenhagen (November 1976).
- 9) S. Doplicher, R. Haag and J.E. Roberts - Commun.Math.Phys. 15, 175 (1969), and literature quoted therein.
- 10) J. Kogut and L. Susskind - Phys.Rev. D11, 395 (1973).
- 11) K.D. Rothe and J.A. Swieca - "Fields and Observables in the Massive Schwinger Model", PUC Rio de Janeiro Preprint (June 1976).
- 12) S. Mandelstam - Phys.Rev. D11, 3026 (1975).
- 13) J.A. Swieca - "Solitons and Confinement", PUC Rio de Janeiro Preprint (June 1976).