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SATURATION OF GAUGE INVARIANT SCHWINGER MODEL
CORRELATION FUNCTIONS BY INSTANTONS

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A B S T R A C T

We draw some conclusions from the saturation of QED_2 correlation functions by non-quasi-classical instantons.

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In recent attempts ^{1),2)} to obtain physical information about QED₄ correlation functions by using Euclidean instanton configurations ³⁾, quasi-classical arguments played an important rôle. Such a procedure does not work for the best known confinement model : Schwinger's ⁴⁾⁻⁶⁾ QED₂. In that case we have shown ⁷⁾ that an understanding of the chiral vacuum structure ^{8),9)} in terms of Euclidean "winding field" configuration can be obtained if one does the fermion integration and the external field Green function computation before one inserts the classical field equation. In that work we only emphasized the use of these new field configurations for large distances. By a slight extension of our methods one can see, however, that there exist winding field configurations which completely saturate the Euclidean integral in function space. These configurations are in fact solutions of the inhomogeneous field equation, so in addition to their "winding" property they represent true instantons. They are, however, not the ordinary quasi-classical instantons of the previous references since the fermion integration is an essential quantum theoretical step. These "induced" instantons are, we believe, a concrete realization of a speculative remark contained in a recent paper ¹⁰⁾.

Before we draw some general conclusions let us sketch briefly the computational steps for QED₂. Consider the two-point correlation function ⁷⁾

$$\langle J_-(x) J_+(y) \rangle = Z^{-1} \int [dA] \exp \{ -S[A] - \Gamma^{[1]}[A] \} \\ \times \text{tr} G(x,y;A) \frac{1-\gamma_5}{2} G(x,y;A) \frac{1+\gamma_5}{2} \quad (1)$$

with

$$J_{\mp}(x) = N \left[\bar{\psi} \frac{1 \pm \gamma_5}{2} \psi \right](x)$$

The $\Gamma^{[1]}$ is obtained ⁷⁾ from the functional fermion determinant, whereas $G(x,y;A)$ is the (explicitly known) fermion Green function in an external field. Insertion of $\Gamma^{[1]}$ and G leads to :

$$\langle J_-(x) J_+(y) \rangle = \langle J_-(x) J_+(y) \rangle_0 Z^{-1} \int [dA] \exp. S_{eff}[A; x, y] \quad (2)$$

where the first factor represents the zero mass spinor contractions and the effective action is given by :

$$\begin{aligned}
 S_{\text{eff}}[A; x, y] = & \frac{1}{4} \int F_{\mu\nu}^2(z) d^2z + \text{gauge terms} \\
 & + \frac{e^2}{2\pi} \int A_\mu (\partial_\mu \partial_\nu - \frac{\partial^2}{\partial z^2}) A_\nu d^2z \\
 & - 2e \varepsilon_{\lambda\mu} \int A_\mu \partial_\lambda [\mathcal{D}(x-z) - \mathcal{D}(y-z)] d^2z
 \end{aligned} \tag{3}$$

The already mentioned classical field equation of this effective Lagrangian (which contains all orders in $\hbar$) is

$$\begin{aligned}
 (-\partial^2 + m^2) A_\mu(z) + \partial_\mu \partial_\nu A_\nu(z) + \text{gauge terms} \\
 = 2e \varepsilon_{\lambda\mu} \partial_\lambda [\mathcal{D}(x-z) - \mathcal{D}(y-z)]
 \end{aligned} \tag{4}$$

All the differentiations act on the running variable z . The integration gives :

$$\begin{aligned}
 A_\mu^{\text{cl}}(z) = \frac{2\pi}{e} \varepsilon_{\lambda\mu} \partial_\lambda \left[\mathcal{D}(x-z) - \Delta(x-z; m^2) \right. \\
 \left. - \mathcal{D}(y-z) + \Delta(y-z; m^2) \right]
 \end{aligned} \tag{5}$$

with $m^2 = e^2/\pi$.

Insertion of (5) into the interaction gives immediately :

$$S_{\text{eff}}^{\text{cl}} = -4\pi [\Delta(x-y; m^2) - \mathcal{D}(x-y)] \tag{6}$$

The linear fluctuation terms in the functional integral are absent, and the quadratic fluctuation terms lead to a x - y independent factor which cancels against z^{-1} , i.e., we obtain the known correlation function.

Note that (5) describes a sum of instanton and anti-instanton.

A similar consideration applies to the higher correlation functions of gauge invariant local composite spinor fields.

This saturation by instantons only describes the chiral degeneracy and its breaking or in our terminology ⁷⁾ the "chiral-condensation" phenomenon. The "colour-condensation" ⁷⁾ on the other hand cannot be described by instanton configurations but rather requires a new configuration in the Euclidean integration which will be discussed in a forthcoming publication.

Does this simple exercise teach us any lesson for QED_4 ? We believe that also for the non-Abelian case the integration of fermions and their external field dependence is an issue which should be settled before the introduction of instanton configurations. In the realistic case one cannot hope for a closed form for $\Gamma^{[1]}$ and G . But we do nourish the hope that one can obtain a local effective $S_{\text{eff}}[A; x, \dots]$ which is asymptotically correct for large distances or low momentum modes of A_μ .

We find it not inconceivable to expect an exact asymptotic (large distance) saturation by suitable non-Abelian instantons. For the colour confinement one again has to go beyond these configurations.

H. Kleinert kindly informed us that he also obtained the saturation of the Schwinger model correlation functions by instantons in his functional approach.

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