MIXING OF P WAVE AXIAL VECTOR RESONANCES

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ABSTRACT

An analysis is given of the mixing of the axial vector kaon resonances by use of new data from SLAC. Mass values, decay widths and branching ratios of the SLAC work may be compatible with $SU(3)$ and/or "pair creation model" classifications for a mixing angle $\lambda \approx -35^\circ \pm 10^\circ$. The mass values and coupling strengths proposed by Basdevant and Berger, from a different parametrization of the same data, can be fitted in an $SU(3)$ and/or broken $SU(6)_W$ model with a larger mixing angle $\lambda \approx -80^\circ$. Such a mixing would mean that the lower kaonic state is closely related to the 1^+ -nonet in contrast to expectations from duality and/or one-gluon exchange mixing schemes. We also discuss the mixing of the charmed D meson states.

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The new partial wave analysis of the $K^{\pm}\pi^+\pi^-$ system in $K^{\pm}p \rightarrow K^{\pm}\pi^+\pi^-$ at 13 GeV ¹⁾ confirms the existence of strange axial vector states in the 1.3 - 1.4 GeV mass region. According to the SLAC analysis there is evidence of the existence of two $J^P = 1^+$ K resonances, one at ~ 1300 MeV ($\Gamma \approx 150 \pm 70$ MeV) decaying predominantly into ρK , and the other at ~ 1400 MeV ($\Gamma \approx 142 \pm 15$ MeV) decaying predominantly into $K^*\pi$. Predominant coupling to ρK for one state and to $K^*\pi$ for the other is an interesting feature, since it suggests a large mixing between the strange states of the $J^{PC} = 1^{++}$ and 1^{+-} axial vector nonets.

Nevertheless, in Ref. 2) it has been pointed out that one can also have a description of the data by Deck amplitudes superimposed on only one resonance in the $K\pi\pi$ final state. An acceptable fit to the SLAC data has been obtained with resonance parameters $m = 1340$ MeV, $\Gamma = 150$ MeV and the relative coupling $K^*\pi/K\rho \approx 2/3$. The existence of a broad (≈ 300 MeV) resonance at 1400 MeV is not excluded by this analysis.

According to the quark model there are two axial vector strange resonances in the 1.3 - 1.4 GeV mass region which can mix, e.g., through their common ρK and $K^*\pi$ channels. The aim of the present note is to reinvestigate the classification problem of the axial vector kaonic states.

1. SU(3) ANALYSIS ³⁾

The helicity amplitudes $[M(+), M(0)]$ of the $A \rightarrow VP$ and $B \rightarrow VP$ decays can be parametrized as follows:

$$M_A(+)=\frac{1}{2\sqrt{3}}a_{a,bc}(2s_A-d_A) \quad (1a)$$

$$M_A(0)=\frac{1}{\sqrt{3}}a_{a,bc}(s_A+d_A) \quad (1b)$$

$$M_B(+)=\frac{1}{2\sqrt{3}}b_{a,bc}(2s_B+d_B) \quad (1c)$$

$$M_B(0)=\frac{1}{\sqrt{3}}b_{a,bc}(s_B+d_B) \quad (1d)$$

where

$$a_{a,bc} = \text{Tr} \{ P_b^+ [A_a, V_c^+] \} , \quad b_{a,bc} = \text{Tr} \{ P_b^+ \{ B_a, V_c^+ \} \}$$

and P, A, V denote the $SU(3)$ wave functions. The decay widths are given by

$$\Gamma = \frac{k_{cm}}{m_u^2} \frac{1}{3} (2|M(+)|^2 + |M(0)|^2) \quad (2)$$

From the angular distribution of $B \rightarrow \omega\pi$ it is known that $\sqrt{2} d_B/s_B = +0.25$, i.e., the d wave amplitude is relatively small; furthermore, the relative phase of s_B and d_B can be assumed to be zero. In discussing the partial decay widths it seems to be reasonable to neglect the d wave amplitude. In this case $|s_A|$ can be fitted from $\Gamma(A1 \rightarrow \rho\pi) \approx 300 \text{ MeV}$ and $|s_B|$ from $\Gamma(B \rightarrow \omega\pi) \approx 125 \text{ MeV}$ leading to $s_A = 1.04 \text{ GeV}$, $s_B = 0.906 \text{ GeV}$ and $s_A/s_B = 1.15 \text{ GeV}^*$. The mixing angle is defined by

$$|K_1\rangle = \cos\lambda |K_A\rangle + \sin\lambda |K_B\rangle \quad (3a)$$

$$|K_2\rangle = -\sin\lambda |K_A\rangle + \cos\lambda |K_B\rangle \quad (3b)$$

Using the SLAC data, $m_{K_1} \sim 1.3 \text{ GeV}$, $m_{K_2} \sim 1.4 \text{ GeV}$, and summing up the partial modes $^{**})$ ($K^*\pi$, ρK , ωK), we obtain

$$\Gamma_{tot}(K_i) = c_1^i + c_2^i \cos^2\lambda + c_3^i \sin^2\lambda \quad (4)$$

where $c_1^1 = 0.112$, $c_2^1 = 0.036$, $c_3^1 = 0.052$; $c_1^2 = 0.218$, $c_2^2 = -0.053$, $c_3^2 = -0.003$. The width $\Gamma(K_1 \rightarrow \rho K)$ has been calculated by integrating over a Breit-Wigner distribution because of the low kinetic energies. According to the SLAC analysis $\Gamma_1 = 150 \pm 70 \text{ MeV}$, $\Gamma_2 = 142 \pm 15 \text{ MeV}$. The ratio

$$\left(\frac{\Gamma_2}{\Gamma_1}\right)_{exp} \approx 0.95 \pm \frac{1.01}{0.34}$$

does not put severe limitations on λ ; one obtains $-30^\circ < \lambda < 80^\circ$. From the

*) We choose zero relative phase between $s_A(K_A^+ \rightarrow K^{*0}\pi^+)$, $s_B(K_B^+ \rightarrow K^{*0}\pi^+)$ and $m_{K_1} < m_{K_2}$.

**) The decay mode $K_{A,B} \rightarrow \mu e$ is left out from the analysis because of the many uncertainties. According to the data ¹⁾ it was probably seen. The coupling strength should be small.

branching ratios $\Gamma_1(K^*\pi)/\Gamma_1(\rho K) < 1/3$, $\Gamma_2(K^*\pi)/\Gamma_2(\rho K) > 3$, one obtains $\lambda = -40^\circ \pm 20^\circ$. This value and the preceding inequality lead to values for λ , Γ_1 , Γ_2 and the branching ratios as follows

$$\lambda = -30^\circ \pm 10^\circ, \quad \Gamma_1 = 85 \pm 15 \text{ MeV}, \quad \Gamma_2 = 160 \pm 30 \text{ MeV}$$

$$\frac{\Gamma_1(\pi K^*)}{\Gamma_1(K\rho)} = 0.13 \pm 0.4 \quad ; \quad \frac{\Gamma_2(\pi K^*)}{\Gamma_2(K\rho)} = 12 \pm \frac{50}{7}$$

and are compatible with the data.

If we accept the resonance parameters given in Ref. 2) [$g_{\rho K}/g_{K^*\pi} = 3/2$, $m_{K_1} = 1.34$, $\Gamma_{\text{tot}}(K_1) = 150 \text{ MeV}$], we obtain instead $\lambda \approx -80^\circ$. Assuming $m_{K_2} = 1.4 \text{ GeV}$, we then find $\Gamma_2 \approx 230 \text{ MeV}$.

We note that a large negative mixing angle is incompatible with simple non-relativistic mixing schemes and duality schemes ⁴⁾, which prefer the lower state to be closely related to the 1^{++} nonet.

2. PAIR CREATION MODEL

The pair creation model proved to be remarkably successful in the analysis of the P wave nonets. [The schemes based on the Melosh transformation lead to weak restrictions ⁵⁾.] For the 1^{++} and 1^{+-} nonets it leads to

$$s_B = \frac{1}{\sqrt{2}} s_A, \quad d_B = -\sqrt{2} d_A \quad (5)$$

i.e., it fixes the relative strengths and phases for the two nonets.

Furthermore, the $s = s_A$ amplitude determines the decay strength of the 0^{++} P wave (S), and the $d = d_A$ amplitude gives the decay strength of the 2^{++} P wave nonet (T). Extracting a barrier factor, $d = (k_{\text{cm}}/\mu)^2 \tilde{d}$, one obtains an acceptable description of the meagre data available on the P wave S, A, B and T nonets, by the values $s = (1.1 \pm 0.2) \text{ GeV}$ and $\tilde{d} = (0.6 \pm 0.15) \text{ GeV}$ with $\mu = 0.5 \text{ GeV}$ ^{*}). With these amplitudes we obtain the same parametrization for $\Gamma_{\text{tot}}(K_1)$ and $\Gamma_{\text{tot}}(K_2)$ as in Eq. (4), but the coefficients are different: $c_1^1 = 0.101$, $c_2^1 = 0.095$, $c_3^1 = 0.056$; $c_1^2 = 0.299$, $c_2^2 = -0.135$, $c_3^2 = -0.003$ (all in GeV units).

*) Our s and d are identical to the ones of Ref. 6), where $s = 0.77$ and $d = 0.6$ has been used; the increase of s is explained by the change of the data on the 0^{++} mesons.

The ratio of the total widths $\Gamma_2/\Gamma_1 \approx 0.95 \pm \frac{1.01}{0.37}$ leads again to $-30^\circ < \lambda < 70^\circ$; the limits on the branching ratios are consistent with $\lambda = -55 \pm 15^\circ$. The allowed regions again have a small overlap, giving

$$\lambda \approx -40^\circ \pm 5^\circ, \quad \Gamma_1 \approx 80 \pm 10, \quad \Gamma_2 \approx 180 \pm 20$$

$$\frac{\Gamma_1(\pi K^*)}{\Gamma_1(\rho K)} \approx 0.35 \pm 0.15; \quad \frac{\Gamma_2(\pi K^*)}{\Gamma_2(\rho K)} = 40 \pm 10,$$

compatible with the SU(3) fit and with the data. As to the fit by Basdevant and Berger, from $(K\rho/K\pi) = 3/2$ it follows that $\tan \lambda = -5\sqrt{2}$, i.e., $\lambda \approx -82^\circ$. The partial decay widths then turn out to be $\Gamma(K_1 \rightarrow K^* \pi) = 68$ MeV, $\Gamma(K_1 \rightarrow \rho K) = 56$ MeV, $\Gamma(K_1 \rightarrow \omega K) = 16$ MeV. Assuming that the mass of the K_2 is 1.4 GeV we have $\Gamma_{\text{tot}}(K_2) = 290$ MeV. [$\Gamma(K_2 \rightarrow K^* \pi) = 166$ MeV, $\Gamma(K_2 \rightarrow \rho K) = 95$ MeV and $\Gamma(K_2 \rightarrow \omega K) = 29$ MeV.] Such a broad K_2 resonance is not excluded by the fit of Ref. 2).

The conclusions are in agreement with those of our SU(3) fit.

3. NON-RELATIVISTIC MIXING SCHEME

One can develop a mixing scheme from one-gluon exchange. The mixing of the P wave kaonic states arises from the Breit-Fermi term

$$H_I = a \left(\frac{1}{m_1^2} \vec{S}_1 \cdot \vec{L} + \frac{1}{m_2^2} \vec{S}_2 \cdot \vec{L} \right) \quad (6)$$

where m_1, m_2 are the masses of the constituent quarks. In the SU(3) limit $H_I \sim (\vec{S}_1 + \vec{S}_2) \cdot \vec{L} = \vec{S} \cdot \vec{L}$, the non-diagonal matrix elements vanish, the kaon states do not mix^{*)}. The mixing of the kaonic states is due to spin-dependent SU(3) breaking forces. The one-gluon exchange scheme leads to additional constraints⁷⁾. The mass matrix has the form

$$M = \begin{pmatrix} M_{AA} & M_{AB} \\ M_{AB} & M_{BB} \end{pmatrix} = \begin{pmatrix} M_0 + \delta, & -\epsilon \\ -\epsilon, & M_0 \end{pmatrix} \quad (7a)$$

where

$$\delta = -\frac{c}{m_1 m_2} \left[\frac{4}{5} + \frac{m_1^2 + m_2^2}{4 m_1 m_2} \right] < 0 \quad (7b)$$

*) More generally, this trivially follows from generalized G parity.

The constant C is proportional to the positive definite overlap integral

$$\int_0^{\infty} \frac{1}{r} \varphi_1^2(r) dr$$

and

$$\epsilon = \sqrt{2} \frac{C}{m_1 m_2} \frac{m_1^2 - m_2^2}{4 m_1 m_2} \quad (7c)$$

With the phase convention chosen above for the relative sign of the K_A and K_B states, we have for the mixing angle

$$\operatorname{tg} \lambda = \frac{1}{2|\epsilon|} \left[(\delta^2 + 4\epsilon^2)^{1/2} - |\delta| \right] \quad (8)$$

From (8) it follows that

$$0 \leq \lambda \leq 35.3^\circ \quad (9)$$

Such a value of λ is in agreement with duality schemes, but is in contradiction with the preceding values ($\lambda = -35^\circ$ or -80° , respectively).

The formula (8) has the remarkable feature that as $m_1 \gg m_2$ $\operatorname{tg} \lambda \approx 1/\sqrt{2}$, independent of the overlap integral.

We now discuss the mixing of the P wave charmed D mesons. It seems reasonable to assume that the $D_1 \rightarrow D_0$ and $D_2 \rightarrow D_0$ decay modes are below threshold, i.e., the only allowed decay mode is $D^* \pi$. Furthermore, it is very likely that D_1 lies very close to or even below the $D_1^* \pi$ threshold. With the mixing of the Breit-Fermi model, for $m_1 \gg m_2$, one has

$$\Gamma(D_1 \rightarrow D^{0*} \pi^+) = \beta \frac{3k_1}{4m_{D_1}^2} |\alpha|^2 \quad (10a)$$

$$\Gamma(D_2 \rightarrow D^{0*} \pi^+) = \beta' \frac{3k_2}{4m_{D_2}^2} |\alpha|^2 \quad (10b)$$

where β is a factor resulting by the folding over the Breit-Wigner distribution. Assuming ψ'' and/or ψ''' dominance in the production of $D_{1,2}$

in e^+e^- annihilation in the energy region $2E \approx (4.0 - 4.4)$ GeV, we calculate the $\Gamma(\psi'' \rightarrow D_1 D)$, $\Gamma(\psi'' \rightarrow \bar{D}_2 D)$ decay widths. After crossing from the $(A,B) \rightarrow VP$ channel to the $V \rightarrow (A,B)P$ channel, the s and d wave amplitudes do not mix, and the relative sign for s_A and s_B remains the same as the one for the $D_A \rightarrow D^* \pi$, $D_B \rightarrow D^* \pi$ amplitudes. Therefore,

$$\Gamma(\psi'' \rightarrow D_1 D) \sim |s|^2 \quad (11a)$$

$$\Gamma(\psi'' \rightarrow D_2 D) \sim |d|^2 \quad (11b)$$

i.e., $\Gamma(\psi'' \rightarrow D_1 D) \gg \Gamma(\psi'' \rightarrow D_2 D)$ ^{*)}. Consequently at energies $2E = 4.0 - 4.5$ GeV production of $D_1 D$ pairs would be predominant as compared to $D_2 D$. Since the subsequent decay of D_1 goes through the s wave it should be relatively broad.

Unfortunately, the relative magnitude of the couplings $g_{\psi'' DD}$ and $g_{\psi'' D_1 D}$ is determined, in a pair creation model, by the ratio of different overlap integrals; therefore it is very model-dependent, although one would expect $|g_{\psi'' DD}|^2 > |g_{\psi'' D_1 D}|^2$. On the other hand, if the ratio $|g_{\psi'' D_1 D}|^2 / |g_{\psi'' DD}|^2$ is not smaller than $\sim \frac{1}{2}$, one would expect as many or even more $D_1 \bar{D}$ pairs than $D \bar{D}$ pairs (without taking into account kinematical reflections).

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*) In the limiting case $m_2 \rightarrow \infty$, $\lambda = 35.3^\circ$, the spin of the charm quark decouples and our results can be reproduced by using angular momentum selection rules for the light quarks ⁸⁾.

REFERENCES

- 1) G.W. Brandenburg et al., Phys. Rev. Letters 36, 703 (1976) and SLAC-PUB-1767, June (1976), unpublished.
- 2) J.L. Basdevant and E.L. Berger, Phys. Rev. Letters 37, 977 (1976).
- 3) R. Gatto and L. Maiani, Phys. Letters 26B, 95 (1967).
- 4) J. Mandula, J. Weyers and G. Zweig, Phys. Rev. Letters 23, 266 (1969).
- 5) H. Burkhardt and A.J. Hey, Nuclear Phys. B108, 514 (1976).
- 6) E.W. Colglazier and J.L. Rosner, Nuclear Phys. B27, 349 (1971).
- 7) R. Barbieri, R. Gatto, R. Kögerler and Z. Kunszt, Nuclear Phys. B105, 125 (1976).
- 8) A. De Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D12, 147 (1975).