CERN LIBRARIES, GENEVA

CM-P00061854

Ref.TH.2233-CERN

HIGH-ENERGY SCATTERING IN A TWO-DIMENSIONAL GAUGE THEORYR.C. Brower ^{*,+)}, J. Ellis, M.G. Schmidt and J.H. Weis ⁺)

CERN - Geneva

ABSTRACT

We study high-energy scattering in two-dimensional quantum chromodynamics in the $1/N_{\text{colour}}$ expansion. To first order, high-energy forward scattering amplitudes are power-behaved with the factorization, signature and exchange degeneracy properties found in dual Regge models. Backward scattering has deviations from naive quark power-counting rules which are related to the form factor and "Regge" behaviours. In the second order of $1/N_{\text{colour}}$ vacuum exchange diagrams do not give a new Pomeron singularity, but do break exchange degeneracy.

*) Supported in part by the National Science Foundation.

+) A.P. Sloan Foundation Fellow.

The strong interactions may be due to a gauge theory of coloured vector gluons and quarks (QCD). Motivation for QCD comes partly from the approximate Bjorken scaling it yields in deep inelastic scattering, and partly from the possibility that it confines quarks. It is also related to recent approaches to strong interactions using $1/N$ expansions. If confinement is assumed, the expansion in $1/N_c$, the number of colours, leads to the dual loop expansion ¹⁾, while the expansion with fixed ratio of N_c and N_f , the number of flavours, is related to the topological expansion ²⁾. We want to know if QCD conforms to our expectations for the terms in these expansions. For example, the leading term in the $1/N_c$ expansion for meson-meson scattering should have zero width resonances, Regge behaviour and duality, while the next higher term is expected to contain a Pomeron ^{1),2)}. Another interesting area to study is hard scattering constituent ideas ³⁾ about fixed angle and other "hard" hadronic processes.

Here we investigate these questions in the two-dimensional QCD model of 't Hooft ⁴⁾. It is a more realistic laboratory than perturbation theory because the incident particles are bound states and the amplitudes exhibit confinement with an infinite sequence of zero width resonances in leading order of $1/N_c$. We show that in this order the model has power-law high-energy forward scattering with "Regge-like" factorization, signature and exchange degeneracy properties. "Intercepts" are additive in the exchanged quarks $\alpha^{ab}(0) = -\beta_a - \beta_b$ and related to the bound state spectrum and form factor behaviours. Backward scattering amplitudes deviate by related powers from quark power-counting rules ⁵⁾ proposed for fixed angle scattering. The non-planar diagrams appearing in the next order of $1/N_c$ do not yield a Pomeron because of cancellations due to colour gauge invariance and the colour singlet nature of hadrons. We expect that in higher dimensions the Pomeron will appear with intercept $1 + O(N_f/N_c)$, reflecting the existence of real gluonic degrees of freedom. The model does have the expected Regge renormalization effects of order N_f/N_c , as well as exchange degeneracy breaking of order $1/N_c$ in the vacuum channel. We also find Regge-Regge cuts and multi-Regge effects, which will be discussed fully in a forthcoming paper ⁶⁾.

We hope these results may have some relevance to the real world. Naively, since high-energy scattering seems almost two dimensional, the 't Hooft model may give useful intuition about four-dimensional QCD in these processes. Several approaches in fact use two dimensions as a calculational stepping-stone to four. These include a transverse lattice formulation of QCD ⁷⁾, "heavy Pomeron" descriptions ⁸⁾ of high-energy scattering which may also use a transverse lattice ⁹⁾, and string models ¹⁰⁾. However, gluons in

two-dimensional QCD appear only as an instantaneous Coulomb potential. Hence some of their manifestations are poorly approximated, including bound states of glue ("gluonium"), and the Pomeron which is presumably their Regge trajectory.

We now examine meson-meson scattering in first order of $1/N_c$. Rules for constructing bound state scattering amplitudes for two-dimensional QCD in a light-like gauge $A_+ = 0$ have been given by previous authors ^{4),11)}. You first write down ¹¹⁾ the possible quark-antiquark exchange diagrams shown in Fig. 1. Each diagram may have scattering amplitudes T inserted in colour singlet $(q - \bar{q})$ channels ¹¹⁾, as shown in Fig. 2.

Using properties of the bound state wave functions and representations for the colour singlet T matrix, it is possible to perform the integrals over $(+)$ components of loop momenta. This procedure gives a set of diagrams like old-fashioned perturbation theory in which

- i) the quark lines have no propagators;
- ii) the gluons are instantaneous with principal-value propagators

$$\frac{1}{2} \left(\frac{1}{(k_- + i\epsilon)^2} + \frac{1}{(k_- - i\epsilon)^2} \right) \quad (1)$$

- iii) all vertices have truncated wave functions $\phi_n^{a\bar{b}}(x)$ where $x = p_-^a / p_-^a + p_-^{\bar{b}}$ has support only for $0 < x < 1$ and obeys the equation

$$\mu_n^2 \phi_n^{a\bar{b}}(x) = \left(\frac{\gamma_a^{-1}}{x} + \frac{\gamma_b^{-1}}{1-x} \right) \phi_n^{a\bar{b}}(x) - \oint_0^1 \frac{\phi_n^{a\bar{b}}(y) dy}{(y-x)^2} \quad (2)$$

with

$$\gamma_a \equiv \left(\frac{\pi}{g^2 N_c} \right) m_a^2 \equiv \frac{m_a^2}{m^2} \quad (3)$$

where m_a is the quark mass. The wave function

$$\phi_n^{a\bar{b}}(x) \simeq C_{a\bar{b}} x^{\beta_a} \left[(-1)^n C_{b\bar{a}} (1-x)^{\beta_b} \right] \text{ as } x \rightarrow 0 \text{ [1]} \quad (4)$$

where

$$\pi\beta_a \cot \pi\beta_a = 1 - \gamma_a \quad : 0 \leq \beta_a < 1 \quad (5)$$

and obeys the scaling relation

$$\lim_{n \rightarrow \infty} \phi_n^{a\bar{r}}\left(\frac{\bar{z}}{\mu_n^2}\right) = \phi^a(\bar{z}) \quad (6)$$

with

$$\phi^a(\bar{z}) \sim \begin{cases} \bar{z}^{\beta_a} & \text{as } \bar{z} \rightarrow 0 \\ \sin(\bar{z}/\pi) & \text{as } \bar{z} \rightarrow \infty \end{cases} \quad (7)$$

The set of diagrams for A^{st} when $s > 0$ is illustrated in Fig. 3. The diagrams with no T matrices inserted have been cancelled by gluonic terms in the T matrices, while the sums over intermediate bound states remain. Notice how the instantaneous gluons are used in every possible way to reverse the $(-)$ components of the quark momenta and feed them into the truncated wave functions $\phi_n^{a\bar{b}}(x)$.

For forward equal mass scattering the crossed-channel graphs of Fig. 3c do not contribute. The gluon diagram of Fig. 3a yields for equal mass kinematics

$$A_{(a)}^{st} = 4g^2 \int_0^{p_{1-}} dk_- \int_0^{p_{3-}} dk'_- \frac{\phi^{a\bar{r}}\left(\frac{k_-}{p_{1-}}\right) \phi^{b\bar{c}}\left(\frac{p_{2-}-k'_-}{p_{2-}}\right) \phi^{d\bar{e}}\left(\frac{p_{3-}-k'_-}{p_{3-}}\right) \phi^{a\bar{d}}\left(\frac{k_-}{p_{4-}}\right)}{(p_{1-}+p_{2-}-k_- - k'_-)^2} \quad (8)$$

Introducing $x \equiv k_-/p_{1-}$, $z \equiv 1 - k'_-/p_{3-}$, Eq. (8) can be rewritten as

$$A_{(a)}^{st} = 4g^2 \frac{p_{1-}}{p_{2-}} \int_0^1 dx \int_0^1 dz \frac{\phi^{a\bar{r}}(x) \phi^{b\bar{c}}(z) \phi^{d\bar{e}}(z) \phi^{a\bar{d}}(x)}{(z + (1-x)p_{1-}/p_{2-})^2} \quad (9)$$

The first of the four diagrams of Fig. 3b is

$$\begin{aligned}
 A_{(b)_1}^{st} = & -4g^2 \sum_n \left\{ \frac{p_{1-}}{p_{2-}} \int_0^1 dx \phi^{a\bar{b}}(x) \int_0^1 dz \phi^{b\bar{c}}(z) \phi_n^{a\bar{c}} \left(\frac{x}{1+p_{2-}/p_{1-}} \right) \right\} \\
 & \times \frac{1}{(s/m^2 - \mu_n^2)} \times \\
 & \left\{ \frac{p_{1-}}{p_{2-}} \int_0^1 dx' \phi^{a\bar{d}}(x') \int_0^1 dz' \phi^{d\bar{c}}(z') \phi_n^{a\bar{c}} \left(\frac{x'}{1+p_{2-}/p_{1-}} \right) \right\} \quad (10)
 \end{aligned}$$

and the others are similar.

We now go to the high-energy limit, working in the centre-of-mass frame so that $\sqrt{2}p_{1-} \simeq \sqrt{s}$, $\sqrt{2}p_{2-} \simeq m_2^2/\sqrt{s}$. If we then examine the contribution to (9) coming from the region $x \simeq 1$ by introducing η : $x = 1 - \eta(m_2^2/s)$ and use the limiting properties (4) we find a behaviour

$$A_{(a)}^{st} \simeq 4g^2 \left(\frac{s}{m_2^2} \right)^{-\beta_b - \beta_d} \int_0^\infty d\eta \int_0^1 dz \frac{C_{a\bar{b}} C_{a\bar{d}} \eta^{\beta_b + \beta_d} \phi^{b\bar{c}}(z) \phi^{d\bar{c}}(z)}{(\eta + z)^2} \quad (11)$$

Analyzing the high-energy behaviour of $A_{(b)_1}^{st}$, the dominant contribution to Eq. (10) comes from x near 1 because of the gluon denominators. Using $x = 1 - \eta m_2^2/s$ and $x' = 1 - \eta' m_2^2/s$ together with the limiting properties (4) and the scaling property

$$\sum_n \frac{\phi_n^{a\bar{c}}(\frac{3m^2}{s}) \phi_n^{a\bar{c}}(\frac{3m^2}{s})}{(s/m^2 - \mu_n^2)} \simeq \frac{1}{\pi^2} \int_0^\infty \frac{d\lambda}{\lambda - 1 + i\epsilon} \phi^a(\lambda \xi) \phi^a(\lambda \xi') \quad (12)$$

we find that the $A_{(b)}^{st}$ graphs are asymptotically

$$\begin{aligned}
 A_{(b)}^{st} \simeq & -\frac{4g^2}{\pi^2} \left(\frac{s}{m_2^2} \right)^{-\beta_b - \beta_d} \int_0^\infty \frac{d\lambda}{\lambda - 1 + i\epsilon} \times \\
 & \times \left\{ \int_0^\infty d\eta \int_0^1 dz C_{a\bar{b}} \eta^{\beta_b} \phi^{b\bar{c}}(z) \left[\frac{\phi^c(m_2^2(1+\eta)\lambda) - \phi^c(m_2^2(1-\eta)\lambda)}{(\eta + z)^2} \right] \right\} \\
 & \times \left\{ \int_0^\infty d\eta' \int_0^1 dz' C_{a\bar{d}} \eta'^{\beta_d} \phi^{d\bar{c}}(z') \left[\frac{\phi^c(m_2^2(1+\eta')\lambda) - \phi^c(m_2^2(1-\eta')\lambda)}{(\eta' + z')^2} \right] \right\} \quad (13)
 \end{aligned}$$

Let us now discuss the properties and implications of the results (11) and (13) in some detail. The forward amplitude behaves asymptotically like a non-integral power of s which depends additively on the quark and anti-quark exchanged ^{*)}, but is otherwise independent of the external bound states. There is crossed-channel factorization, and if we add in A^{tu} corresponding from Fig. 1, then we recover the signature and exchange degeneracy properties found in dual Regge models. Indeed it is characteristic of dual models that the sum over s channel resonances be Regge behaved: this is a simple consequence of unsubtracted dispersion relations. As the quark mass $m_b \rightarrow \infty$, $\beta_b \rightarrow 1$ so that the contributions (11) and (13) to A^{st} are $O(s^{-2})$ for large quark masses. If A^{st} obeys a dispersion relation as we expect, then it should not fall faster than s^{-1} , and we see from (9) that $A_{(a)}^{st}$ indeed has a fixed power singularity

$$A_{(a)}^{st} \simeq 4g^2 \left(\frac{m^2}{s} \right) \int_0^1 dx \int_0^1 dx' \frac{\phi^{a\bar{b}}(x) \phi^{a\bar{d}}(x) \phi^{c\bar{b}}(x') \phi^{c\bar{d}}(x')}{(1-x)^2} \quad (14)$$

which is multiplicative with the "Regge" pole ^{**))}. We can see a connection between the "intercept" $\alpha^{b\bar{d}}(0) = -\beta_b - \beta_d$ and the bound state spectrum in 't Hooft's model in the limit of small quark masses. The lowest bound state has mass ¹³⁾

$$m_0^{b\bar{d}^2} \simeq m \frac{\pi}{\sqrt{3}} (m_b + m_d) \quad (15)$$

whereas the "intercept" is

$$\alpha^{b\bar{d}}(0) \simeq -\frac{1}{m} \frac{\sqrt{3}}{\pi} (m_b + m_d) \quad (16)$$

We can therefore define a "slope" for light quark trajectories near zero momentum

$$\alpha'_0 \equiv -\frac{\alpha^{b\bar{d}}(0)}{m_0^{b\bar{d}^2}} \simeq \frac{3}{\pi g^2 N_c} = 3\alpha'_{\infty} \quad (17)$$

*) This remains true in unequal mass scattering.

**) This is reminiscent of the collision in dual models at $J = -1$ between leading and sister trajectories ¹²⁾.

where $(\alpha'_{\omega})^{-1}$ is the separation between adjacent high mass excited states. When $m_b = m_d = 0$ the "Regge" behaviour should coincide with the t channel exchange of the zero mass bound state. We find that in this limit the "Regge" terms (11) and (13) are real and non-singular, consistent with the decoupling of the zero mass bound state ^{11), 14)}.

It is possible to give a simple heuristic discussion of the asymptotic behaviours of the A^{st} graphs which is a useful guide for other more complicated diagrams. In Eq. (8) the dominant behaviour ^{*}) comes when the gluonic propagator $(p_{1-} + p_{2-} - k_- - k'_-)^{-2} \sim s$. Because $p_{2-} \sim s^{-\frac{1}{2}}$, k'_- is also $\sim s^{-\frac{1}{2}}$ because of the kinematic restriction on the argument of $\phi^{c\bar{b}}$. The a quark is then forced to be "hot": $k_- = p_{1-} - O(s^{-\frac{1}{2}})$, and these kinematic factors in the integration of (8) are asymptotically constant. The "Regge" behaviour comes from the small amplitudes $\sim x^{\beta_b}, x^{\beta_d}$ for the dribbling of "cold" quarks ($x \rightarrow 0$) by the meson bound states. This picture is reminiscent of Feynman's parton description of Regge behaviour ^{15)**)}.

To complete our discussion of meson-meson scattering in lowest order of $1/N_c$, we should consider A^{us} of Fig. 1, or equivalently the contributions of A^{st} in the backward direction: $\sqrt{2}p_{1-} \sim \sqrt{s}$, $\sqrt{2}p_{2-} \sim m_2^2/\sqrt{s}$, $\sqrt{2}p_{3-} \sim \sqrt{s}$, $\sqrt{2}p_{4-} \sim m_4^2/\sqrt{s}$. A large momentum $\sim \sqrt{s}$ must now be routed across the diagram: in Fig. 3a this can only be across the gluon, whose propagator $(p_{1-} + p_{2-} - k_- - k'_-)^{-2} \sim s^{-1}$ in this case. Kinematic factors are then $O(1/s^2)$ in integrating (8), and these are modified by factors for the "cold" a and c lines giving

$$A_{(a)}^{st} \sim S^{-2-\beta_a-\beta_c} \quad (18)$$

in the backward direction. Analyzing Fig. 3b is more complicated: for example, in the second diagram the "hot" momentum may be routed through either of the gluons, or across the intermediate meson states. Power counting for this last routing works just as if the bound state sum were replaced by a single gluon ⁶⁾. The three "hot" routings then yield backwards behaviours

$$A_{(b)_2}^{st} \sim S^{-2-\beta_a-\beta_d}, S^{-2-\beta_c-\beta_d} \text{ and } S^{-2-\beta_b-\beta_d} \quad (19)$$

*) Except for $\beta_b + \beta_d > 1$.

**) This picture also provides an intuitive basis for the Regge behaviour of deep inelastic structure functions as $x \rightarrow 0$ which occurs in the 't Hooft model.

The behaviours (18) and (19) encompass all the leading backward scattering behaviours, and we see that A^{us} does not threaten Regge behaviour in forward scattering.

Backward scattering is the closest two-dimensional analogue to fixed angle scattering in higher dimensions. Indeed the diagrams of Fig. 3 have the quark content of the constituent interchange model (CIM) ³⁾. The power behaviours (18) and (19) resemble the dimensional counting rules ^{5),*)}, but with modifications related to the model's "Regge" behaviour and to form factors which behave ¹¹⁾ as $(q^2)^{-1-\beta}$ ^{**)} . Because of the different contributions in (18) and (19) it is difficult to interpret fixed angle scattering as a product of two form factors, and the result certainly differs from the CIM where a product of three form factors is advocated. Also, there is no reason for the fixed angle amplitude to be purely real as in many constituent models.

We now turn to the next order of $1/N_c$ where four types of quark diagrams contribute to the meson-meson scattering amplitude ^{***)}. The planar diagram of Fig. 4a is expected to give widths to s channel mesons and to renormalize t channel Regge exchanges. Figure 4b should give widths, have t channel Regge cuts, and may give new direct channel Pomeron (gluonium) bound states as in the dual model. Figure 4c should contain Regge cuts. Finally, Fig. 4d may contain a new Pomeron singularity as well as renormalizing the vacuum Regge exchange.

We first examine whether Fig. 4d indeed gives a Pomeron. The imaginary part of the amplitude is easy to study because it only has contributions from pairs of mesons of masses M_1^2, M_2^2 , and is therefore given by an integral over the square of the amplitude A^{tu} of Fig. 1b

$$\text{Im } A_{(4d)} = \frac{1}{16\pi^2} \int \frac{dM_1^2 dM_2^2}{\lambda^{\frac{1}{2}}(s, M_1^2, M_2^2)} |A^{tu}(s, M_1^2, M_2^2)|^2 \quad (20)$$

For large masses $M_1^2 \sim s$ we may get a contribution which is not controlled by the "Regge" behaviour (11), (13) of A^{tu} . To obtain large masses, both quarks a and b in Fig. 4d must be "hot". It is then easy to see that for M_1^2, M_2^2 both $O(s)$ we have $A^{tu} \sim \text{constant}$, and since the phase space

*) Notice, however, that Brodsky and Farrar have several "hot" gluons in their calculations.

**) The implicit connection between Regge and form factor powers is different from that advocated by Bjorken and Kogut ¹⁶⁾.

***) We omit Zweig-violating insertions on external legs.

integral is $\sim s$ we obtain $\text{Im } A_{(4d)} \sim s$. However, A^{tu} has a total of 11 contributions resembling Fig. 3 in this model, and we find ⁶⁾ their sum vanishes asymptotically. This cancellation can be understood by returning to the basic multiple gluon exchange of Fig. 5 which should generate the Pomeron in QCD ^{*)}. It occurs between pairs of diagrams differing only by the coupling of a "cold" gluon to one or the other of the "hot" quarks a, b , and so reflects the colour singlet nature of the bound states.

Thus although the existence of the high mass bound states provides a potential mechanism for generating the Pomeron with intercept $= 1 + O(N_f/N_c)$, it is absent in two dimensions. Since this diagram renormalizes the vacuum "Regge" intercept upwards, this model resembles that of Chew and Rosenzweig ¹⁹⁾. But we believe the Pomeron cancellation will not operate in higher dimensions. For one thing, colour magnetic or electric dipole scattering is now possible because bound states have finite transverse size. Second, gluons are not just instantaneous Coulomb potentials but dynamical degrees of freedom which tend to form gluonium in higher dimensions. Since gluonium states should be the particles on the Pomeron trajectory, their absences may be linked.

Investigations ⁶⁾ of contributions to Fig. 4d from other regions of M_1^2 and M_2^2 , and of Figs. 4a, b, c confirm the general prejudices about Regge renormalization, cuts and so on. Also, simple "hot-line" analyses yield triple-Regge behaviour for inclusive cross-sections, double Regge behaviour in $(2 \rightarrow 3)$ meson amplitudes and the expected behaviour in the inclusive pionization region. These results support the interpretation of the high-energy limits discussed here as embryonic Regge behaviour.

We conclude that two-dimensional QCD generally confirms prejudices about "soft" processes in high-energy hadronic collisions. The 't Hooft model also provides a unique link between these and "hard" processes like fixed angle scattering, deep inelastic scattering and form factors. If only we knew how to lift the model towards four dimensions.

ACKNOWLEDGEMENTS

We would like to thank Geoff Chew, Julius Kuti and Gabriele Veneziano for stimulating discussions.

*) For a two-gluon exchange model see Ref. 17). For a model using gluons as dynamical degrees of freedom for this and many other purposes, see Ref. 18).

REFERENCES

- 1) G. 't Hooft, Nuclear Phys. B72, 461 (1974).
- 2) G. Veneziano, CERN Preprint TH.2200 (1976) and references therein.
- 3) D. Sivers, S.J. Brodsky and R. Blankenbecler, Physics Reports 23C, 1 (1976) and references therein.
- 4) G. 't Hooft, Nuclear Phys. B75, 461 (1974).
- 5) S.J. Brodsky and G. Farrar, Phys. Rev. Letters 31, 1153 (1973);
V.A. Matveev, R.M. Muradyan and A.N. Tavkhelidze, Nuovo Cimento Letters 7, 719 (1973).
- 6) R.C. Brower, J. Ellis, M.G. Schmidt and J.H. Weis, CERN Preprint (in preparation).
- 7) W.A. Bardeen and R.B. Pearson, Phys. Rev. D14, 547 (1976).
- 8) V.N. Gribov, Nuclear Phys. B106, 189 (1976).
- 9) D. Amati, M. Le Bellac, G. Marchesini and M. Ciafaloni, CERN Preprint TH.2152 (1976);
R.C. Brower, M.A. Furman and K. Subbarao, U.C. Santa Cruz Preprint "Quantum spin model for Reggeon field theory" (1976).
- 10) W.A. Bardeen, I. Bars, A.J. Hanson and R.D. Peccei, Phys. Rev. D13, 2364 (1976);
I. Bars, Phys. Rev. Letters 36, 1521 (1976).
- 11) C.G. Callan, N. Coote and D.J. Gross, Phys. Rev. D13, 1649 (1976);
M.B. Einhorn, FNAL Preprint Pub-76/22-THY (1976).
- 12) P. Hoyer, N.A. Törnqvist and B.R. Webber, Phys. Letters 61B, 191 (1976).
- 13) G. 't Hooft, ITP, Utrecht Preprint "Gauge theory for strong interactions" (1975).
- 14) A.W. Smith, DAMTP, Cambridge Preprint 76/17 (1976).
- 15) R.P. Feynman, "Photon-Hadron Interactions" (W.A. Benjamin, Reading) (1972).
- 16) J.D. Bjorken and J.B. Kogut, Phys. Rev. D8, 1341 (1973).
- 17) F.E. Low, Phys. Rev. D12, 163 (1975).
- 18) S. Nussinov, Phys. Rev. Letters 34, 1286 (1975) and Phys. Rev. D14, 246 (1976).
- 19) G.F. Chew and C. Rosenzweig, Nuclear Phys. B104, 290 (1976).

FIGURE CAPTIONS

Fig. 1 : Contributions to meson-meson scattering in first order of $1/N_c$.

Fig. 2 : Contributions to A^{st} indicating insertions of T matrices.

Fig. 3 : Decomposition of A^{st} using the rules described in the text.

Fig. 4 : Contributions to meson-meson scattering in second order of $1/N_c$.

Fig. 5 : A typical multiple gluon exchange diagram which underlies
Fig. 4d.

FIG.1

FIG. 2

FIG.3

FIG.4

FIG.5