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AXIAL VECTOR MASS SPECTRUM AND MIXING ANGLES

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A B S T R A C T

Spectral sum rules of the axial-vector current and axial-vector current-pseudoscalar field are used to study the axial-vector mass spectrum and mixing angles, as well as the decay constants and mixing angles of the pseudoscalar mesons. In general, the result is quite persuasive for the existence of the $J^{PC} = 1^{++}$ multiplet in which one has a canonical D-E mixing.

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For some time, the status of the axial-vector multiplets has been somewhat unclear, though theoretical reasoning based on either the quark model or SU(6) predicts the existence of two multiplets, i.e., one to which $A_1(J^{PC} = 1^{++})$ belongs and the other centred around $B(J^{PC} = 1^{+-})$. However, recent reports^{1,2)} concerning the existence of two axial-vector strange mesons, Q_1 and Q_2 , renew the interest in the axial-vector system. Also the observation of three or four intermediate states in the radiative decay of $\psi'(3.7)$ suggests a possible candidate of the $c\bar{c}$ axial vector ψ_A .

As of now, only the B meson is experimentally identified with definitive quantum numbers. The spin-parity of the Q_1 and Q_2 is determined to be 1^+ . Of the two $Y = I = 0$ mesons, the D meson is relatively well established, compared with the E meson, with known $C = +$ and favoured $J^P = 1^+$. On the other hand, the E-D mixing (assuming E exists) ought to be canonical like the cases of the 1^{--} and 2^{++} multiplets, if the quark-gluon mechanism is correct.

If the potential between the quarks is independent of the spin, the two axial-vector multiplets 1^{++} and 1^{+-} are expected to be degenerate. In reality, the existence of Q_1 and Q_2 implies that they are probably made of some mixture of the two pure axial-vector Q_A and Q_B belonging to the $J^{PC} = 1^{++}$ and 1^{+-} family, respectively. The mass difference between Q_A and Q_B is expected to be of the order of the mass-difference between A_1 and B. But, due to the ever-confusing status of A_1 , it is difficult to determine the Q_A and Q_B masses, though they should be not too different from the Q_1 and Q_2 masses. It is expected, however, that the Q_A - Q_B mixing is maximal^{*)3)} due to the different decay modes of Q_1 and Q_2 , i.e. to ρK and $K^*\pi$ final states, respectively. We further note that the SU(6) mass relation $m_{A_1}^2 + m_E^2 = 2m_{Q_A}^2$ is fairly well satisfied (better than 5% accuracy) by A_1 , E and $Q_A = Q_1$, thus suggesting forcefully that the E and $Q_1 = Q_A$ are members of the 1^{++} family. However, we run into a rather anomalous result if the group theoretical approach⁴⁾ is extended to the 1^{++} family; not only the Schwinger mass formula^{**) fails completely but also the mass-mixing angle obtained by the Gell-Mann-Okubo formula is too big.}

) In Ref. 3 it is shown that the mass degeneracy of the 1^{++} and 1^{+-} leads to a 45° mixing angle between Q_1 and Q_2 , to explain the suppression of $Q_1 \rightarrow K^\pi$ decay mode.

**) The Schwinger mass formula for the 1^{--} ($L=0$) and for the 0^{++} , 2^{++} ($L=1$) multiplets has been discussed in Ref. 4 by an SU(4) group theoretical approach to the meson mass spectrum. The relevant hypothesis for the validity of the formula is that in each multiplet one of the SU(3) singlet states is essentially a charm-anticharm state.

In this paper we present the mass relations and mixing angles for the 1^{++} family based on the spectral sum rules of the axial-vector currents and 0^+ fields with the $(3, \bar{3}) + (\bar{3}, 3)$ breaking. In such a framework, the 0^+ fields are related to the divergence of 1^+ currents by the partial conservation. Also there are mixings between the currents or the fields to be contrasted to just the mass-mixing in the Wigner-Eckart approach. The spectral sum rules for the axial-vector currents and the pseudoscalar fields introduce the current- and field-mixing angles naturally and accommodate the role of the pseudoscalar multiplet properly so as to take into account the corrections of the vacuum and symmetry breakings.

Following Refs. 5 and 6, we write the two spectral sum rules of Weinberg

$$J_{\alpha, \beta} = \int_0^\infty dm^2 \left[\rho_{\alpha, \beta}^{(1)}(m^2) m^{-2} + \rho_{\alpha, \beta}^{(0)}(m^2) \right] = A \delta_{\alpha, \beta} \quad (1)$$

$$\begin{aligned} K_{\alpha, \beta} &= \int_0^\infty dm^2 \rho_{\alpha, \beta}^{(1)}(m^2) = \\ &= B \delta_{\alpha, \beta} + C d_{8, \alpha, \beta} + D d_{15, \alpha, \beta} + F \delta_{\alpha, 0} \delta_{\beta, 0} \end{aligned} \quad (2)$$

where $\rho_{\alpha, \beta}^{(1)}$ and $\rho_{\alpha, \beta}^{(0)}$ are the spin 1 and spin 0 spectral functions of the $SU(4)$ axial-vector currents $A_\mu^{(\alpha)}$ ($\alpha = 0, 1, 2, \dots, 15$) in the propagator

$$\begin{aligned} \int d^4x e^{-iqx} \langle 0 | T(A_\mu^{(\alpha)}(x) A_\nu^{(\beta)}(0)) | 0 \rangle = \\ = -i \int_0^\infty dm^2 (q^2 + m^2)^{-1} \left[(g_{\mu\nu} + \frac{q_\mu q_\nu}{m^2}) \rho_{\alpha, \beta}^{(1)}(m^2) + q_\mu q_\nu \rho_{\alpha, \beta}^{(0)}(m^2) \right] \end{aligned} \quad (3)$$

+ Schwinger terms.

In the current-mixing model, the symmetry is broken by certain current operator terms and different current operators have in general different mixing angles. For example, the $E-D-\psi_A$ mixing and the $\eta-\eta'-\eta_c$ mixing are due to $A_\mu^{(8)}$, $A_\mu^{(0)}$ and $A_\mu^{(15)}$ when the spectral functions $\rho_{\alpha\beta}^{(1)}$ and $\rho_{\alpha\beta}^{(0)}$ are saturated by the single-particle states. In what follows, we use, instead of $A_\mu^{(0)}$ and $A_\mu^{(15)}$

$$A_{\nu}^{(\pm)} = 1/2 \left[\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix} A_{\nu}^{(0)} \pm \begin{pmatrix} 1 \\ \sqrt{3} \end{pmatrix} A_{\nu}^{(15)} \right] \quad (4)$$

and define the mixing angles by the vacuum-to-one particle matrix elements

$$\left. \begin{aligned} \langle 0 | A_{\nu}^{(8)} | E \rangle &= \frac{m_E^2}{a_8} \cos \theta_8^A \in_{\nu}^{(E)} \\ \langle 0 | A_{\nu}^{(8)} | D \rangle &= \frac{-m_D^2}{a_8} \sin \theta_8^A \in_{\nu}^{(D)} \\ \langle 0 | A_{\nu}^{(8)} | \Psi_A \rangle &= 0 \end{aligned} \right\} \quad (5)$$

$$\left. \begin{aligned} \langle 0 | A_{\nu}^{(+)} | E \rangle &= \frac{m_E^2}{a_+} \sin \theta_+^A \in_{\nu}^{(E)} \\ \langle 0 | A_{\nu}^{(+)} | D \rangle &= \frac{m_D^2}{a_+} \cos \theta_+^A \in_{\nu}^{(D)} \\ \langle 0 | A_{\nu}^{(+)} | \Psi_A \rangle &= 0 \end{aligned} \right\} \quad (6)$$

$$\left. \begin{aligned} \langle 0 | A_{\nu}^{(-)} | E \rangle &= \langle 0 | A_{\nu}^{(-)} | D \rangle = 0 \\ \langle 0 | A_{\nu}^{(-)} | \Psi_A \rangle &= \frac{m_{\Psi_A}^2}{a_-} \in_{\nu}^{(\Psi_A)} \end{aligned} \right\} \quad (7)$$

and

$$\left. \begin{aligned} \langle 0 | A_{\nu}^{(8)} | \eta \rangle &= F_8 \cos \theta_8^P p_{\nu}(\eta) \\ \langle 0 | A_{\nu}^{(8)} | \eta' \rangle &= -F_8 \sin \theta_8^P p_{\nu}(\eta') \\ \langle 0 | A_{\nu}^{(8)} | \eta_c \rangle &= 0 \end{aligned} \right\} \quad (8)$$

$$\left. \begin{aligned} \langle 0 | A_{\nu}^{(+)} | \eta \rangle &= F_+ \sin \theta_+^P p_{\nu}(\eta) \\ \langle 0 | A_{\nu}^{(+)} | \eta' \rangle &= F_+ \cos \theta_+^P p_{\nu}(\eta') \\ \langle 0 | A_{\nu}^{(+)} | \eta_c \rangle &= 0 \end{aligned} \right\} \quad (9)$$

$$\left. \begin{aligned} \langle 0 | A_{\nu}^{(-)} | \eta \rangle &= \langle 0 | A_{\nu}^{(-)} | \eta' \rangle = 0 \\ \langle 0 | A_{\nu}^{(-)} | \eta_c \rangle &= F_- p_{\nu}(\eta_c) \end{aligned} \right\} \quad (10)$$

The vacuum-to-one particle matrix elements of the other axial vector currents are defined similarly, i.e.

$$\langle 0 | A_\mu^{(3)} | A_1 \rangle = \frac{m_{A_1}^2}{a_{A_1}} \epsilon_\mu (A_1) \quad (11)$$

$$\langle 0 | A_\mu^{(3)} | \pi \rangle = F_\pi p_\mu (\pi)$$

etc. In Eqs. (5) to (10) we have assumed that ψ_A and η_c are pure $c\bar{c}$ states. This is why the use of $A^{(\pm)}$ is more convenient in defining the mixing angles. The narrow width of the $\psi(3.100)$ and $\psi(3.700)$ give an indirect support to this approximation. This allows to reduce the six mixing angles within each multiplet to only two angles. In any way, the presently available experimental data do not allow to treat the general case and/or estimate the above approximation. By setting the first sum rule (1) equal to the isospin current, one gets $A = m_\rho^2/f_\rho^2$ and

$$\frac{m_{A_1}^2}{a_{A_1}^2} = P_\pi, \quad \frac{m_{Q_1}^2 + m_{Q_2}^2}{a_Q^2} = \frac{m_{Q_A}^2 + m_{Q_B}^2}{a_Q^2} = P_K$$

$$\frac{1}{a_8^2} (m_E^2 \cos^2 \theta_8^A + m_D^2 \sin^2 \theta_8^A) = P_8 \quad (12)$$

$$\frac{1}{a_+^2} (m_E^2 \sin^2 \theta_+^A + m_D^2 \cos^2 \theta_+^A) = P_+$$

$$\frac{1}{a_8 a_+} (m_E^2 \cos \theta_8^A \sin \theta_+^A - m_D^2 \sin \theta_8^A \cos \theta_+^A) = F_8 F_+ \sin(\theta_8^P - \theta_+^P)$$

where

$$P_i = \frac{m_\rho^2}{f_\rho^2} - F_i^2 \quad (i = \pi, K, 8 \text{ and } +) \quad (13)$$

Also the second sum rule (2) results

$$\frac{m_{A_1}^4}{2 a_{A_1}^2} - \frac{m_{Q_1}^4 + m_{Q_2}^4}{a_Q^2} = \frac{3}{2\sqrt{2}} \frac{1}{a_8 a_+} (m_E^4 \cos \theta_8^A \sin \theta_+^A - m_D^4 \sin \theta_8^A \cos \theta_+^A) \quad (14)$$

$$\frac{4}{3 a_Q^2} (m_{Q_1}^4 + m_{Q_2}^4) - \frac{1}{3 a_{A_1}^2} m_{A_1}^4 = \frac{1}{a_8^2} (m_E^4 \cos^2 \theta_8^A + m_D^4 \sin^2 \theta_8^A)$$

If the last term in Eq.(2) ($F \delta_{\alpha,0} \delta_{\beta,0}$) is suppressed, the following relation can be derived

$$\frac{2}{3 a_Q^2} (m_{Q_1}^4 + m_{Q_2}^4) + \frac{1}{3 a_{A_1}^2} m_{A_1}^4 = \frac{1}{a_+^2} (m_E^4 \sin^2 \theta_+^A + m_D^4 \cos^2 \theta_+^A) \quad (15)$$

where

$$m_{Q_1}^4 + m_{Q_2}^4 = m_{Q_A}^4 + m_{Q_B}^4 - \frac{1}{2} \sin 2\alpha (m_{Q_A}^2 - m_{Q_B}^2)^2 \quad (16)$$

α being the Q_A - Q_B mixing angle. Note that (16) is practically independent of α as the last term is no more than one percent of $m_{Q_A}^4 + m_{Q_B}^4$. In what follows we make use of only the relations (12) and (14). Relation (15), which depends on the second sum rule for singlet current $A_\mu^{(+)}$, is not used. When the solutions of (12) and (14) are checked against (15), this relation turns out to be quite wrong, although it has proved to be reasonable for the vector meson family.

In order to proceed with (12) and (14), we need to know the decay constants F_π, F_K, F_8 , etc. For this purpose, we make use of the spectral sum rules for the current-field combinations derived by Auvil and Deshpande⁷⁾ from the Wilson expansion in the $(3, \bar{3}) + (\bar{3}, 3)$ chiral model. In addition to their sum rules, i.e. Eqs. (2) and (3) in Ref. 7, one can get further sum rules so long as the dimension of the singlet axial vector current is less than 7/2 for the combinations of $A_\mu^{(+)}$ and the 8th pseudoscalar field and $A_\mu^{(+)}$ and the singlet pseudoscalar field, which are in their notation as follows:

$$\int_0^\infty dm^2 \left(\rho_A^{33} - \rho_A^{44} - \frac{3}{2\sqrt{2}} \rho_A^{+8} \right) = 0$$

$$\int_0^\infty dm^2 \left(2\rho_A^{44} + \rho_A^{33} - 3\rho_A^{++} \right) = 0$$
(17)

Then we get in addition to their Eqs. (4) and (5) two more relations

$$1 - F_K \sqrt{Z_K} = \frac{3}{2\sqrt{2}} \sqrt{Z_8} F_+ \sin(\varphi - \theta_+^P)$$

$$1 + 2F_K \sqrt{Z_K} = 3\sqrt{Z_0} F_+ \cos(\varphi - \theta_+^P)$$
(18)

where ϕ and θ are the pseudoscalar field-mixing angles defined by

$$\langle 0 | v_8 | \eta \rangle = \sqrt{Z_8} \cos \varphi, \quad \langle 0 | v_8 | \eta' \rangle = -\sqrt{Z_8} \sin \varphi$$

$$\langle 0 | v_0 | \eta \rangle = \sqrt{Z_0} \sin \varphi, \quad \langle 0 | v_0 | \eta' \rangle = \sqrt{Z_0} \cos \varphi$$
(19)

If we take the two field-mixing angles to be the same, $\phi = \theta$, we get all the SU(3) renormalization constants Z_i ($i = \pi, K, 8$) to be the same. This appears to be a reasonable approximation, as discussed in Refs. 8 and 9. In this way we determine F_8/F_π , F_+/F_π , F_K/F_π , the mass m_K of the scalar strange meson K and the mixing angles θ_8^P , θ_+^P and ϕ , all in terms of F_K/F_π . With these decay constants we proceed to solve (12) and (14). But, due to the experimental uncertainty of the masses of A_1 , Q_A and Q_B , we use $m_{Q_B}^2 = m_B^2 + \Delta$ and $m_{A_1}^2 = m_{Q_A}^2 - \Delta$, Δ being a free parameter of the order of the splitting between the K^* and A_2 in the 2^{++} multiplet. A systematic search for the solutions of (12) and (14) is made with $m_B = 1.218$ GeV, 1.228 GeV, 1.238 GeV and $m_\rho^2/f_\rho^2 = 0.0224$ GeV², 0.024 GeV² and 0.026 GeV² *). The conditions

$$\theta_8^A \sim 35^\circ, \quad \theta_{SU3}^A \sim \frac{1}{2} (\theta_8^A + \theta_+^A)$$

*) From $\rho \rightarrow e^+e^-$ we obtain $m_\rho^2/f_\rho^2 = (3/4 \pi \alpha^2) m_\rho \Gamma_{\rho \rightarrow e^+e^-} = (22.7 + 2.6) \times 10^{-3}$ GeV².

where θ_{SU3}^A is the angle given by the Gell-Mann-Okubo mass formula, determine the solution $m_{Q_A}^2$ and also the parameters F_K/F_π and Δ . The result of our analysis is given in the Table. Note that $m_K^2 - m_\delta^2 = 0.39 \text{ GeV}^2$ suggests that partial conservation of the vector current is quite reasonable also for the scalar strange meson field. If the Q_1 mass is 1.27 GeV we obtain: $m_{Q_2} = 1.36 \text{ GeV}$ and $\alpha = 35$.

To conclude, the spectral sum rules (12) and (14) with current-mixing model give very reasonable mixing angles and mass relations for the axial-vector multiplet as well as the decay constants and mixing angles for the pseudoscalar mesons.

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TABLE

input	result
$m_B = 1.228 \text{ GeV}$	$\theta_8^A = 35.7^\circ$ $m_{Q_A}^2 = 1.69 \text{ GeV}^2$ $F_K/F_\pi = 1.14$
$m_D = 1.286 \text{ GeV}$	$\theta_+^A = 28.5^\circ$ $m_{Q_B}^2 = 1.78 \text{ GeV}^2$ $F_8/F_\pi = 1.2$
$m_E = 1.416 \text{ GeV}$	$\theta_{SU3}^A = 31.4^\circ$ $m_{A_1} = 1.2 \text{ GeV}$ $F_+/F_\pi = 1.35$
$m_K^2 / m_\pi^2 = 13.5$	$\theta_8^P = 11.9^\circ$ $a_{A_1} = 9.6$ $F_K/F_\pi = -0.17$
$m_\eta^2 / m_\pi^2 = 16.5$	$\theta_+^P = 10.0^\circ$ $a_Q = 11.6$ $Z_+/Z_\pi = 0.66$
$m_{\eta'}^2 / m_\pi^2 = 50.3$	$\theta_{SU3}^P = 10.7^\circ$ $a_8 = 12.7$ $Z_K/Z_\pi = 0.77$
$m_\rho^2 / f_\rho^2 = 0.024 \text{ GeV}^2$	$\varphi = 4.2^\circ$ $a_+ = 14.4$ $m_K = 1.16 \text{ GeV}$
$F_\pi^2 = 8.47 \times 10^{-3} \text{ GeV}^2$	$\Delta = .27 \text{ GeV}^2$

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