CERN LIBRARIES, GENEVA

CM-P00061849

Ref.TH.2226-CERN

ENERGY BOUNDS FOR CHARM AND FOR HEAVY LEPTON PRODUCTION
IN e^+e^- ANNIHILATION

O. Nachtmann ^{*)} and A. Pais ⁺⁾
CERN -- Geneva

A B S T R A C T

Bounds are derived for the mean charged to mean neutral pion (and kaon) energy ratios, valid in arbitrary phase space domains, for charm and for heavy lepton pair production. Absolute bounds for the mean neutrino and/or π^0 energy in terms of the beam energy are also given. For charm production, D^* effects are included.

^{*)} Permanent address : Institut für Theoretische Physik,
Universität Heidelberg.

⁺⁾ Permanent address : Rockefeller University, New York.

At this time there exists evidence of varying degree for the production in e^+e^- annihilation of charmed meson pairs ¹⁾ and of heavy lepton pairs ²⁾. It is the purpose of this Letter to report on some inclusive decay characteristics of such pairs. In theoretical analyses of charmed meson decays ³⁾ and of semi-hadronic heavy lepton decays ⁴⁾ the main emphasis to date has been on exclusive (two, three-body) channels ⁵⁾. In such studies it is often necessary to introduce model-dependent assumptions. We believe it may be of some interest to see to what extent such considerations can be supplemented by inclusive arguments based on fairly minimal assumptions, not only because the particles concerned have so many decay channels open to them but also because there is often a considerable loss of information since the decays occur in flight and since they may involve numerous neutral particles. In this note we concentrate mainly on this neutral particle aspect. We shall derive a variety of bounds on the average energy of neutrinos and/or neutral pions for the processes at hand.

Let us first state our assumptions. We take the heavy leptons ($L^\pm$) to be of sequential type with a zero mass associated neutrino. The leptonic decay is assumed to be an arbitrary V, A mixture. Semi-hadronic decays satisfy $|\Delta I| = 1(\frac{1}{2})$ for $|\Delta S| = 0(1)$ (as is conventional). For charmed mesons we consider the zero strangeness isodoublet D^+, D^0 and the unit strangeness F^+ (and their conjugates) and assume the following selection rules : $\Delta S/\Delta C = 1$ for all decays ; $\Delta C/\Delta Q = 1$ and $\Delta I = 0$ for the semi-leptonic decays ; and $|\Delta I| = |\Delta I_3| = 1$ for the non-leptonic decays. All these charm decay properties are part of the GIM model ⁶⁾ if one neglects the Cabibbo angle.

In what follows it makes no difference whether or not π 's, K 's are produced in resonant states, with the exception of course of η production. We first state all results ignoring η 's and discuss the η problem at the end.

A. - BOUNDS WHICH HOLD IN ARBITRARY DOMAINS OF PHASE SPACE

These bounds allow for limited acceptance angles, minimal allowed pion energy, etc. These results are also independent of the energy of the decaying particle. Let E_o^π, E_{ch}^π denote the total energy which goes into neutral, charged pions, respectively, and $E(K^0)$ the total energy which goes into K^0 's (and similarly for other K 's). Let $\langle E \rangle$ stand for the average of E .

$L^+ \rightarrow \bar{\nu}_L + \text{hadrons}$ 7)

$$\Delta S = 0 : \quad 0 \leq \langle E_o^\pi \rangle \leq \langle E_{ch}^\pi \rangle, \quad (1)$$

$$0 \leq \langle E(K^0) \rangle \leq 3 \langle E(K^+) \rangle, \quad (2)$$

$$0 \leq \langle E(K^-) \rangle \leq 3 \langle E(\bar{K}^0) \rangle. \quad (3)$$

$$\Delta S = 1 : \quad \langle E_o^\pi \rangle = \frac{1}{2} \langle E_{ch}^\pi \rangle, \quad (4)$$

$$0 \leq \langle E(K^0) \rangle \leq 2 \langle E(K^+) \rangle, \quad (5)$$

$$0 \leq \langle E(K^-) \rangle \leq 2 \langle E(\bar{K}^0) \rangle. \quad (6)$$

Charm ; semi-leptonic decays 8)

$$\text{All } D'_s; F'_s : \quad \langle E_o^\pi \rangle = \frac{1}{2} \langle E_{ch}^\pi \rangle \quad (7)$$

$$D^0 : \quad 0 \leq \langle E(\bar{K}^0) \rangle \leq 2 \langle E(K^-) \rangle \quad (8)$$

$$D^+ : \quad 0 \leq \langle E(K^-) \rangle \leq 2 \langle E(\bar{K}^0) \rangle \quad (9)$$

$$F^+ : \quad \langle E(K^-) \rangle = \langle E(\bar{K}^0) \rangle \quad (10)$$

Charm ; non-leptonic decays

$$D^0 : \quad 0 \leq \frac{\langle E_o^\pi \rangle}{\langle E_{ch}^\pi \rangle} \leq \infty ; \quad 0 \leq \frac{\langle E(\bar{K}^0) \rangle}{\langle E(K^-) \rangle} \leq \infty, \quad (11)$$

$$D^+ : \quad 0 \leq \langle E_o^\pi \rangle \leq \frac{3}{2} \langle E_{ch}^\pi \rangle ; \quad 0 \leq \langle E(K^-) \rangle \leq 4 \langle E(\bar{K}^0) \rangle, \quad (12)$$

$$F^+ : \quad 0 \leq \langle E_o^\pi \rangle \leq \langle E_{ch}^\pi \rangle ; \quad 0 \leq \langle E(K^-) \rangle \leq 3 \langle E(\bar{K}^0) \rangle. \quad (13)$$

Remarks

a) - Equations (1)-(13) also hold if we replace each mean energy $\langle E \rangle$ by the corresponding ⁹⁾ mean particle number $\langle N \rangle$, e.g., for the semi-leptonic decays of D's and F's the average number of neutral pions equals half the average number of charged pions.

b) - Consider the reaction sequences

$$e^+e^- \rightarrow D^0 \bar{D}^0 \rightarrow \underline{\text{one}} e (\text{or } \mu) + \text{rest}, \quad (14)$$

$$\rightarrow D^+ D^- \rightarrow \text{id.} \quad (15)$$

$$\rightarrow F^+ F^- \rightarrow \text{id.} \quad (16)$$

$$\rightarrow L^+ L^- \rightarrow \text{id.} \quad (17)$$

which are of such great diagnostic value. Equations (14)-(16) arise from one semi-leptonic + one non-leptonic decay, Eq. (17) from one leptonic + one semi-hadronic decay. Then

$$\begin{aligned} \langle E_0^\pi \rangle &\leq \left(\frac{3}{2} ; 1 ; 1 \right) \cdot \langle E_{ch}^\pi \rangle \quad \text{and} \\ \langle N_0^\pi \rangle &\leq \left(\frac{3}{2} ; 1 ; 1 \right) \cdot \langle N_{ch}^\pi \rangle \end{aligned} \quad (18)$$

for reactions (15), (16), (17) respectively, while there are no such non-trivial bounds for Eq. (14). Unfortunately, this last situation also applies to all reactions of the type ¹⁰⁾

$$e^+e^- \rightarrow DD^* \text{ or } D^*D^* \rightarrow \underline{\text{one}} e (\text{or } \mu) + \text{rest}.$$

c) - The equations (1)-(13) are, of course, elementary consequences of isospin algebra and positivity requirements.

B. - ABSOLUTE ENERGY BOUNDS

These same ingredients are also needed but are not sufficient if one asks the question : what is the bound on the mean energy of particles of a certain species in terms of the available energy ? (we call these absolute energy bounds). In that case we must also implement the energy and

momentum conservation conditions. Moreover, we must take into account certain optimal kinematic configurations ; for this last reason, the absolute bounds cannot be implemented with arbitrary phase space cuts.

A similar problem was recently discussed ¹¹⁾ in connection with the isospin conserving processes $e^+e^- \rightarrow$ pions. We follow related methods. To begin with, we consider decays at rest of the various particles. The results are as follows (M =mass of parent particle, m_K =K mass, m_π = pion mass).

$$L \rightarrow e \nu_e \nu_L : \langle E_\nu \rangle / M = 0.65 \quad (19)$$

$$L \rightarrow \nu_L + \text{hadrons } (\Delta S = 0)$$

$$\langle E_\nu \rangle / M \leq \frac{1}{2} \left(1 - \frac{m_\pi^2}{M^2} \right) \quad (20)$$

$$\langle E_0^\pi \rangle / M \leq \frac{1}{2} \quad (21)$$

$$\langle E_\nu + E_0^\pi \rangle / M \leq \frac{3}{4} \left(1 - \frac{4m_\pi^2}{3M^2} \right) \quad (22)$$

$$L \rightarrow \nu_L + \text{pions only}$$

$$\langle E_\nu \rangle / M \geq 0 \quad (23)$$

$$\langle E_0^\pi \rangle / M \geq 0 \quad (24)$$

$$\langle E_\nu + E_0^\pi \rangle / M > \frac{11 - \sqrt{41}}{40} \approx 0.11 \quad (25)$$

$$L \rightarrow \nu_L + \text{hadrons } (\Delta S = 1)$$

$$\langle E_\nu \rangle / M \leq \frac{1}{2} \left(1 - \frac{m_K^2}{M^2} \right) \quad (26)$$

$$\langle E_0^\pi \rangle / M \leq \frac{1}{3} \left(1 - \frac{m_K}{M} \right) \quad (27)$$

$$\langle E_\nu + E_0^\pi \rangle / M < \frac{2}{3} \left(1 - \frac{\sqrt{3}}{2} \frac{m_K}{M} \right) \quad (28)$$

$$\text{Any } D \rightarrow e \nu + \text{hadrons}$$

$$\langle E_\nu \rangle / M \leq \frac{1}{2} \left(1 - \frac{m_K^2}{M^2} \right) \approx 0.46 \quad (29)$$

$$\langle E_0^\pi \rangle / M \leq \frac{1}{3} \left(1 - \frac{m_K}{M} \right) \approx 0.24 \quad (30)$$

$$\langle E_\nu + E_0^\pi \rangle / M < \frac{2}{3} \left(1 - \frac{\sqrt{3}}{2} \frac{m_K}{M} \right) \approx 0.51 \quad (31)$$

$$F^{\pm} \rightarrow e \nu + \text{hadrons}$$

$$\langle E_{\nu} \rangle / M \leq \frac{1}{2} \left(1 - \frac{4m_{\pi}^2}{M^2} \right) \quad (32)$$

$$\langle E_{\pi}^{\pi} \rangle / M \leq \frac{1}{3} \quad (33)$$

$$\langle E_{\nu} + E_{\pi}^{\pi} \rangle / M \leq \frac{2}{3} \left(1 - \frac{2m_{\pi}^2}{M^2} \right) \quad (34)$$

$$D^{\pm} \rightarrow \text{hadrons}$$

$$\langle E_{\pi}^{\pi} \rangle / M < \frac{3}{5} \left(1 - \frac{m_K}{M} \right) \approx 0.44 \quad (35)$$

$$D^0, \bar{D}^0 \rightarrow \text{hadrons}$$

$$\langle E_{\pi}^{\pi} \rangle / M < \frac{9 + \sqrt{41}}{20} \left(1 - \frac{m_K^2 + 2m_K m_{\pi}}{M^2} \right) \approx 0.68 \quad (36)$$

$$F^{\pm} \rightarrow \text{hadrons}$$

$$\langle E_{\pi}^{\pi} \rangle / M < \frac{1}{2} \quad (37)$$

Remarks

a) - In the derivation of these bounds, Eqs. (1), (4), (7), (11), (12) and (13) have to be used.

b) - The bounds given in Eqs. (20)-(37) all are valid without neglect of the pion mass. Wherever we have used the $\leq$ sign, we imply that the bound in question can be saturated. In all other cases, improvements are possible at considerable labour. These improvements would be quite small in all instances except perhaps for Eqs. (35) and (36).

c) - Under circumstances, the K mass can give effects as large as $\sim 25\%$ as compared with the case where m_K is neglected. The inclusion of the K mass is somewhat subtle as may be shown by one example. Take Eq. (35) and imagine for a moment that m_K were a free parameter. Then we can easily show that for $D^{\pm} \rightarrow \text{hadrons}$:

$$\langle E_{\pi}^{\pi} \rangle / M < \text{Min} \left\{ \frac{3}{5} \left(1 - \frac{m_K}{M} \right); \frac{1}{3} \left(1 + \sqrt{\frac{2}{5}} \right) \left(1 - \frac{m_K^2 + 2m_K m_{\pi}}{M^2} \right) \right\},$$

and one notes that, for the physical K mass, the first of the two choices wins out, whereas the second choice would have been better if we had neglected the K mass.

We enquire next what these bounds imply for L and charm production with a total energy E in the two beams.

1) - Obviously, for L pair production, the total hadronic energy is always $\leq E/2$, for charm pair production it is always $\geq E/2$.

2) - Next we wish to make use in the laboratory frame of the Eqs. (19)-(37) which refer to the decay rest frame. Since the one-particle inclusive decay distributions in the rest frame are isotropic for the (spinless) charmed mesons as well as for the heavy leptons (where we neglect any possible longitudinal polarization of the e^+e^- beams) it follows that the rest frame expressions may be applied to the laboratory frame if we replace everywhere expressions like $\langle E_\nu \rangle/M$ by $2\langle E_\nu \rangle/E$. We first consider the average energy $\langle E_\ell \rangle$ for $\ell = e$ or μ . Then we have for the reactions (14)-(17) :

$$\frac{\langle E_\ell \rangle}{E} \left\{ \begin{array}{ll} = \frac{7}{40} \approx 0.18 & : L, \\ \leq 0.25 & : \text{ charmed mesons.} \end{array} \right. \quad (38)$$

3) - Consider next the average neutral energy $\langle E_\nu + E_0^\pi \rangle$. For heavy leptons we have

$$\left. \begin{array}{l} \frac{13}{40} \approx 0.32 \\ \frac{37 - \sqrt{41}}{80} \approx 0.38 \end{array} \right\} \leq \frac{\langle E_\nu + E_0^\pi \rangle}{E} \leq \frac{7}{10}, \quad (39)$$

where the lower bound 0.32 is general, while the value 0.38 obtains if all $|\Delta S| = 1$ decays as well as $K\bar{K}$ production in semi-hadronic $\Delta S = 0$ decays of L were negligible. For charm production one finds

$$\frac{\langle E_\nu + E_0^\pi \rangle}{E} \leq \begin{array}{ll} 0.48 & : D^+ D^-, \\ 0.60 & : D^0 \bar{D}^0, \\ 0.59 & : F^+ F^- \end{array} \quad (40)$$

Our initial motivation for deriving Eqs. (38)-(40) was to see if energy bounds could serve as additional discrimination between the production of heavy leptons and of charm of various kinds. One notes that there is some but no dramatic distinction between these bounds for the various cases.

- 4) - We make one remark on the cases where both $L\bar{L}$ decay semi-hadronically and both charmed particles decay non-leptonically. One has

$$\frac{\langle E_o^\pi \rangle}{E} \leq \begin{array}{ll} 1/2 & : L^+ L^-, \\ 0.44 & : D^+ D^-, \\ 0.68 & : D^0 \bar{D}^0, \\ 1/2 & : F^+ F^-. \end{array} \quad (41)$$

For isospin conserving processes $e^+e^- \rightarrow$ pions one has ¹¹⁾ $\langle E_o^\pi \rangle \leq 0.77 E$. Thus for L - or for elastic charm production - the neutral pion energy bounds are actually stronger than for isospin conserving channels.

- 5) - There is evidence ¹⁾ that the production of D mesons mainly proceeds via DD^* channels, at the energies so far explored. Here D^* is presumably ¹⁰⁾ the vector counterpart of the pseudoscalar D . There is very little phase space for $D^* \rightarrow D\pi$ so that the branching ratios for these strong decays strongly violate isospin. Our bounds can also be applied to DD^* and D^*D^* production if we use prescribed $D^* \rightarrow D\pi$ branching ratios, as for example those of Ref. 10). Barring branching ratio information for semi-leptonic versus non-leptonic decays of D^+ and D^0 one finds :

$$\frac{\langle E_\nu + E_o^\pi \rangle}{E} \leq \begin{array}{ll} 0.58 & : D^{ch} \bar{D}^{ch*}, \\ 0.60 & : D^0 \bar{D}^{0*}, \\ 0.60 & : D^* \bar{D}^* \text{ of either charge,} \end{array} \quad (42)$$

for the "one-lepton" final configurations, as in Eqs. (14), (15).

- 6) - Equation (42) provides a first example of the application of our methods to inelastic charm production. These methods can in fact be brought to bear on the reactions

$$e^+e^- \rightarrow (D^0 \bar{D}^0 \text{ or } D^+ D^- \text{ or } F^+ F^-) + \text{anything} \quad (43)$$

but here the analysis is much more involved. In this Letter we shall content ourselves by noting that $\langle E_\nu + E_o^\pi \rangle$ for the reactions (43) can be at least as large as

$$\frac{\alpha (E^2 - 2EM + 2M^2) + E(E - 2M)}{2(E - M)} + \beta M \quad (44)$$

where

$$\alpha = 0.68 ; 0.51 ; 0.67 ;$$

$$\beta = 0.51 ; 0.44 ; 0.50 ;$$

for neutral D's ; charged D's ; F's, respectively.

C. - ABSOLUTE BOUNDS AND THE η PROBLEM

Of course, the problem of η production arises in e^+e^- annihilation whether or not L's or charm are produced. The question cannot be ignored ¹²⁾ since, in η decay, $\langle E_o^\pi \rangle / M_\eta \simeq 0.8$. However, for the reactions (14)-(17) (and also for the D^* cases) the problem is at least simplified by the circumstance that the number of η 's which can at most be produced is limited. We may perhaps be fortunate in that decay branching ratios into η channels will turn out to be small, but we shall not rely on this in the subsequent few comments on this issue.

1) - Case of D pair production

In the decay of a D or a $\bar{D}$ at most two η 's can be produced ; and for that case the decay phase space is already small. Even apart from this last circumstance, one can show that the case of single η production causes the greater strain on the previously given bounds.

For the decays $D^0, \bar{D}^0 \rightarrow K + \eta + \dots$ the η channels do not affect the π^0 bound given in Eq. (36). For $D^0, \bar{D}^0 \rightarrow \nu e K \eta + \dots$ the bound in Eq. (31) would be raised to 0.65. Hence appreciable η production could raise the neutral D bounds in Eqs. (40) and (42) to 0.67. For charged D's the bounds in Eqs. (31) and (35) would become 0.65 and 0.53, respectively. This would raise the $D^\pm$ bound in Eq. (40) to 0.59.

2) - Case of F pair production

Here as many as three η 's could be produced in either semi-leptonic or non-leptonic decays. The presence of η 's would change the bounds in Eqs. (34) and (37) to 0.89 and 0.80, respectively, and therefore the F bound in Eq. (40) would be raised to 0.85 : as was to be expected, appreciable η production in charm decay has potentially a much greater impact on F than on D channels.

3) - Heavy lepton production

Again as many as three η 's could be produced in semi-hadronic $\Delta S = 0$ decays (for $M_L \simeq 2$ GeV). The effect of the presence of η 's is to change the bound in Eq. (22) to 0.89, hence the upper bound in Eq. (39) to 0.77. The lower bounds in that equation remain unaffected.

In this Letter we have mainly emphasized the implications of our bounds for the reactions $e^+e^- \rightarrow$ single lepton + anything. As more detailed information becomes available on charm or heavy lepton production, there will, of course, be many more implications (including tests of the standard charm model) of the bounds given here.

ACKNOWLEDGEMENTS

We are grateful to the Theoretical Physics Division of CERN for its warm hospitality.

REFERENCES

- 1) G. Goldhaber et al. - Phys.Rev.Letters 37, 255 (1976).
- 2) M.L. Perl et al. - SLAC-PUB-1781 (1976), to appear in Physics Letters.
- 3) See, e.g.,
M.B. Einhorn and C. Quigg - Phys.Rev. D12, 2015 (1976), also for earlier references.
- 4) See, e.g.,
T. Hagiwara, S.Y. Pi and A.I. Sanda - Rockefeller University Preprint COO-2232B-105 (1976), also for earlier references.
- 5) Some comments on multipion decays and inclusive semi-leptonic decays were made by :
M.K. Gaillard, B.W. Lee and J.L. Rosner - Revs.Modern Phys. 47, 277 (1975).
- 6) S.L. Glashow, J. Iliopoulos and L. Maiani - Phys.Rev. D2, 1285 (1970).
- 7) The bounds for $L^- \rightarrow \nu_L + \text{hadrons}$ are obtained from Eqs. (1)-(6) by replacing each particle by its charge symmetric counterpart.
- 8) Equations (8)-(13) also hold if, on each line, each particle is replaced by its antiparticle. These equations also remains true if one substitutes $\bar{K}^0 \rightarrow K^+$, $K^- \rightarrow K^0$.
- 9) Number bounds related to Eqs. (1)-(3) were given by :
A. Pais and S.B. Treiman - Phys.Rev. D14, 293 (1976).
- 10) Here we follow the notations and results of :
A. de Rujulà, H. Georgi and S.L. Glashow - Phys.Rev.Letters 37, 398 (1976).
- 11) J.S. Bell, G. Karl and C.H. Llewellyn Smith - Phys.Letters 52, 363 (1974).
- 12) The possible presence of η 's also would affect the Eqs. (1), (4), (7), (12) and (13). These equations remain valid in phase space regions free from pions due to η decay. Elsewhere, all these equations are to be replaced by $\langle E_0^\pi \rangle \lesssim 2,38 \langle E_{ch}^\pi \rangle$.