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A QUEST FOR A WHOLLY SCALING VARIABLE

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A B S T R A C T

We study quark-parton and hadron target mass corrections to Bjorken scaling using the parton model, the light-cone expansion, and operator product expansion techniques motivated by asymptotic freedom. We find descriptions of the approach to scaling in terms of a unique scaling variable with a parton model interpretation even for massive quarks, but with model-dependent spectral conditions. We also discuss limitations on the applicability of our analysis.

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The possibility of deep inelastic scaling was first suggested ¹⁾ on the basis of current commutators, which were subsequently given a physical interpretation via the quark parton model ²⁾. Given the approximate validity of Bjorken scaling, it is worth while to consider corrections to the naïve picture, either for $Q^2 \rightarrow \infty$ as in asymptotically free gauge theories ³⁾, or at small Q^2 due to mass effects. Questions raised at small Q^2 include the rôle of target masses, the understanding of precocious scaling, and the effect of new thresholds associated with the excitation of heavy quarks ⁴⁾. These problems have recently been discussed by Georgi and Politzer ^{5),6)}, who argue that in asymptotically free theories, where $\bar{g}^2(Q^2)/4\pi^2$ is plausibly $\ll 1$ for $Q^2 \simeq m_p^2 \approx 1$, it is possible to do perturbation theory in the strong coupling constant, and compute quark and target mass effects exactly in lowest order. They used operator product expansions ⁷⁾ to derive a new scaling variable to summarize mass corrections at finite Q^2 . However, their variable does not have a simple parton interpretation, which seems strange in view of the usual similarities in the assumptions and results of the parton ²⁾ (PM), light-cone ⁸⁾ (LC) and operator product expansion ⁷⁾ (OPE) descriptions of deep inelastic scattering.

In this paper we argue that parton, light-cone and operator product expansion analyses at finite Q^2 in fact all yield the same scaling variable ^{*)}

$$\xi = \frac{(Q^2 + m_f^2 - m_i^2) + \sqrt{(Q^2 + m_f^2 - m_i^2)^2 + 4m_i^2 Q^2}}{2(\gamma + \sqrt{\gamma^2 + Q^2 m_T^2})} \quad (1)$$

which describes subasymptotic mass corrections. However, our PM and LC analyses impose a kinematic restriction $\xi \gtrsim m_i/m_T$ which is absent in the OPE analysis. We find use of the variable (1) questionable when $\xi \rightarrow 0$ or 1, or when $m_i \ll m_T$ ^{**) .}

For simplicity we illustrate our points in a model with scalar currents $J(x)$ which are composites of self-conjugate scalar boson constituent fields :

$$J(x) = : \phi_f(x) \phi_i(x) : \quad (2)$$

*) m_i, m_f and m_T are the initial parton, final parton and target masses, respectively. $Q^2 \equiv -q^2$ and $\gamma \equiv p \cdot q$ are the usual kinematic variables.

**) Hereafter we will set $m_T = 1$.

Analyzing the light-cone singularities of the commutator of two such currents (2), the disconnected piece never contributes to deep inelastic scattering, while the "cat's ears" piece

$$: \phi_f(x) \phi_i(x) \phi_f(0) \phi_i(0) :$$

does not contribute to the current commutator if interactions between the quarks are neglected ^{*)}. The deep inelastic structure function is then given as usual by

$$\begin{aligned} W(Q^2, \nu) &= \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle p | [J(x), J(0)] | p \rangle \\ &= \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \Delta(x, m_f^2) \langle p | : \phi_i(x) \phi_i(0) : | p \rangle \\ &\quad + (i \leftrightarrow f) \end{aligned} \quad (3)$$

where

$$\Delta(x, m_f^2) = \frac{-i}{(2\pi)^3} \int d^4k e^{-ik \cdot x} \epsilon(k_0) \delta(k^2 - m_f^2) \quad (4)$$

To the extent that $\phi_i(x)$ can be regarded as a free field ^{*)}

$$B_i(x^2, x \cdot p) \equiv \langle p | : \phi_i(x) \phi_i(0) : | p \rangle \quad (5)$$

obeys a free field equation

$$(\partial^2 + m_i^2) B_i(x^2, x \cdot p) = 0 \quad (6)$$

and hence can be represented as

$$B_i(x^2, x \cdot p) = \int d^4k e^{ik \cdot x} \delta(k^2 - m_i^2) f_i(k \cdot p) \quad (7)$$

Combining (3), (4) and (7), we find

$$\begin{aligned} W(Q^2, \nu) &\propto \int d^4k f_i(k \cdot p) \delta(k^2 - m_i^2) \delta((k+q)^2 - m_f^2) \epsilon(k_0, q_0) \\ &\quad + (i \leftrightarrow f) \end{aligned} \quad (8)$$

*) Motivation for this approximation in the deep Euclidean region was proposed by Georgi and Politzer ^{5), 6)} on the basis of asymptotic freedom.

This equation (8) is just what we would have written down in a covariant formulation ¹⁰⁾ of the naïve parton model ^{2),11)} with both initial and final quark-partons essentially free (and hence on mass-shell) - as in the simple-minded impulse approximation. Thus the light-cone and naïve parton formulations are equivalent ¹²⁾.

Calculating from equation (8) we find

$$W(Q^2, \nu) \propto \frac{1}{\sqrt{\nu^2 + Q^2}} \left[F_i(E_+) + F_i(-E_-) + i \leftrightarrow \right] \quad (9)$$

where $F_i(z) \equiv \int_{-z}^{\infty} dk^0 f_i(k^0)$, with k^0 the parton energy in the target rest frame. The limits $E_{\pm}$ are specified by the on-mass-shell conditions in Eq. (8); they are

$$E_{\pm} = \frac{1}{2Q^2} \left[-\nu(Q^2 + m_f^2 - m_i^2) \pm \sqrt{\nu^2 + Q^2} \sqrt{(Q^2 + m_f^2 - m_i^2)^2 + 4m_i^2 Q^2} \right] \quad (10)$$

The mass-shell condition $\delta(k^2 - m_i^2)$ further ensures $k_0 \geq m_i$ and hence $F_i(E_+) \neq 0$ only if $E_+ \geq m_i$. It is expedient to re-express this function in terms of

$$\xi \equiv E_+ + \sqrt{E_+^2 - m_i^2} \quad (11)$$

which has the interpretation $\xi = k^0 + |\underline{k}|$ and again obeys $\xi \geq m_i$. Substituting (10) into (11) yields the expression (1) for ξ . The physical spectral condition $Q^2 \leq 2\nu$ becomes the condition $\xi \lesssim 1$ asymptotically corresponding to the mass of the system "recoiling" against the struck parton being ≥ 0 . Clearly F_i should have support only for $m_i \leq \xi \leq 1$ ^{*}). The $F_i(-E_-)$ term in (9) will then contribute to the structure function for small ν and Q^2 . However, this contribution can be eliminated by restricting $\nu \gtrsim \frac{1}{2}$ in the application of formula (9). We then find that the PM and LC approaches give the form :

^{*}) Subasymptotically, the physical spectral condition becomes

$$m_i \leq \xi \leq \frac{Q^2 + m_f^2 - m_i^2 + \sqrt{(Q^2 + m_f^2 - m_i^2)^2 + 4m_i^2 Q^2}}{2 \left(\frac{Q^2}{2} + \sqrt{\frac{Q^4}{4} + Q^2} \right)}$$

Clearly a Q^2 dependent spectral condition cannot be achieved by $F(\xi)$. It can only be achieved by a breakdown of the form (9) near the elastic limit due to $O(\bar{g})$ effects becoming important. We discuss this difficulty later.

$$W(Q^2, \nu) = \frac{1}{\sqrt{\nu^2 + Q^2}} \mathcal{F}_i(\xi) + (i \leftrightarrow f) : m_i \leq \xi \leq 1 \quad (12)$$

Another approach, motivated by the renormalization group, is to expand ⁷⁾ $T(J(x)J(0))$ as a series of local operators

$$O_i^{\mu_1 \dots \mu_n}(0) \propto \frac{1}{2^n} \phi_i(0) \overleftrightarrow{\partial}_{\mu_1} \dots \overleftrightarrow{\partial}_{\mu_n} \phi_i(0)$$

The $O_i^{\mu_1 \dots \mu_n}(0)$ are then re-expressed in terms of traceless operators $\Theta_i^{\mu_1 \dots \mu_n}(0)$ using the free field equations of motion for ϕ .

$$T(J(x)J(0)) = \sum_{n,i} C_n^i(x^2, g, m) \Theta_i^{\mu_1 \dots \mu_n} x_{\mu_1} \dots x_{\mu_n} \quad (13)$$

Applying the expansion (13) between quark states in the deep Euclidean region, we may use the renormalization group ¹³⁾ to determine the coefficients $C_n^i(x^2, g, m)$ as a function of the effective coupling $\bar{g}(Q^2)$ and the quark masses m . We find that the Fourier transform of the C_n^i are, to zeroth order in $\bar{g}$,

$$\mu_n^i(Q^2) = \frac{2}{Q^2} \left[\frac{2Q^2}{(Q^2 + m_f^2 - m_i^2) + \sqrt{(Q^2 + m_f^2 - m_i^2)^2 + 4m_i^2 Q^2}} \right]^{n+1} \quad (14)$$

where we have assumed the initial quarks obey the free field equation.

Applying Eq. (13) between proton states in the physical region, we find ¹⁴⁾

$$A_n^i \mu_n^i(Q^2) = \frac{2}{Q^2} \int_{Q^2/2}^{\infty} d\nu W(\nu, Q^2) \left[\frac{Q^2}{\nu + \sqrt{\nu^2 + Q^2}} \right]^{n+1} \quad (15)$$

where A_n^i is the reduced matrix element of Θ_1^n between proton states.

Then, by resumming Eq. (15), we may determine the structure function dependence on quark and proton masses. We find a form for $W(Q^2, \nu)$ identical with Eq. (12) for $\nu > \frac{1}{2}$, except that there is no restriction $\xi \geq m_1$. Thus the PM and LC results are obtained as special cases of the OPE formula, with a spectral restriction which corresponds to a constraint on the matrix elements. A_n^i . This happens because the PM and LC approaches are applied to the absorptive part of the amplitude and so are enmeshed in parton spectral conditions, whereas the OPE analysis (motivated by the renormalization group) is applied only to the real part of the amplitude in the deep Euclidean region.

The corresponding results for the physical cases of vector currents and fermion quark-partons in electro- and neutrino-production :

$$J_\mu(x) = : \bar{\psi}_f(x) (a \gamma_\mu + b \gamma_\mu \gamma_5) \psi_i(x) :$$

are ⁹⁾ :

$$W_1 = (a^2 + b^2) \frac{Q^2}{2\sqrt{\nu^2 + Q^2}} \left[\left(1 + \frac{m_f^2 + m_i^2}{Q^2}\right) \mathcal{F}_i(\xi) + \mathcal{G}_i \right] - (a^2 - b^2) \frac{m_i m_f}{\sqrt{\nu^2 + Q^2}} \mathcal{F}_2(\xi) \quad (16.a)$$

$$W_2 = (a^2 + b^2) \frac{Q^4}{2(\nu^2 + Q^2)^{3/2}} \left[\left(\frac{R}{Q^2}\right)^2 \mathcal{F}_i(\xi) + 3 \mathcal{G}_i \right] \quad (16.b)$$

$$W_3 = 2ab \frac{Q^2}{(\nu^2 + Q^2)} \left[\left(\frac{R}{Q^2}\right) \mathcal{F}_i(\xi) + \frac{1}{\sqrt{\nu^2 + Q^2}} \int_{\xi}^1 d\xi' \left(1 - \frac{m_i^2}{\xi'^2}\right) \mathcal{F}_2(\xi') \right] \quad (16.c)$$

with

$$\mathcal{G}_i = \frac{R}{Q^2 \sqrt{\nu^2 + Q^2}} \int_{\xi}^1 d\xi' \left(1 - \frac{m_i^2}{\xi'^2}\right) \mathcal{F}_i(\xi') + \frac{1}{\nu^2 + Q^2} \int_{\xi}^1 d\xi' \left(1 - \frac{m_i^2}{\xi'^2}\right) \int_{\xi'}^1 d\xi'' \left(1 - \frac{m_i^2}{\xi''^2}\right) \mathcal{F}_i(\xi'') \quad (16.d)$$

and

$$R = \left[(Q^2 + m_f^2 - m_i^2)^2 + 4 m_i^2 Q^2 \right]^{1/2}$$

The same spectral conditions apply as in the spin zero case : $m_i \leq \xi \leq 1$ in the LC and PM approaches, no such restriction applies in the OPE approach.

The variable ξ is not novel. The form

$$\xi = \frac{Q^2}{\nu + \sqrt{\nu^2 + Q^2}}$$

relevant to light quarks $m_i \approx m_f \approx 0$ has been proposed by Nachtmann ¹⁴⁾ and occurs in the work of Broadhurst ¹⁵⁾. The form $\xi = (Q^2 + m_f^2)/2\nu$ relevant to $Q^2, m_f^2 \gg m_p^2$ has been proposed and used recently ¹⁶⁾. The variable (1) has been derived from the parton model by Frampton ¹⁷⁾, though he did not establish the equivalence with light-cone and OPE results or give the forms (16) for the structure functions. However, the variable of Georgi and Politzer ^{5), 6), *} does not include target mass effects correctly.

When are the results (1), (16) likely to be useful approximations? On general grounds we would expect them to break down at small ξ , a region dominated by Regge effects and hence intrinsically involving the way quarks form crossed channel bound states, so that approximate freedom is unlikely to be a good assumption. Indeed, Broadhurst ¹⁵⁾ has exhibited paradoxes arising in electroproduction from free-field assumptions and a Regge intercept $\alpha(0) > -1$: e.g., $\sigma_L/\sigma_T < 0$ for ξ near 0, Q^2 small.

Problems also arise as $\xi \rightarrow 1$. We have checked that the Adler sum rule is obeyed when W_2 is given by (16.a), but at the price of choosing a threshold which is fixed in ξ . This yields an unphysical threshold in $W^2 = (q+p)^2$: for example $\xi = 1$ yields $W^2 = 0$ for all Q^2 if $m_i = m_f = 0$, while for general m_i, m_f the W^2 threshold varies with Q^2 . Since the experimental structure functions vanish rapidly as $x \equiv Q^2/2\nu \rightarrow 1$, the numerical effects on the Adler sum rule at finite Q^2 of choosing a physical W^2 threshold are small, but the formulae (16) are presumably completely inapplicable for ξ near 1. On general grounds we would expect the direct channel bound state aspects of hadrons to be relevant in this limit. This is true in a quark-vector gluon model, where we find that gauge curl terms ¹⁸⁾ contribute $\propto n\bar{g}$ to $u_n(Q^2)$ for large n . Thus neglecting them is a non-uniform approximation, which standard inversion ¹⁹⁾ arguments suggest will break down as $\xi \rightarrow 1$. This problem worsens as one probes higher powers of (m^2/Q^2) ^{**)}.

It would also appear from the PM and LC spectral condition $\xi \geq m_i$ that using the results (16) is dicey when $\xi \lesssim m_i$, and hence for all ξ when $m_i \geq m_p$. If m_i or m_f is $\ll m_p$, then the effects of neglected interactions

*) Georgi and Politzer seem to have made an analytical error. It is not true as they assert in Section V of Ref. 6) that including target mass effects simply involves replacing Q^2 in the results of their Section IV by $Q^2 + m_f^2$: see for example their equation (4.9).

**) Similar conclusions have recently been reached by other authors ²⁰⁾.

may well be bigger than the m_i^2/Q^2 effects included in (16). For example, in many formulations of the parton model ^{10),11)} the initial quark is off mass-shell by an amount $O(\langle p_T \rangle) \approx 300$ MeV. Thus in the PM and LC pictures the $O(m_i^2/Q^2)$ effects included in (1) and (16) should be taken with a pinch of salt, and the formulae may well be inapplicable when $\xi \lesssim O(\langle p_T \rangle) \sim 1/3$, another manifestation of trouble in the Regge region.

What about $O(m^2/Q^2)$ effects in the OPE picture in the context of asymptotic freedom ^{5),6)} ? As remarked earlier, renormalization group arguments cannot be applied directly to the large Q^2, ν behaviour, but are instead applied to the OPE (13), where the operators are normalized between quark states states of momentum $p^2 = -\mu^2$.

Georgi and Politzer ^{5),6)} argue that when $\bar{g}$ is small one may compute $C_n(x^2, 0, m)$ [cf., Eq. (13)] to all orders in the quark masses m via the free quark model Compton amplitude, as in the first part of this paper. Unfortunately, we note that to zeroth order in $\bar{g}$, the result will only be gauge invariant if the initial quark is on its mass-shell, i.e., $-\mu^2 = m_i^2$. With this choice one can, unfortunately, no longer justify the renormalization group equations for there may appear additional terms corresponding to the mass insertion terms of the Callan-Symanzik equations ¹³⁾.

If one persists with the original normalization scheme at $p^2 = -\mu^2$ then we must, from gauge invariance arguments, drop all terms of order μ^2/Q^2 , or m_i^2/Q^2 . However, for neutrino production, with $m_f^2 \gg m_i^2$, μ^2 the renormalization group techniques work and give answers which may be believed to order (m_f^2/Q^2) . Similarly, order (m_p^2/Q^2) effects may be believed if $m_p^2 \gg m_f^2, \mu^2$.

Since μ^2 is a normalization point, one may ask whether there is any meaning to our result (16), even in these cases. What is relevant here is the expansion in Eq. (1). For this expansion to make sense it is not sufficient for C_n to be expanded in a (small) parameter $\bar{g}$ but also that O^n should be reasonably normalized so as not to spoil the validity of the $\bar{g}$ expansion. For this reason it may make sense to choose μ^2 of order of the mean momentum of a parton in the proton, say $(300 \text{ MeV})^2$.

The conclusion of this critique is that general arguments suggest that the results (1), (16) are inapplicable for $\xi \rightarrow 0$, $\xi \rightarrow 1$, and that $O(m_i^2/Q^2)$ terms are unreliable when $m_i \lesssim 300$ MeV. Renormalization group arguments further suggest that corrections are likely to be $O(\mu^2/Q^2)$ with μ a normalization mass where $\bar{g}$ is already small. Only if μ^2 can be chosen $\ll m_p^2$ will $O(m_p^2/Q^2)$ terms in (16) be reliable. Otherwise only the $O(m_f^2/Q^2)$ terms when $m_f^2 \gg m_p^2, \mu^2$ may be believable.

More details of our analysis of the J_{μ} fermion case (16), together with an analysis of $e^+e^- \rightarrow \text{hadron} + X$ and more on the renormalization group, will be given elsewhere ⁹⁾.

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NOTE ADDED

Subsequently to the writing of this paper, as erratum by Georgi and Politzer has appeared [Phys.Rev.Letters 37, 68 (1976)]. Their corrected variable agrees with our Eq. (1).

R E F E R E N C E S

- 1) J.D. Bjorken, Phys.Rev. 179, 1547 (1969).
- 2) R.P. Feynman, "Photon-Hadron Interactions", (Benjamin, N.Y., 1972);
J.D. Bjorken and E.A. Paschos, Phys.Rev. 158, 1975 (1969).
- 3) H.D. Politzer, Phys.Reports 14C, 129 (1974).
- 4) S.L. Glashow, J. Iliopoulos and L. Maiani, Phys.Rev. D2, 1285 (1970);
M.K. Gaillard, B.W. Lee and J.L. Rosner, Revs.Modern Phys. 47, 277 (1975);
A. Benvenuti et al., Phys.Rev.Letters 36, 1478 (1976).
- 5) H. Georgi and H.D. Politzer, Phys.Rev.Letters 36, 1281 (1976).
- 6) H. Georgi and H.D. Politzer, Harvard preprint, "Freedom at Moderate
Energies : Masses in Colour Dynamics" (1976).
- 7) K.G. Wilson, Phys.Rev. 179, 1499 (1969).
- 8) Y. Frishman, Phys.Rev.Letters 25, 966 (1970);
R.A. Brandt and G. Preparata, Nuclear Phys. B27, 541 (1971).
- 9) R. Barbieri, J. Ellis, M.K. Gaillard and G.G. Ross, CERN preprint in
perspiration (1976).
- 10) P.V. Landshoff, J.C. Polkinghorne and R.D. Short, Nuclear Phys. B20,
225 (1971).
- 11) S.D. Drell, D.J. Levy and T.-M. Yan, Phys.Rev. D1, 1035 (1970).
- 12) R.L. Jaffe, Phys.Rev. D5, 2622 (1972);
J.C. Polkinghorne, Nuovo Cimento 8A, 572 (1972).
- 13) N. Christ, B. Hasslacher and A.H. Mueller, Phys.Rev. D6, 3543 (1972).
- 14) O. Nachtmann, Nuclear Phys. B63, 237 (1973) and B78, 455 (1974).
- 15) D.J. Broadhurst, Open University, England, preprint OUT-4102-1,
"Static Properties and Structure Functions of a Bag of Finite-
Mass Quarks" (1975).
- 16) R.M. Barnett, Phys.Rev.Letters 36, 1163 (1976).
- 17) P.H. Frampton, UCLA preprint UCLA/76/TEP/6 (1976).
- 18) D.J. Broadhurst, J.F. Gunion and R.L. Jaffe, Phys.Rev. D8, 566 (1973);
R.L. Jaffe and C.H. Llewellyn Smith, Phys.Rev. D8, 2506 (1973).
- 19) G. Parisi, Phys.Letters 50B, 367 (1974);
D.J. Gross, Phys.Rev.Letters 32, 1071 (1974).
- 20) R.K. Ellis, R. Petronzio and G. Parisi, Roma preprint, "Mass Dependent
Corrections to the Bjorken Scaling Law" (1976);
R. Barbieri, R. Gatto, P. Menotti and I. Vondramin, private communication.