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PIONIC ATOMS AND THE NEUTRON DISTRIBUTION IN NUCLEI

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ABSTRACT

We calculate the widths and shifts of the atomic levels due to the strong pion-nucleus interaction. A formula with two phenomenological parameters is presented. These are determined from pionic data on ^{28}Si , ^{32}S , and ^{40}Ca . The data on the 2p levels are used to predict the r.m.s. radii of the neutron distributions in some nuclei.

Geneva - 18 February 1976

(Submitted to Physics Letters)

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At low energies the pion-nucleon interaction is well described by two partial waves. One of the features of pionic atoms is that in the p-wave the pion interacts predominantly with the neutron component of the nuclear matter. This is partly because the isospin $3/2$ (πN) amplitude dominates, and partly because the spin-averaged amplitudes come into play. The corresponding averaged scattering volumes are¹⁾

$$\begin{aligned} a_{\pi^- n}^{\text{p-wave}} &= 0.128 \text{ (m}_{\pi}^{-3}) , \\ a_{\pi^- p}^{\text{p-wave}} &= 0.012 \text{ (m}_{\pi}^{-3}) . \end{aligned} \quad (1)$$

This gives a chance to extract the r.m.s. radius of the neutron distribution from the data on strong interaction corrections to the energy levels in pionic atoms.

Measurements have been performed over the last few years, and a high level of precision was reached, especially for the $1s$ level^{2,3)}. The physics of the pionic atom has been explained by the optical potential approach^{4,5)}. This approach was also successful on a quantitative level, considering the main bulk of data, but at the expense of introducing two complex phenomenological parameters describing the effect of the virtual absorption of the pion on a correlated pair of nucleons.

In Ref. 6 we have obtained a formula which relates the strong energy shifts and broadenings of the atomic levels directly to the (πN) scattering matrix below threshold. The absorption effect is described by only one complex phenomenological parameter $q = q_r - iq_i$. This reduction of the number of free parameters allows some conclusions to be drawn from the data.

We describe the formula very briefly. Let us consider a pion interacting with a nucleon being in a single-particle level α in a nucleus. The relevant free scattering matrix in the centre of mass of the pair is

$$t^{\Gamma}(E_n + \epsilon_{\alpha} - p^2/2M) , \quad (2)$$

where $\Gamma = T, J, L$ stands for the πN quantum numbers. The energy E_n ,

$$E_n = \epsilon_n + \Delta\epsilon_n - i \Gamma_n/2 , \quad (3)$$

is the energy of the atomic level ϵ_n shifted by the strong interaction ($\Gamma_n, \Delta\epsilon_n \ll \epsilon_n$). The energy of the c.m. motion is subtracted in the argument of t . All the contributions there are negative, and the atomic data ($E_n \approx 0$) are determined by an average of the scattering matrix below threshold:

$$\tau^\Gamma = \int d\vec{p} \sum_{\alpha} t^\Gamma(\epsilon_\alpha - p^2/2M) W_\Gamma^2(\alpha, p), \quad (4)$$

where W^2 is the distribution of the c.m. momentum of the atomic pion and a nucleon on a single-particle level ϵ_α . In the low-energy approximation, and in the limit of zero range forces, we have

$$t^{\text{TJL}}(E) = a_{\text{TJL}} / [1 - i k(E)^{2L+1} a_{\text{TJL}}], \quad (5)$$

and the effect of the subthreshold extrapolation is very small as the scattering lengths and volumes are very small. Thus $\tau \approx a$, and the only important exception occurs in the s-wave state owing to the cancellation of the proton [$a_{\pi p}^{\text{s-wave}} = 0.082 (m_\pi^{-1})$] and the neutron [$a_{\pi n}^{\text{s-wave}} = -0.092 (m_\pi^{-1})$] scattering length⁷⁾. The corresponding $\tau^0 = \tau_{\pi p}^0 + \tau_{\pi n}^0$ is different from the threshold value by a factor of ~ 3 due to the average

$$\langle \epsilon_\alpha - p^2/2M \rangle \approx -25 \text{ MeV}.$$

It should be noticed here, that this extrapolation of t to below threshold energies holds for the free (πN) system, and corrections due to renormalization of the π and N propagators in nuclear medium (binding corrections) are included later (Eq. 9).

The single nucleon scattering leads to the impulse approximation formula

$$\Delta\epsilon - i\Gamma/2 = -(\tau^s \omega_s + \tau^p \omega_p), \quad (6)$$

where the ω 's are the overlap integrals for s- and p-wave interaction:

$$\omega_s[\rho] = \frac{2\pi}{\mu_{\pi N}} \int d\vec{r} \rho(r) \psi^2(r) \quad (7)$$

$$\begin{aligned} \omega_p[\rho] = \frac{2\pi}{\mu_{\pi N}} & \left(3 \left(\frac{m_\pi}{m_\pi + m_N} \right)^2 \int d\vec{r} |\psi(\vec{r})|^2 \sum_{\alpha} \vec{\partial} \phi_{\alpha}^*(\vec{r}) \vec{\partial} \phi_{\alpha}(\vec{r}) \right. \\ & + 3 \left[1 - A^{-1} \left(\frac{m_\pi}{m_\pi + m_N} \right) \right]^2 \int dr r^2 \rho(r) \left\{ \frac{\ell + 1}{2\ell + 1} [\psi'(r) - \frac{\ell}{r} \psi(r)]^2 \right. \\ & \left. \left. + \frac{\ell}{2\ell + 1} [\psi'(r) + \frac{\ell + 1}{r} \psi(r)]^2 \right\} \right). \end{aligned} \quad (8)$$

The summation over the proton and neutron contribution is understood in Eq. (6); $\rho(r)$ is the nuclear density distribution, and $\psi(r)$ and $\phi(r)$ are the pion and nucleon wave functions, respectively.

The impulse approximation produces $\sim 4/5$ of the shift in the $\ell \geq 1$ atomic states, but fails in the $1s$ state. The necessary corrections are due to the effects of multiple scattering on a pair and the virtual absorption processes.

The first effect is given by two terms which we specify for the s-wave interaction:

$$D^{\text{dir}} + D^{\text{exch}} = \frac{2\pi}{\mu_{\pi N}} \sum_{\mu, \nu} \int d\vec{r} d\vec{r}' \tau_{\mu} \rho_{\mu}(r) \tau_{\nu} \rho_{\nu}(r) g_{\pi}(\vec{r} - \vec{r}') \times \\ \times [1 - \delta_{\mu\nu} D^F(r - r')] f_{\text{HC}}(r - r'), \quad (9)$$

where g_{π} is the pion Green's function, the f_{HC} and D^F are the hard core and Fermi gas correlation functions, respectively. The second term in Eq. (9) comes from antisymmetrization and produces the dominant effect. This happens after summation over the isospin indices ν, μ is performed. The first term, which is approximately (in the case of $N = Z$) proportional to τ^{0^2} , and the exchange term due to $\pi^-(p, p)$, $\pi^-(n, n)$, $\pi^- \rightarrow \pi^0 \rightarrow \pi^-(n, p)$ processes, contribute $\{\tau_p^{0^2} + \tau_n^{0^2} + (\tau_p^0 - \tau_n^0)^2/2\}$, which is by far dominant.

The similar formula for the p-wave scattering produces $\Delta g_{\pi} \sim \delta(r - r')$ and yields no effect due to the hard core correlations functions.

All the effects related to Eq. (9) have been accounted for in the optical potential approach⁴⁾.

The virtual absorption on a correlated pair is assumed to be of a very short range, state-independent, and due to an initial Yukawa-like p-wave capture on a proton. The approximate formulae are given by Eq. (8) with an additional ρ^{eff} under the integrals

$$q\sigma = (q_r - iq_i) \omega_p [\rho^{\text{proton}} \times \rho^{\text{eff}}], \quad (10)$$

where

$$\rho^{\text{eff}} = \frac{3}{4} \rho^{\text{neutron}} + \frac{1}{4} \rho^{\text{proton}},$$

according to the number of the spin states of the closely correlated p, n and p, p pairs.

The above-defined quantities are related to the level shift by a formula, obtained in Ref. 6:

$$\Delta\epsilon - i\Gamma/2 = \frac{-(\tau_{\omega_s}^s + \tau_{\omega_p}^p)}{1 + \frac{(D^{\text{dir}} + D^{\text{exch}}) + q\sigma}{\tau_{\omega_s}^s + \tau_{\omega_p}^p}} \quad (11)$$

This expression contains two phenomenological parameters q_r, q_i and the πN scattering lengths and volumes. As mentioned before, the cancellation in $\tau^0 = \tau_p^0 + \tau_n^0$ renders this quantity poorly known. The scattering lengths from Ref. 7 yield $\tau^0 \approx -0.03 \pm 0.005 (m_\pi^{-1})$.

Equation (11) was used first to fit the two parameters q_r and q_i to the pionic data. The pionic 2p- and 3d-level data of the nuclei ^{28}Si , ^{32}S , and ^{40}Ca were used since their charge distributions are well known^{8,9}). The assumption was made that their neutron distributions are identical to the proton distributions. As the s-wave pion-nucleus interaction is very sensitive to badly known corrections, involving detailed knowledge of the nucleon states (see discussion below), we consider the pionic 1s-level data as being of little interest for quantitative information on q_r and q_i .

The 2p level is rather insensitive to τ^0 . So τ^0 was kept within the range mentioned above; we chose $\tau^0 = -0.038 (m_\pi^{-1})$ to make the fit better. The result of the fit for the 2N parameters is

$$q_r = -0.115 (m_\pi^{-6}) ,$$

$$q_i = 0.39 (m_\pi^{-6}) .$$

The comparison of the calculation with experiment is shown in Table 1 for the above elements. The result for q_i compares well with the optical potential constant $\text{Im } C_0 \sim 1/4 q_i$ ⁴⁾. The average τ 's for the p-wave interaction are obtained with Eq. (1) and differ slightly from the scattering lengths, being however, within the errors of these.

The 1s-level data and the comparison with our calculations are shown in Table 2. The dominant part of the numerator of Eqs. (11) or (6) comes from the s-wave interaction. Owing to the subtraction in τ^0 the p-wave contributes a negative correction of about 30%. The denominator of Eq. (11) is dominated by $D^{\text{exch}}/\tau\omega \sim -0.6$ in the case of $N = Z$. Thus there is a strong cancellation of this term and the unity. The effect is seen as an enhancement of the pion-nucleus scattering length averaged per proton $(\Delta\epsilon - i \Gamma/2)/\omega_s^{\text{proton}}$, in comparison with the average pion-nucleon scattering length $\tau^0/2$. The first quantity is given in col. 6 of Table 2. It is relatively independent of the nucleus, provided we stay within families $N = Z, Z + 1$. The small damping with increased atomic number Z is due to the increasing effect of the direct multiple scattering which produces a positive contribution to the denominator of Eq. (11). Because of the cancellation in the denominator, the second-order effects of Eq. (9) such as different neutron and proton densities and short-range correlations, become important. The

hard core defect volume $I = 1 \text{ fm}^3$ was used in the calculations. No information can be obtained from the $1s$ -level data, and we treat it as a check for the theory. The comparison with the data is shown in cols. 2-5 of Table 2. The theory works satisfactorily for the elastic interaction, and agreement within 5% is obtained. The absorption is less well reproduced, the agreement with the data not being better than 10% for nuclei heavier than B. For lighter nuclei the theory does not reproduce the experimental absorption. This failure for low- Z elements is apparently due to non-realistic nuclear wave function derivatives, which we generate by the local Fermi gas model, and which we use in Eqs. (8) and (10). A more refined nuclear model is needed at this point.

The $2p$ level is more promising. One reason is that the derivatives of the mesonic wave function in Eqs. (8) and (10) dominate over the derivatives of the nucleonic wave functions, which are not known very well. The other reason is the sensitivity to the neutron distribution. This may best be seen by the shift $\Delta\epsilon_{2p}$. As $\psi_{2p} \sim r$ and $\psi'_{2p} \sim 1$ the contribution from the s -wave interaction comes mostly from the surface region, and the p -wave interaction is more of a volume effect. Thus, if one changes the neutron distribution to a more extended one, the cancellation of the s - and p -wave contributions to $\tau\omega$ is stronger. This is clearly seen in the case of the isotope effects in Ca, as the apparently stronger interaction in ^{44}Ca (more neutrons) results in a smaller shift than in ^{40}Ca (cf. Tables 1 and 3). We use this effect to interpret the data in terms of the extended neutron distribution. The shift is, to a good approximation, related directly to the root mean square radius of the neutron distribution R_n . The results depend on the range of the pion-nucleon forces. We assumed it to be roughly equal to the proton electromagnetic radius, and it is the charge density which we use in the calculations. The effect of the extended nucleon (or the range of force) comes through the relation of the r.m.s. radii:

$$R_{\text{charge}}^2 = R_{\text{bare}}^2 + r_{\text{em}}^2 (\text{force range}) \quad (12)$$

and is very small for the $2p$ level. The uncertainty due to this relation amounts to about 1% of R for the p levels. However, for the higher levels, $3d$ and $4f$, the higher moments of the distribution are involved and the uncertainty rises to about 4% and 8%, respectively.

In Table 3 we present the results of our analysis. Using the parameters obtained above, we fitted the difference of the r.m.s. radii of the proton and neutron distributions, $R_n - R_p$ for the nuclei ^{44}Ca , ^{58}Ni , ^{60}Ni , and ^{208}Pb . The charge distributions were taken from the literature. Columns 3 and 4 show the calculated and experimental shifts $\Delta\epsilon$ and widths Γ . Column 5 shows $R_n - R_p$, col. 7

gives the reference for the charge distributions used in the fits, and col. 6 shows their r.m.s. radii R_p . For the neutron distribution we took the parameters of the proton distribution and changed the half-density radius only.

The error bars of $R_n - R_p$ correspond to the errors of the experimental pionic data only. A change of 1% in the charge radius R_p results in a change of $R_n - R_p$ of about 2%. The influence of the surface parameter t was also checked (in the case of ^{44}Ca): keeping R_p fixed and changing t by 10% would result in a change of $R_n - R_p$ of 2% only. Thus the method seems not to be very sensitive to the exact shape (within reasonable limits) of the distributions of neutrons or protons. Therefore we are rather confident that, within the framework of the present theory, the results of the method are reliable. However, the bad knowledge of the p-wave πN scattering lengths and the fitted q_r render the results less significant. An assumed uncertainty for $a_p^{\text{p-wave}}$ and $a_n^{\text{p-wave}}$ of 0.01 fm, which would cover almost all recent phase-shift results, and the quoted uncertainty of q_r would reflect in an additional error of $R_n - R_p$ of ± 0.29 fm for the nuclei of Table 3. The experimental errors of the 2p, 3d, and 4f level data do not allow to restrict or to test the values of $a^{\text{p-wave}}$ and q_r accurately enough. New, more precise measurements would be very helpful.

The results for $R_n - R_p$ agree reasonably with the measurements of Ref. 10 and with the calculations of Refs. 11, which were obtained by optical model calculations for proton elastic scattering. A qualitatively similar effect is also seen in the K^- -meson absorption in nuclei¹²⁾.

There exists the problem of renormalization of the πN scattering volumes in nuclear matter due to the Pauli exclusion principle. On the phenomenological level we see no substantial renormalization which would be significant, at the present level of precision, to $\Delta\epsilon$. For theoretical arguments related to this we refer to Refs. 13 and 14. The statement concerns $a_{\pi p}^{\text{p-wave}} + a_{\pi n}^{\text{p-wave}}$ relevant to $N = Z$ nuclei and $a_{\pi n}^{\text{p-wave}}$ relevant in the case of 4f levels in atoms with large neutron excess. In the last case the level shift, as may be seen from Table 3, is not sensitive to the neutron radius but depends essentially on the actual value of $a_{\pi n}^{\text{p-wave}}$. On the other hand, the R_n as deduced from the 2p-level shift come out as an effect of the cancellation of the s- and p-wave πN amplitudes. Let us also stress that the shifts depend very weakly on the assumptions related to the level widths.

We would like to express our gratitude to Prof. M. Ericson and Prof. T.E.O. Ericson for their help and criticism.

REFERENCES

- 1) H. Pilkuhn, W. Schmidt, A.D. Martin, C. Michael, F. Sterner, B.R. Martin, M.M. Nagels and J.J. DeSwart, Nuclear Phys. B65, 460 (1973).
- 2) G. Backenstoss, Annu. Rev. Nuclear Sci. 20, 467 (1970).
- 3) L. Tauscher, Hadronic atoms, Proc. Sixth Internat. Conf. on High-Energy Physics and Nuclear Structure, Santa Fe, 1975 (AIP, Washington, 1975), p.541.
J. Hüfner, L. Tauscher and C. Wilkin, Nuclear Phys. A231, 455 (1974).
- 4) M. Ericson and T.E.O. Ericson, Ann. Phys. 36, 323 (1966).
- 5) L. Kisslinger, Phys. Rev. 98, 761 (1955).
- 6) L. Tauscher and S. Wycech, On the pionic atom, to be published.
- 7) D.V. Bugg, A.A. Carter and J.R. Carter, Phys. Letters 44B, 278 (1973).
- 8) G.L. Li, M.R. Yearian and I. Sick, Phys. Rev. C 9, 1861 (1974).
I. Sick, private communication.
- 9) B.B.P. Sinha, G.A. Peterson, R.R. Whitney, I. Sick and J.S. McCarthy, Phys. Rev. C 7, 1930 (1973).
- 10) H. Foeth, M. Holder, E. Radermacher, A. Staude, P. Darriulat, J. Deutsch, K. Kleinknecht, C. Rubbia, K. Tittel, M.I. Ferrero and C. Grosso, Phys. Letters 31B, 544 (1970).
- 11) G.W. Greenless, G.J. Pyle and Y.C. Tang, Phys. Rev. 171, 1115 (1968).
G.W. Greenless, W. Makoske and G.J. Pyle, Phys. Rev. C 1, 1145 (1970).
J.W. Negele and D. Vantherin, Phys. Rev. C 5, 1472 (1972).
J.L. Friar and J.W. Negele, Nuclear Phys. A212, 93 (1973).
Review: D.F. Jackson, Rep. Progr. Phys. (GB) 37, 55 (1974).
- 12) E.M.S. Burhop, Nuclear Phys. B1, 434 (1967).
- 13) C.B. Dover, D. Ernst and R.M. Thaler, Phys. Rev. Letters 32, 557 (1974).
- 14) S. Barshay, G.E. Brown and M. Rho, Phys. Rev. Letters 32, 787 (1974).

Table 1

Shifts $\Delta\epsilon$ and width Γ as obtained with the fitted parameters $q_r = -0.115 \pm 0.024 \mu_\pi^{-6}$ and $q_i = 0.39 \pm 0.06 \mu_\pi^{-6}$. The experimental values are given in brackets. The average τ 's were kept fixed at the values $\tau^0 = -0.038 \mu_\pi^{-1}$, $\tau_{\pi^-n}^p = 0.132 \mu_\pi^{-3}$, $\tau_{\pi^-p}^p = 0.013 \mu_\pi^{-3}$.

Element	$-\Delta\epsilon_{2p}$ (keV)	Γ_{2p} (keV)	Γ_{3d} (eV)
^{28}Si	0.263 (0.288 $\pm$ 0.150)	0.288 (0.180 $\pm$ 0.080)	0.07 0.13 $\pm$ 0.08
^{32}S	0.566 (0.547 $\pm$ 0.090)	0.679 (0.790 $\pm$ 0.140)	0.21 0.23 $\pm$ 0.15
^{40}Ca	1.821 (1.876 $\pm$ 0.150)	2.199 (2.230 $\pm$ 0.120)	1.44 (0.70 $\pm$ 0.30)

Table 2

The 1s-level shifts and widths. Column 6 gives the experimental pion-nucleus scattering length divided by the number of protons. The calculated values are obtained with the fitted parameters. (See Table 1)

(1) Element	(2) $\Delta\epsilon^{\text{exp}}$ [keV]	(3) $\Delta\epsilon^{\text{calc}}$ [keV]	(4) Γ^{exp} [keV]	(5) Γ^{calc} [keV]	(6) $\frac{\Delta\epsilon - i\Gamma/2}{\omega_{\text{proton}}^s}$ [μ_π^{-1}]
^4He	0.0757(20)	0.069	0.045(3)	0.055	4.42 - i 1.31
^6Li	0.324(3)	0.391	0.195(12)	0.048	3.84 - i 1.16
^7Li	0.570(4)	0.592	0.195(13)	0.048	6.75 - i 1.13
^9Be	1.595(9)	1.609	0.591(14)	0.171	6.02 - i 1.12
^{10}B	2.977(85)	2.81	1.59(11)	0.97	4.76 - i 1.26
^{11}B	3.839(85)	3.48	1.79(12)	0.72	6.06 - i 1.41
^{12}C	5.870(92)	5.43	3.14(12)	2.61	4.61 - i 1.23
^{14}N	9.92(14)	9.23	4.34(24)	5.62	4.28 - i 0.95
^{16}O	15.03(24)	14.96	7.64(49)	7.55	4.15 - i 1.05
^{19}F	24.46(35)	25.16	9.4(1.5)	6.3	4.43 - i 0.85
^{20}Ne	33.34(50)	32.10	14.5(3.0)	10.8	4.08 - i 0.89
^{23}Na	49.9(7)	47.7	10.3(4.0)	16.8	4.50 - i 0.46

Table 3

Differences of neutron and proton distribution r.m.s. radii $R_n - R_p$ (col. 5) as obtained from a fit to the experimental pionic data on the 2p-level shift $\Delta\epsilon$ (col. 3) and width (col. 4). The calculated values are listed in parentheses. The r.m.s. radius of the charge distribution R_p is listed in col. 6.

Element	Level	$\Delta\epsilon$ (keV)	Γ (keV)	$R_n - R_p$ (fm)	R_p (fm)	Ref.
^{44}Ca	2p	1.19 ± 0.10 (1.27)	2.07 ± 0.15 (1.88)	0.49 ± 0.07	3.519(33)	a
^{58}Ni	2p	5.75 ± 0.40 (5.45)	8.5 ± 1.3 (10.7)	0.62 ± 0.06	3.725(15)	b
^{60}Ni	2p	5.35 ± 0.51 (5.10)	8.5 ± 1.5 (10.5)	0.63 ± 0.06	3.755(23)	b
^{208}Pb	4f	1.82 ± 0.25 (1.82)	1.1 ± 0.3 (1.39)	$+0.0 \pm 0.5$	5.5097(6)	c

- a) Ref. 9 and K.J. van Oostrum, R. Hofstadter, G.K. Nöldeke, M.R. Yearian, B.C. Clark, R. Herman and D.G. Ravenhall, Phys. Rev. Letters 16, 528 (1966).
- b) J.R. Ficenec, W.P. Trower, J. Heisenberg and I. Sick, Phys. Letters 32B, 460 (1970).
- c) D. Kessler, H. Mes, A.C. Thompson, H.L. Anderson, M.S. Dixit, C.K. Hargrove and R.J. McKee, Phys. Rev. C 11, 1719 (1975).