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ZWEIG RULE VIOLATING DECAYS OF THE NEW PARTICLES

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ABSTRACT

The specific decay modes of the new ψ particles and their cascade products are considered according to an approximate scheme of sequential pole dominance proposed by Freund and Nambu, which is dictated by the dual model dynamics. Predictions and comparison with available data are presented. While asymptotically free gauge theory can trace the origin of violation of Zweig rule and the smallness of ψ width, it can only give an estimate of the over-all violation of the rule and is not well suited for each decay channel separately. The sequential pole model fulfils this task.

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The understanding of the origin of the Zweig-Iizuka ¹⁾ rule is of great theoretical interest. In the asymptotically free gauge theory ²⁾ this rule is accounted for by considering the Zweig rule violating decays as going through colour gluons. When the gluons are energetic, their couplings to quarks of φ and ψ are small, and one can treat the problem perturbatively by considering only the lowest three (or two) colour gluons as the intermediate states.

Recently Freund and Nambu ³⁾ have proposed a model for the decay processes violating Zweig rule which emerges naturally from the dual dynamics. In the same way as the Pomeron term P is responsible for the disconnected diagram for $K\pi$ scattering (Fig. 1a), the decay of, say, $\varphi \rightarrow \rho\pi$ proceeds via an intermediate state $J^P = 1^-$ called O . This O state in the dual dynamics is the first daughter of the Pomeron trajectory and is assumed to be an SU_4 singlet which mixes the φ and the ω (Fig. 1b). Since the slope of Pomeron is $\alpha'_P \simeq 1/3 - 1/2$, for the mass of this O state one expects $m_O^2 \simeq 2 - 3$. The transition $O \rightarrow V$ ($V = \omega, \varphi, \psi$) is given by the couplings $f_{O \rightarrow V} : f_{O \rightarrow \psi} = f_{O \rightarrow \varphi} = (1/\sqrt{2})f_{O \rightarrow \omega} \equiv f$. The implications of this model for the hadronic productions and decays of particles on tower $J^P = 1^-$ which mix through the first Pomeron daughter, have been considered ⁴⁾ with quantitative success.

In the field theoretic treatment of the dual dynamics, the O meson string is a quarkless energy concentration of the gluons. So there is a similarity between this model and the approach of the asymptotically free gauge theory. It may be that, in certain approximations, the two approaches are complementary in the sense that while the latter is successful in describing the over-all violation of the Zweig rule, the sequential pole model dominance is suitable when each specific decay channel is considered separately. From the theoretical side, it is amusing that from the Veneziano topological perturbation expansion for unitarization of the dual resonance model, the Freund-Nambu ansatz for the breaking of Zweig's rule indeed emerges ⁵⁾.

In this note we exploit the consequences of the Freund-Nambu model for the hadronic and the radiative cascade decays of the new mesons. These decays bring into consideration the tower of $J^P = 0^+$ mesons, and therefore the second daughter of Pomeron with positive charge conjugation which we call O_S (i.e., scalar O) ^{*}). It is necessary to study the present model in

^{*}) One may speculate that if the glue states resonate ⁶⁾ (gluonium) as is the case, e.g., in the MIT bag model ⁷⁾, the O and O_S states can be identified with these gluonic states.

detail, as it may bring to a better understanding of the origin of the Zweig rule, especially when there exist very few models with the field theoretical background which claim to explain the rule.

We adapt the charm scheme and the SU_4 as the underlying symmetry. In case there will be need for more than four quarks (say, six quarks), the underlying group can be enlarged. The results will remain unchanged, if SU_4 is the subgroup of the new symmetry group.

Consider the cascade decay

$$\psi'(3.684) \longrightarrow \psi(3.095) + \pi^+ \pi^- \quad (1)$$

Within the model, the process goes as the following sequences (Fig. 1c): first ψ' goes to ψ and ϵ_c (ϵ_c is the $J^P = 0^+$ state with $c\bar{c}$ quark content) for which connected quark diagram can be drawn. The scalar ϵ_c then mixes with the usual ϵ ($m_\epsilon = 700$ MeV, $\Gamma_\epsilon = 400-700$ MeV) through the intermediate state O_s , which is the state lying on the second daughter trajectory of Pomeron. With almost parallel daughter trajectories, the mass of O_s is almost like the O state, $m_{O_s}^2 \simeq m_O^2$. The O_s state is again an SU_4 singlet which gives ^{*)}

$$f_{O_s - \epsilon_c} = f_{O_s - \epsilon_\lambda} = \frac{1}{\sqrt{2}} f_{O_s - \epsilon} = f_\epsilon \quad (2)$$

for the $O_s - S$ transition amplitudes. Guided by the experimental mass distribution of $\pi\pi$ system and the theoretical argument of chiral symmetry ⁸⁾, we take derivative coupling for $\epsilon\pi\pi: \mathcal{L} = (g_{\epsilon\pi\pi}/m_\epsilon) \phi_\epsilon \partial_\mu \vec{\phi} \partial^\mu \vec{\phi}$. The decay width of (1) is given as

$$\Gamma(\psi' \rightarrow \psi \pi^+ \pi^-) = \int_{2m_\pi}^{m_{\psi'} - m_\psi} \frac{d\Gamma}{dM} dM, \quad (3)$$

$$\frac{d\Gamma}{dM} = 2M \frac{m_{\epsilon_c}}{\pi} \cdot \frac{\Gamma(\psi' \rightarrow \psi \epsilon_c)_M}{(M^2 - m_{\epsilon_c}^2)^2 + m_{\epsilon_c}^2 \Gamma_{\epsilon_c}^2} \cdot \frac{f_{O_s - \epsilon_c}^2 f_{O_s - \epsilon}^2}{(M^2 - m_{O_s}^2)^2 + m_{O_s}^2 \Gamma_{O_s}^2} \cdot \frac{\Gamma(\epsilon \rightarrow \pi^+ \pi^-)_M}{(M^2 - m_\epsilon^2)^2 + m_\epsilon^2 \Gamma_\epsilon^2}, \quad (4)$$

) As a first approximation, we assume ideally mixed 0^+ states. The ϵ_λ meson consists then of $\lambda\bar{\lambda}$ quarks. We are guided by the observation that the $S^(993)$ scalar meson, although just above the threshold, decays predominantly to $K\bar{K}$. We take this meson with $\Gamma = 50$ MeV as candidate for ϵ_λ . This is a particular choice for the quark assignment of the confused $0^+, I=0$ meson spectrum. Other choices would not change much our numerical evaluations.

$$\Gamma(\psi' \rightarrow 4\epsilon_c)_M = (m_{\psi'} g_{\psi' 4\epsilon_c})^2 \frac{1}{3} \left[2 + \left(\frac{m_{\psi'}^2 + m_{\psi'}^2 - M^2}{2m_{\psi'} m_{\psi'}} \right)^2 \right] \frac{1}{8\pi} \frac{m_{\psi'}}{m_{\psi'}} \left[\left(\frac{m_{\psi'}^2 + m_{\psi'}^2 - M^2}{2m_{\psi'} m_{\psi'}} \right)^2 - 1 \right]^{\frac{1}{2}} \quad (5)$$

from an effective coupling $\mathcal{L} = m_{\psi'} g_{\psi' \psi \epsilon_c} V_{\mu}^{\psi} V^{\mu} \phi_{\epsilon_c}$ and

$$\Gamma(\epsilon \rightarrow \pi^+ \pi^-)_M = \frac{2}{3} \frac{M^3}{64\pi} \left(\frac{g_{\epsilon \pi \pi}}{m_{\epsilon}} \right)^2 \left(1 - \frac{4m_{\pi}^2}{M^2} \right)^{\frac{1}{2}} \left(1 - \frac{2m_{\pi}^2}{M^2} \right)^2 \quad (6)$$

$d\Gamma/dM$ is the mass distribution of the $\pi\pi$ system with the invariant square mass $M^2 = (k+k')^2$. For definiteness, we use the values $m_{\epsilon} = 700$ MeV, $\Gamma_{\epsilon} = 400$ MeV and $m_{0_S}^2 = 2$ (GeV)². For the total width Γ_{ϵ_c} , we use the asymptotically free theory arguments $\Gamma_{\epsilon_c}/\Gamma_{\psi'} \sim 1/\alpha_s$ with $\alpha_s \simeq 0.2-0.3$ and take therefore $\Gamma_{\epsilon_c} = 1$ MeV. The mass m_{ϵ_c} we take as one of the experimental values ⁹⁾ or the theoretical calculations based on the charmonium picture ^{10), 11)} to be $m_{\epsilon_c} = 3.41$. Fixing the width $\Gamma(\psi' \rightarrow \psi \pi^+ \pi^-)$ to the experimental value of 72 keV, we obtain the mass distribution $d\Gamma/dM$ as plotted in Fig. 2. If for the coupling $g_{\psi' \psi \epsilon_c}$ we use the SU_4 relation $g_{\psi' \psi \epsilon_c} = \sqrt{2} g_{\rho' \rho \epsilon}$ with the value $g_{\rho' \rho \epsilon}^2/4\pi = 2$ ⁸⁾ [which gives $\Gamma(\rho' (1600) \rightarrow \rho \epsilon \rightarrow \rho \pi^+ \pi^-) = 135$ MeV], for the transition amplitude f_s we obtain a value of 0.7 (GeV)². Notice that this value is larger than the transition amplitude $f \simeq 0.16$ for the vector mesons, in accord with asymptotically free expectations. With this value of f_s the calculated width Γ_{0_S} comes out large and of the order of the mass of 0_S . Had we abandoned the SU_4 relation and instead taken a higher value for $g_{\psi' \psi \epsilon_c}$, we would have had smaller value for f_s and $\Gamma_{0_S} (\sim f_s^4)$.

The calculated widths are listed in the table. The input has been the experimental width $\Gamma(\psi' \rightarrow \psi \pi^+ \pi^-)$. The dominant diagrams for the processes listed in the table are typically as in Figs. 1d, 1f and 1g. The diagram 1e with assumed $(g_{\omega \omega \epsilon}^2/4\pi) = (g_{\rho \rho \epsilon}^2/4\pi) \simeq (g_{\rho' \rho \epsilon}^2/4\pi) = 2$ is suppressed compared with 1d. The chosen value 1 MeV for Γ_{ϵ_c} is crucial only for the decays $\psi' \rightarrow \gamma \pi^+ \pi^-$ and $\psi' \rightarrow \gamma K \bar{K}$ where M , the mass of $\pi\pi$ or $K\bar{K}$ system, passes the pole of ϵ_c (cf. Fig. 1f). The decay mass distributions of $\pi\pi$ systems in $\psi''(4.1) \rightarrow \psi \pi^+ \pi^-$ and $\psi \rightarrow \gamma \pi^+ \pi^-$ are plotted in Figs. 3a and 3b.

We see from the table that there are many ratios which do not depend on the relative importance of the diagonal ($g_{\psi \psi \epsilon_c}$) and off-diagonal ($g_{\psi' \psi \epsilon_c}$) coupling constants entering the quantities C_i ($i=4, 8, 13, 15$). Take, for instance, the ratio $[\Gamma(\psi \rightarrow \varphi \pi^+ \pi^-)/\Gamma(\psi \rightarrow \omega \pi^+ \pi^-)] = (0.15 C_4 \text{ keV}/0.25 C_4 \text{ keV}) = 0.6$,

to be compared with the experimental value ¹²⁾ of 0.2 ± 0.1 . The number 0.6 comes out (except for slight kinematic differences) because $(f_{0-\psi}/f_{0-\omega})^2 = \frac{1}{2}$. Otherwise, the both decays are the same order in Zweig violation. In the asymptotically free treatment the situation seems analogous, as the two processes would go according to Figs. 1h and 1i. So there is only one extra gluon compared with $\psi \rightarrow \omega\pi\pi$. However, since the integration in the case of $\psi\pi\pi$ may involve extra numerical factors due to possibility of exchanged gluon with small invariant mass, the experimental result may not be so surprising ^{*}).

Another ratio is $\Gamma(\psi \rightarrow \gamma\pi^+\pi^-)/\Gamma(\epsilon_c \rightarrow \gamma\psi) = 0.04$ which is independent of diagonal and off-diagonal couplings, but depends on the chosen $g_{\psi'\psi\epsilon_c}^2/4\pi = 4$ value. From the table we have $\Gamma(\psi \rightarrow \gamma\pi^+\pi^-) = 3.6 C_{13}$ keV. If we require the upper limit 1 keV for this width, then we should have $g_{\psi\psi\epsilon_c}/g_{\psi'\psi\epsilon_c} = -1.13$. This value then gives $\Gamma(\epsilon_c \rightarrow \gamma\psi) = 92 C_{13}$ keV = 26 keV with $g_{\psi'\psi\epsilon_c}^2/4\pi$ assumed. If we accept that a considerable part of the missing decays of ψ' is due to its radiative decays and take it to be $\psi' \rightarrow \gamma\pi\pi$ with, say, $\Gamma(\psi' \rightarrow \gamma\pi^+\pi^-) = 24$ keV, we should have $g_{\psi'\psi'\epsilon_c}/g_{\psi'\psi\epsilon_c} = -1.23$. This value then gives $\Gamma(\psi' \rightarrow \gamma\epsilon_c) = 178 C_{15}$ keV = 10 keV. With these values, as examples, one can fix all the coefficients C_i of the table. For the couplings of ψ and ψ' to photon, $(\gamma_V^2/4\pi)^{-1} = \Gamma(V \rightarrow e^+e^-)[12/(\alpha^2 M_V)]$, we have used the experimental values $\Gamma(\psi \rightarrow e^+e^-) = 4.8$ keV and $\Gamma(\psi' \rightarrow e^+e^-) = 2.2$ keV and assumed $\gamma_\psi/\gamma_{\psi'} > 0$. We have also used the values $\gamma_\rho^2/4\pi = 3.9$ and $\gamma_\rho^2/4\pi = 0.65$. Further, we can see that from the experimental ratio $\Gamma(\psi' \rightarrow \rho\pi)/\Gamma(\psi \rightarrow \rho\pi) \approx 1/3$ one obtains $f_{0-\psi'}^2/f_{0-\psi}^2 \approx 1$.

We see that the ratios of diagonal to off-diagonal couplings $g_{\psi\psi\epsilon_c}/g_{\psi\psi'\epsilon_c}$ and $g_{\psi'\psi'\epsilon_c}/g_{\psi'\psi\epsilon_c}$ are crucial in having predictions for the decay rates themselves. In addition, reasonable values for the decay rates require these ratios to be of negative sign and of order of unity ¹³⁾. To clarify these points, one has to calculate the overlap integrals over the wave functions obtained ^{10), 11)} from a dynamical picture like charmonium.

Putting aside the question of diagonal and off-diagonal couplings which certainly deserve further study, we can conclude that as an order of magnitude the predictions of the model are satisfactory.

It is a pleasure to thank J. Ellis, Y. Frishman, R. Kögerler and P. Minkowski for enlightening discussions.

^{*}) We are grateful to Y. Frishman for mentioning the relevance of diagram 1h over the other ones.

List of processes considered in the model

Decays	Calculated widths, keV	Assumed	Mass distributions peaked at M=	Notations
1 $\psi' \rightarrow \psi \pi^+ \pi^-$	72	input everywhere	m_ψ	
2 $\psi'' \rightarrow \psi' \pi^+ \pi^-$	1.5	$(g_{\psi' \psi \epsilon} / g_{\psi' \psi \epsilon})^2 = 1$	"	
3 $\psi'' \rightarrow \psi \pi^+ \pi^-$	3.2×10^3	$(g_{\psi'' \psi \epsilon} / g_{\psi' \psi \epsilon})^2 = 1$	"	
4 $\psi \rightarrow \omega \pi^+ \pi^-$	0.25 C_4		"	$C_4 = (1 + 1.2 \frac{f_{0-4}}{f_{0-4}} \frac{g_{\psi \psi \epsilon}}{g_{\psi' \psi \epsilon}})^2$
5 $\psi \rightarrow \phi \pi^+ \pi^-$	0.15 C_4		"	
6 $\psi \rightarrow \omega K^+ K^-$	0.03 C_4		m_{ψ_λ}	
7 $\psi \rightarrow \phi K^+ K^-$	0.025 C_4		"	
8 $\psi' \rightarrow \omega \pi^+ \pi^-$	0.42 C_8		m_ψ	$C_8 = (1 + 1.69 \frac{f_{0-4}}{f_{0-4}} \frac{g_{\psi' \psi \epsilon}}{g_{\psi \psi \epsilon}})^2$
9 $\psi' \rightarrow \phi \pi^+ \pi^-$	0.33 C_8		"	
10 $\psi' \rightarrow \omega K^+ K^-$	0.19 C_8		m_{ψ_λ}	
11 $\psi' \rightarrow \phi K^+ K^-$	0.07 C_8		"	
12 $\rho' \rightarrow \rho \pi^+ \pi^-$	135×10^3	$(g_{\rho' \rho \epsilon} / 4\pi)^2 = 2$	m_ψ	$C_{13} = (1 + 1.84 \frac{\delta_{\psi'}}{\delta_{\psi}} \frac{g_{\psi \psi \epsilon}}{g_{\psi' \psi \epsilon}})^2$
13 $\psi \rightarrow \delta \pi^+ \pi^-$	3.6 C_{13}		"	$C_{15} = (1 + \frac{\delta_{\psi}}{\delta_{\psi'}} \frac{g_{\psi \psi' \epsilon}}{g_{\psi' \psi \epsilon}})^2$
14 $\psi \rightarrow \delta K^+ K^-$	0.82 C_{13}		m_{ψ_λ}	$C_{17} = (1 + \frac{m_{\rho}}{m_{\rho'}} \frac{\delta_{\rho'}}{\delta_{\rho}} \frac{g_{\rho \rho \epsilon}}{g_{\rho' \rho \epsilon}})^2$
15 $\psi' \rightarrow \delta \pi^+ \pi^-$	435 C_{15}		m_{ψ_ϵ} , very sharp	$C_{18} = (1 + \frac{\delta_{\rho}}{\delta_{\rho'}} \frac{g_{\rho \rho' \epsilon}}{g_{\rho' \rho \epsilon}})^2$
16 $\psi' \rightarrow \delta K^+ K^-$	106 C_{15}		"	$C_{18} = (1 + \frac{\delta_{\rho}}{\delta_{\rho'}} \frac{g_{\rho \rho' \epsilon}}{g_{\rho' \rho \epsilon}})^2$
17 $\rho \rightarrow \delta \pi^+ \pi^-$	0.41 C_{17}	$(g_{\rho' \rho \epsilon} / 4\pi)^2 = 2$	m_ψ	
18 $\rho' \rightarrow \delta \pi^+ \pi^-$	76 C_{18}	"	"	
19 $\epsilon_c \rightarrow \delta \gamma$	92 C_{13}	$(g_{\psi' \psi \epsilon} / 4\pi)^2 = 4$	"	
20 $\psi' \rightarrow \delta \epsilon_c$	178 C_{15}	"	"	

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FIGURE CAPTIONS

- Fig. 1 : Diagrams for the decays listed in the table.
- Fig. 2 : Decay mass distribution of $\psi' \rightarrow \psi \pi^+ \pi^-$ with absolute normalization.
- Fig. 3 : Decay mass distributions of (a): $\psi'' \rightarrow \psi \pi^+ \pi^-$ and (b): $\psi \rightarrow \gamma \pi^+ \pi^-$ with arbitrary normalization.

FIG.1

FIG. 2

FIG. 3