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THE $\Delta I = \frac{1}{2}$ RULE IN UNIFIED GAUGE MODELS WITH THE GIM MECHANISM

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ABSTRACT

We consider non-leptonic decays in the framework of unified gauge theories for the weak and electromagnetic interactions. For processes involving a pion, using PCAC we find an enhancement of the one-body operators. The enhancement is large enough to explain the observed $\Delta I = \frac{1}{2}$, $\Delta S = 1$ enhancement in K decays.

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The origin of the $\Delta I = \frac{1}{2}$ rule has been a puzzle for many years. The most spectacular example of the rule is in $K \rightarrow 2\pi$ decays. The decay $K^+ \rightarrow \pi^+ \pi^0$ proceeds entirely through $\Delta I = \frac{3}{2}$ and is found to be strongly suppressed relative to the decays $K_S \rightarrow \pi^+ \pi^-$ or $K_S \rightarrow \pi^0 \pi^0$ which may proceed via both $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$. In particular, the amplitudes are (using Bjorken and Drell normalization)

$$\begin{aligned} A(K^+ \rightarrow \pi^+ \pi^0) &= G_F \sin \theta_C m_K^2 m_\pi \quad 0.14 \\ A(K_S \rightarrow \pi^+ \pi^-) &= G_F \sin \theta_C m_K^2 m_\pi \quad 3.2. \end{aligned} \quad (1)$$

Enhancement of the $\Delta I = \frac{1}{2}$ piece of the weak Hamiltonian is also found in $K \rightarrow 3\pi$ and in hyperon decays.

Recently Gaillard and Lee ¹⁾ and Altarelli and Maiani ²⁾ have investigated $\Delta S = 1$ non-leptonic decays in the framework of a unified gauge theory for the weak and electromagnetic interactions. The strong interactions are supposed to be described by an asymptotically free gauge theory (AFGT) coupled to colour. They found a relative enhancement of the $\Delta I = \frac{1}{2}$ to the $\Delta I = \frac{3}{2}$ piece of the weak Hamiltonian due to the appearance of anomalous dimensions of the two-body operators contributing. The size of this enhancement is dependent on the W boson mass M_W and the strong coupling constant g evaluated at a subtraction point μ ; it could be as large as 5. In order to explain the additional enhancement observed (approximately a further factor of 4) it must be assumed to come from the (unknown) operator matrix elements. Moreover, for $K_S \rightarrow \pi\pi$ the two-body octet matrix element is forbidden in the exact $SU(3)$ limit ³⁾. Whether this is a difficulty in practice is as yet unknown for there are no techniques for evaluating these matrix elements.

In models with right-handed currents coupling the p' to the n or the λ there is an additional, pure $\Delta I = \frac{1}{2}$, piece of the non-leptonic Hamiltonian. It has been suggested ⁴⁾⁻⁶⁾ that this could explain the $\Delta I = \frac{1}{2}$ enhancement without a further enhancement factor from the operator matrix elements. Indeed, the anomalous dimensions give rise to an enhancement factor which is larger ^{5),6)} than that found in models with only left-handed currents. However, the two-body operators appearing in this case involve the new p' quark, and matrix elements of this operator between conventional hadrons may be expected to be small ⁷⁾. Moreover, models with the p' coupled appreciably to the n in a right-handed manner are in disagreement with current algebra predictions for $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ ⁸⁾.

It is the purpose of this note to point out that, for decays involving a pion, there is, independent of anomalous dimensions, an enhancement of the one-body operator contribution. With a reasonable value for $m_{p'}$, the mass of the p' quark the two-body contribution could indeed generate all of the $\Delta I = \frac{1}{2}$ transitions. This applies to purely left-handed models of the Weinberg-Salam ⁹⁾ type using the GIM ¹⁰⁾ mechanism. There is no need to invoke new currents nor to rely on enhancements in (unknown) operator matrix elements. Indeed, since the one-body operators contributing are vector and axial vector currents, their matrix elements may be reliably estimated and the contribution of these operators is then expressible in terms of $m_{p'}$ and known quantities. Moreover, the operator matrix elements do not vanish in the limit of exact $SU(3)$ for $K_S \rightarrow 2\pi$ decays.

In a unified gauge model of the weak and electromagnetic interactions, the effective non-leptonic $\Delta S = 1$ weak interaction is of the form

$$\mathcal{L}_{\text{eff}} = -f^2 \int d^4x \mathcal{D}_F(x; M_W^2) T(j_{\mu_N}^+(x) j_S^\mu(0)) \quad (2)$$

where j_{uN} and j_S^μ are the charged strangeness conserving and strangeness changing currents which, in the Weinberg-Salam $SU(2) \times U(1)$ model are given by

$$\begin{aligned} j_{\mu_N} &= (\bar{p} \cos \theta - \bar{p}' \sin \theta) \gamma_\mu (1 - \gamma_5) n \\ j_{\mu_S} &= (\bar{p} \sin \theta + \bar{p}' \cos \theta) \gamma_\mu (1 - \gamma_5) \lambda \end{aligned} \quad (3)$$

Here p' is the charmed quark introduced by Glashow, Iliopoulos and Maiani ¹⁰⁾.

We work in the 't Hooft-Feynman gauge where the effects of the unphysical scalars may be neglected and the W boson propagator $D_F(x; M_W^2)$ then ensures that the integral in Eq. (2) is dominated by short distances. The time-ordered product of two currents may be expanded in the form

$$T[j_{\mu_N}^+(x) j_S^\mu(0)] = \sum_n \mathcal{T}_n(x, g, m) O_n(0) \quad (4)$$

where $O_n(0)$ are operators and $\mathcal{T}_n(x, g, m)$ are the corresponding Wilson coefficients.

In AFGT the short-distance form of $\mathcal{F}_n(x, g, m)$ may be estimated, since the $\mathcal{F}$'s satisfy a renormalization group equation, the solution of which is given by

$$\mathcal{F}_n(\mu x, g, m) \xrightarrow{\mu x \ll 1} \mu^{D_n} \mathcal{F}_n(x, \bar{g}, \bar{m}) \left[1 - \frac{g^2}{8\pi^2} \ln(\mu \sqrt{-x^2}) \right]^{\gamma_n} \quad (5)$$

Here $\bar{g}$ and $\bar{m}$ are the effective strong coupling constant and effective mass evaluated at the Euclidean point μx . g and m are these quantities evaluated at x . D_n is the dimension of the operator O_n .

In an AFGT, $\bar{g}$ and $\bar{m} \rightarrow 0$ as $u \rightarrow 0$, so $\mathcal{F}_n(x, \bar{g}, \bar{m})$ will reliably be given by a perturbation expansion in $\bar{g}$ and $\bar{m}$. The lowest order terms are just those of the free quark model.

Which operators arise in the expansion Eq. (4)? The leading one is the one-body operator and, evaluating $\mathcal{F}(x, \bar{g}, \bar{m})$ for an intermediate p or p' state we find, keeping only terms even in x which can survive the symmetric x integration

$$\begin{aligned} T \left[j_{\mu N}^+(x) j_S^\mu(0) \right] &= \sin \Theta_c \cos \Theta_c \left\{ -\frac{2x_\alpha x_\beta}{x^4} - \frac{2x_\alpha x_\beta \bar{m}_p^2}{x^2} \right\} \\ \bar{n}(0)(1+\gamma_5) \gamma_\alpha \partial_\beta \lambda(0) A^{\gamma_1} &+ \sin \Theta_c \cos \Theta_c \left\{ -\frac{2x_\alpha x_\beta}{x^4} - \frac{2x_\alpha x_\beta \bar{m}_p^2}{x^2} \right\} \\ \frac{1}{6} x_\gamma x_\delta \bar{n}(0)(1+\gamma_5) \gamma_\alpha \partial_\beta \partial_\gamma \partial_\delta \lambda(0) A^{\gamma_2} &+ O(\bar{g}^2) \\ + p' \text{ contribution} &+ \text{higher order operators} \end{aligned} \quad (6)$$

where $A = [1 - (g^2/8\pi^2) \ln(\mu \sqrt{-x^2})]$ and γ_1 and γ_2 may be calculated in terms of the group parameters.

Upon inserting Eq. (6) in Eq. (2) and performing the x integration, the first two terms yield the (dimension 4) one-body operator $\bar{n}(0)(1+\gamma_5) \not{x} \lambda(0)$ with a divergent coefficient. This will be removed by wave function renormalization in a "natural" theory¹¹⁾ where the strong gauge group commutes with the weak and electromagnetic gauge group. The remaining operators of dimension > 4 have no counter terms and give rise to finite, computable effects. Thus the third and fourth terms in Eq. (6) give rise to the operator $\bar{n}(0) \not{x}^2 \not{x} \lambda(0)$ after the x integration.

However, since the p' contribution contributes to Eq. (6) with an opposite sign to the p contribution, the third term is cancelled and the fourth term gives a contribution of order $G_F \sin \theta_c [(m_p^2 - m_{p'}^2)/M_W^2]$. Due to the additional $1/M_W^2$ this may be neglected.

It follows that the dimension 6 two-body operators

$$\bar{n}(0) \gamma_\mu (1-\gamma_5) p(0) \bar{p}(0) \gamma^\mu (1-\gamma_5) \lambda(0)$$

and

$$\bar{n}(0) \gamma_\mu (1-\gamma_5) p'(0) \bar{p}'(0) \gamma^\mu (1-\gamma_5) \lambda(0)$$

give the leading contribution. These operators can give $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{1}{2}$ contributions. It was found that the anomalous dimensions γ_n appearing in Eq. (5) are different for the two contributions. For a model with three colours

$$\begin{aligned} \gamma_{\frac{3}{2}} &= -0.24 \\ \gamma_{\frac{1}{2}} &= 0.48 \end{aligned} \quad (7)$$

This yields an octet enhancement which depends on the parameters M_W and g . If we take $g^2/4\pi \simeq 1$ at a subtraction point of 1 GeV and $M_W \simeq 100$ GeV, the $\Delta I = \frac{1}{2}$ enhancement would be 3 and the $\Delta I = \frac{3}{2}$ suppression would be 1.7. This is expected to be an approximate upper bound for the effect, since $g^2/4\pi$ and M_W may be smaller but are unlikely to be (much) larger than the values chosen.

The over-all enhancement of $\Delta I = \frac{1}{2}$ to $\Delta I = \frac{3}{2}$ is only 5, so it is insufficient by itself to account for the observed 20-fold enhancement. If this is the correct explanation, the remainder of the enhancement must come from the (unknown) matrix elements of the four-body operators.

It has been suggested ⁴⁾⁻⁶⁾ that models in which there is an additional right-handed current may be capable of explaining the $\Delta I = \frac{1}{2}$ rule completely. The corresponding anomalous dimension is more than twice the $\gamma_{\frac{1}{2}}$ found in Eq. (7), leading to an enhancement of more than 12 ^{5),6)}. However, as mentioned above, this enhancement may be illusory since the matrix element of the new operator involving p' quarks between conventional hadrons is expected to be small.

The above analysis assumes that the x integration over the W boson propagator in Eq. (2) may be performed before taking matrix elements between the initial and final states. However, for processes involving a pion, PCAC together with the methods of current algebra suggest that this is not the case.

Using PCAC in the form

$$\phi_a(x) = \frac{1}{i F_\pi m_\pi^2} \partial_\mu A_a^\mu(x)$$

we find

$$\langle \beta | \pi_{q_a}^a | \mathcal{L}_{eff}(0) | \alpha \rangle = \frac{m_\pi^2 - q_a^2}{F_\pi m_\pi^2} \int d^4 y e^{i q_a \cdot y}$$

$$\langle \beta | T [\partial_\rho A_a^\rho(y) \mathcal{L}_{eff}(0)] | \alpha \rangle$$

$$\xrightarrow{q_a^2 \rightarrow 0} \frac{if^2}{F_\pi} q_{a\rho} \int d^4 y e^{i q_a \cdot y} \int d^4 x D_F(x; M_W^2)$$

$$\langle \beta | T [A_a^\rho(y) j_{\mu_N}^+(x) j_s^\mu(0)] | \alpha \rangle$$

$$+ \frac{f^2}{F_\pi} \int d^4 y e^{-i q_a \cdot y} \int d^4 x D_F(x; M_W^2)$$

$$\times \langle \beta | T \left\{ [A_a^0(y), j_{\mu_N}^+(x)] \delta(x_0 - y_0) j_s^\mu(0) \right. \\ \left. + j_{\mu_N}^+(x) [A_a^0(y), j_s^\mu(0)] \delta(y_0) \right\} | \alpha \rangle$$

(8)

In an AFGT the equal time commutation relations are just those of the free quark model

$$[A_a^0(y), j_{\mu_b}(x)] \delta(x_0 - y_0) = -i c_{abc} j_{\mu_c}(x) \delta^4(x - y) \quad (9)$$

where j_{μ_b} are the (V-A) combinations of SU(4) currents, and c_{abc} are the SU(4) structure constants. Using this we find

$$\begin{aligned}
 \langle \beta | \pi_{q_a}^a | \mathcal{L}_{eff} | \alpha \rangle &= \frac{if^2}{F_\pi} q_{a\rho} \int d^4 y e^{iq_a \cdot y} \\
 &\int d^4 x D_F(x; M_W^2) \langle \beta | T \{ A_a^\rho(y) j_{\mu_N}^+(x) j_s^\mu(0) \} | \alpha \rangle \\
 &+ \frac{if^2}{F_\pi} \int d^4 x D_F(x; M_W^2) \left[e^{iq_a \cdot x} c_{anc} \langle \beta | T \{ \right. \\
 &j_{\mu_c}^+(x) j_s^\mu(0) \} | \alpha \rangle + c_{asc} \langle \beta | T \{ j_{\mu_N}^+(x) j_c^\mu(0) \} \\
 &\left. | \alpha \rangle \right] \quad (10)
 \end{aligned}$$

where Π stands for the combination of $SU(4)$ currents in $j_{u\Pi}^+$ as given by Eq. (3).

Due to the W boson propagator, the second term of Eq. (10) should be dominated by small x . The contribution of various operators to this term may be evaluated in an AFGT as discussed above. However this time, due to the presence of the factor $e^{-iq_a x}$, the current operator does not give zero contribution upon performing the symmetric x integration. For the case $c=\Pi$, we have for this contribution

$$\begin{aligned}
 T \{ j_{\mu_N}^+(x) j_s^\mu(0) \} &= \frac{1}{2\pi^2} \sin \theta_c \cos \theta_c \frac{2x_\alpha}{x^2} (\bar{m}_p^2 - \bar{m}_{p'}^2) \\
 &\bar{n}(0) (1 + \gamma_5) \gamma^\alpha \lambda(0) \quad (11)
 \end{aligned}$$

Here we only have the term proportional to the p and p' effective mass squared, since the term independent of this mass cancels between the p and the p' contributions.

The short-distance piece of the first term in Eq. (10) involving the triple product of currents may also contribute to the one-body operator term. However, its contribution is proportional only to $\bar{m}_p^2$. Since we expect $\bar{m}_p^2 \ll \bar{m}_{p'}^2$, we will ignore such terms for the moment F1 . The remaining term proportional to $\bar{m}_{p'}^2$, since it arises from a p' intermediate state, clearly is pure $\Delta I = \frac{1}{2}$. We may substitute Eq. (11) in Eq. (10) and perform the x integration. This leads to an effective Lagrangian proportional to $q_{a\gamma} \bar{n}(0) (1 + \gamma_5) \gamma^\alpha \lambda(0)$ which vanishes in the soft pion limit $q_a \rightarrow 0$.

The effect of such a term in $K \rightarrow 2\pi$ decays can be obtained by following the analysis of Holstein¹²⁾. He expresses $K \rightarrow 2\pi$ amplitudes in terms of pole terms, and terms independent and terms linear in the pion momenta. These terms are then fixed by demanding consistency in the soft pion limit with the $K-\pi$ matrix element. This analysis goes through in this case, and the presence of linear terms in the pion momenta in the $K-\pi$ matrix element coming from the term $q_{a\alpha} \bar{n}(0)(1+\gamma_5)\gamma^\alpha \lambda(0)$ leads, for $K_S \rightarrow \pi^+\pi^-$, to the amplitude

$$A(K_S \rightarrow \pi_a^+ \pi_b^-) = \frac{G_F}{\sqrt{2}} \sin \theta_c \cos \theta_c \frac{\tilde{m}_{p'}^2}{16\pi^2 F_\pi^2} \quad (12)$$

$$(q_a + q_b)_\alpha \langle 0 | \{ \bar{n}(0)(1+\gamma_5)\gamma^\alpha \lambda(0) - \bar{\lambda}(0)(1+\gamma_5)\gamma^\alpha n(0) \} | K_S \rangle$$

Note that the operator matrix element does not vanish in the $SU(3)$ symmetric limit, in contrast to the two-body operators discussed above. Evaluating this matrix element we find

$$A(K_S \rightarrow \pi^+\pi^-) = G_F \sin \theta_c \cos \theta_c m_K^2 m_\pi \frac{1}{8\pi^2} \left[\frac{\tilde{m}_{p'}^2}{m_\pi^2} \right] \quad (13)$$

where

$$\tilde{m}_{p'}^2 = m_{p'}^2(\mu') \left[1 + \frac{g^2}{8\pi^2} b \ln(\mu' \sqrt{-x^2}) \right]^{-1} \quad (14)$$

and we have evaluated the anomalous dimension for the three quartet model and $\langle -x^2 \rangle$ is the mean value of $-x^2$ in the integral Eq. (10).

Comparing Eq. (13) with Eq. (1) we find the two-body operator could account for the whole amplitude, provided $\tilde{m}_{p'} = 2.8$ GeV. Ignoring, for the moment, the effect of the anomalous dimension term in Eq. (14), we find this value is not in disagreement with the bounds found from analyses of rare K meson decays and from the $K_L - K_S$ mass difference¹³⁾. The strongest bound came from the $K_L - K_S$ mass difference and gave $m_{p'} < 5$ GeV.

It is difficult to make a precise connection between $m_{p'}$ and the actual mass of charmed particles F^2). However, if we were able to build up the amplitude from the sum of charmed and uncharmed particle exchanges, then the mass-independent term would vanish leaving a term proportional, presumably, to the (squared) mass difference between charmed and uncharmed states summed over all contributing exchanges. The quantity $m_{p'}^2 - m_p^2$ must therefore correspond to some mean value of the squared mass difference between physical charmed and uncharmed states evaluated at the subtraction point μ .

The effects of the strong interactions may be estimated by including the anomalous dimension term. However, μ' should not be taken less than $m_{p'}$, since for charmed states scaling is expected to set in later than for uncharmed states. Using g and M_W as above, we find an enhancement of order 1.6. This would give $m_{p'}(\mu') = 3.5$ GeV. As mentioned above, this should be an approximate upper bound for the strong interaction effect. If M_W is less than 100 GeV or $g^2/4\pi(\mu)$ less than 1, then we would get a value for $m_{p'}(\mu')$ closer to 2.8 GeV.

It seems therefore that with $m_{p'}(\mu')$ of the order of 2.8 GeV, we may explain via the resultant two-body operator the predominant $\Delta I = \frac{1}{2}$ decays of $K \rightarrow 2\pi$. Is this consistent with the usual current algebra predictions relating $K \rightarrow 2\pi$ to $K \rightarrow 3\pi$?

The term arising from Eq. (11) vanishes in the soft pion limit $q_a \rightarrow 0$ and we are ascribing a large $\Delta I = \frac{1}{2}$ amplitude to terms linear in q_a upon taking this limit. Thus writing

$$\langle \pi_{q_a}^+ | H_W^{\frac{1}{2}} | K_k^+ \rangle = A_{\frac{1}{2}} (M_K^3 / 2F_\pi) + B_{\frac{1}{2}} (M_K / 2F_\pi) k \cdot q_a$$

we are assuming $B_{\frac{1}{2}} \gg A_{\frac{1}{2}}$. However, given such an amplitude, it is straightforward to check that the relation of $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ amplitudes to it, are given by the usual soft pion limit procedure. Thus the analysis of Holstein¹²⁾ proceeds as before. He finds the $\Delta I = \frac{1}{2}$ contribution to all $K \rightarrow 2\pi$ and $K \rightarrow 3\pi$ processes depends only on the sum $A_{\frac{1}{2}} + B_{\frac{1}{2}}$. It is therefore consistent to have the entire contribution coming from the $B_{\frac{1}{2}}$ term vanishing in the soft pion limit.

As mentioned above, the one-body operator contributing to $\Delta I = \frac{3}{2}$ processes is proportional to $\bar{m}_p^2$. If $\bar{m}_p$ is small¹⁴⁾ ($\ll m_\pi$), $\Delta I = \frac{3}{2}$ processes will probably proceed through the four-body operator contribution.

With only left-handed currents, the conventional current algebra relations between $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ will not be disturbed.

We now turn to hyperon decays. We first observe that the one-body operator discussed above will also contribute here, but that its contribution obeys the octet rules and the Lee-Sugawara relations for both the S and P wave amplitudes. Once again the absolute magnitude of the contribution is determined, given $\tilde{m}_p$, and in the Table we give the results for $\tilde{m}_p = 2.8$ GeV. The amplitudes A and B are defined by

$$T(B \rightarrow B'\pi) = \bar{\psi}_{B'} (A + B\gamma_5) \psi_B$$

with Bjorken and Drell normalization.

The Table also gives the experimental determination of A and B (up to an over-all undetermined sign).

The remarkable thing ¹⁵⁾ about these results is that the parity violating amplitudes A are well fitted by the one-body term alone. In addition to the octet rule relations, this implies a fit to the normalization and to the F/D ratio. The B amplitudes are not fitted at all, but we know that for them other terms, arising from baryon pole terms, etc., are expected to be important. It is possible to fit these amplitudes by including such pole terms, but the meaning of such a fit is at present unclear.

Of course, it may be that the fit to A is spurious (even the sign may be wrong). Naive calculations of the expected two-body operator matrix elements between baryon states suggest they should also be important ¹⁶⁾. What we have shown here is that for $\tilde{m}_p$ of the order of 3 GeV, the one-body operator contributions are important and must be included in any discussion of hyperon decay.

In conclusion, we have found, using PCAC, a contribution to non-leptonic decays involving a pion in addition to the usual two-body operator contribution. For $\tilde{m}_p = 2.8$ GeV we found that $K \rightarrow 2\pi$, $K \rightarrow 3\pi$, and the parity violating amplitudes in hyperon decay could be given entirely by this new one-body contribution. The parity conserving amplitudes are not fitted by this term alone. However, they are expected to have significant contributions also from hyperon pole terms, and it is unclear at present whether they require the presence of significant two-body contributions.

There remains also the question of $\Delta s = 0$ non-leptonic decays. One-body operators should also be important in processes involving (virtual) pions. This may help to explain the observed enhancement of the parity violating contribution due to pion exchange.

For charmed particle decays involving a pion, the intermediate states leading to the one-body operator are n and λ . There is thus no equivalent enhancement due to heavy quark mass, and the decays may be expected to proceed principally via the two-body operators ¹⁷⁾.

Finally, what would be the effect of right-handed currents coupling the n or the λ to the p ⁴⁾⁻⁶⁾? Again we expect one-body operators to contribute, but this time we find a dependence on m_p, m_λ or m_p, m_n and not m_p^2 . The enhancement will be less than in the left-handed case.

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Table

Hyperon decay amplitudes (units $\sqrt{m_\pi}$ sec)

	A_{Σ^-}	A_{Σ^+}	A_{Σ^0}	A_{Λ^0}	A_{Ξ^-}
Theory	2.32	0	-1.64	1.30	-2.11
Expt.	1.89 ± 0.03	0.06 ± 0.02	-1.53 ± 0.14	1.53 ± 0.02	-2.07 ± 0.02
	B_{Σ^-}	B_{Σ^+}	B_{Σ^0}	B_{Λ^0}	B_{Ξ^-}
Theory	-7.12	0	5.04	11.5	-4.3
Expt.	-0.72 ± 0.01	19.08 ± 0.34	11.52 ± 1.81	10.50 ± 0.33	6.68 ± 0.7

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