Ref.TH.2098-CERN

ON THE BAND STRUCTURE IN THE MASS DISTRIBUTION OF HADRON PAIRS

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ABSTRACT

The recently observed exponential decrease and band structure in the invariant mass distributions of inclusively produced hadron pairs is interpreted as a direct consequence of single particle inclusive momentum distributions. Any model describing these should be able to explain the new data. A simple analytical model description is given which can reproduce the data. We believe that the observed behaviour is not related to the small distance structure of nucleons, because the parameters of the usual hadronic regime are sufficient to explain the phenomena.

Ref.TH.2098-CERN

13 November 1975

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In a recent letter "Structure of nucleons at small distances" ¹⁾, the authors report on the measurement of invariant mass m and total transverse momentum $p_{\perp}$ of inclusively produced two-body systems:

$$p + Be \rightarrow \pi^- + p + X \quad (1a)$$

$$p + Be \rightarrow \begin{cases} \pi^+ + \pi^- + X \\ p + \bar{p} + X \\ K^- + p + X \end{cases} \quad (1b)$$

$$p + Be \rightarrow \begin{cases} K^- + \pi^+ + X \\ \bar{p} + \pi^+ + X \end{cases} \quad (1c)$$

observed at BNL (28.6 GeV). They find for all reactions

$$\left. \frac{d^2\sigma}{dm dx} \right|_{x \approx 0} \propto \exp(-5m); \quad 2 \leq m \leq 5 \text{ GeV} \quad (2)$$

and furthermore $(1a):(1b):(1c) \approx 1:1/10:1/100$, while inside each group the differences are small.

A rule, allegedly due to Feynman ²⁾ summarizes the observed "band structure" as follows: assign the number zero to the proton, one to π and K^+ , two to $\bar{p}$ and K^- , then in the top band (1a) these numbers add to 1, in the middle (1b) to 2, and in the lower one (1c) to 3. A quark counting model ³⁾ has been proposed in order to give to this rule a physical meaning by identifying the above numbers with the numbers of newly created quarks required to build up the emitted hadrons. A band structure similar to the observed one is obtained if the probability α of producing the pair $(u\bar{u} + d\bar{d} + \gamma s\bar{s})$ is $\alpha \approx 0.5$, independently of the quark momenta.

In this note we wish to explain at once band structure, exponential drop in m and the $p_{\perp}$ distribution, without invoking small distance behaviour or the parton-quark picture; our motivation is the exponential drop in m , which suggests that the small distance regime has not yet set in; note that we are at $\sqrt{s} \approx 7.5 \text{ GeV}$ and that the biggest measured mass $m \approx 5 \text{ GeV}$, if it decays into two pions, gives each of them $p_{\perp} \approx 2.5 \text{ GeV}/c$.

We claim that the inclusive single particle distributions are sufficient to understand the new data qualitatively; then we give a simple analytical model description. We start from three observations:

- i) in the given experimental situation the two particles are emitted at $x \approx 0$ ($p \approx p_{\perp}$) and with sufficient difference in transverse rapidity to consider them as uncorrelated (except near the kinematical limit);
- ii) the $p_{\perp}$ distribution of each member i of the pair is roughly

$$f(p_{\perp i}, m_i) \approx \exp(-b \sqrt{p_{\perp i}^2 + m_i^2}) \quad (3)$$

From (i) and (ii) follows (for $x_1 + x_2 \approx 0$)

$$\frac{d^2}{df_{\perp 1} df_{\perp 2}} \propto \exp(-b(\sqrt{p_{\perp 1}^2 + m_1^2} + \sqrt{p_{\perp 2}^2 + m_2^2})) = \exp(-b' \sqrt{(p_{\perp 1} - p_{\perp 2})^2 + m^2})$$

$$b' = \frac{\sqrt{p_{\perp 1}^2 + m_1^2} + \sqrt{p_{\perp 2}^2 + m_2^2}}{E_1 + E_2} \cdot b \leq b$$
(4)

This explains the exponential slope;

- iii) the absolute magnitudes of single particle inclusive distributions in pp collisions ⁴⁾, read off at small x and fixed $\sqrt{p_{\perp 1}^2 + m_1^2}$, have the following order-of-magnitude relations between each other

$$p : \pi^+ : \pi^- : K^+ : K^- : \bar{p} \approx 1 : \frac{1}{10} : \frac{1}{10} : \frac{1}{10} : \frac{1}{100} : \frac{1}{100} \quad (5)$$

This, combined with the multiplication hypothesis (4), yields a grouping in bands separated from each other by a factor ≈ 10 ; the band structure (1a,b,c) is reproduced and further predictions follow trivially by making new combinations.

Note that our band separation agrees with that of Ref. 3) and that the model ³⁾, applied to single particle spectra, gives the orders of magnitude listed in (5) (more precisely, $1:0.61/10:0.47/10:0.42/10:0.51/100:0.35/100$).

It is then natural that any model which gives a reasonable fit to single particle spectra, should also explain the new data. As an example, consider the thermodynamical formulation of the statistical bootstrap model 5),*), in which everything can be expressed by simple analytical expressions. For easy reference we quote equations from CERN lectures on "Thermodynamics of strong interactions" 6).

1. Mass distribution: Equation (8.8) of Ref. 6) predicts for the inclusive production rate per collision of a discrete mass m

$$\mathcal{V}(m, T) = V_0 z_m \frac{m^2 T}{2\pi^2} K_2\left(\frac{m}{T}\right) \xrightarrow{m \gg T} V_0 z_m \left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}} \quad (6)$$

This equation holds if the production of m does not require (by conservation laws) associated production of other particles, otherwise further factors arise; $V_0 = 4\pi/(3m_\pi^3)$ is the Fermi volume and $T \leq T_0$ the temperature of the emitting fireball (cluster); finally z_m is the number of internal states of m ; as here the mass is not discrete, we have $z_m \rightarrow Z(m)dm$:

$$Z(m)dm = \text{number of internal states in } \{m, dm\} \quad (7)$$

We derive $Z(m)$ from invariant two-body phase space R_2^{IPS} , given explicitly in Ref. 7), Eq. (3.5) and defined as follows:

$$R_2^{\text{IPS}}(Q^2, Q\omega, \omega^2, m_1, m_2) d^4Q = \text{number of states in } \{Q, d^4Q\} \quad (8)$$

Q is the total four momentum, ω the four volume. Evaluating with $Q^2 = m^2$ we obtain in the volume rest frame

$$\begin{aligned} Z(m)dm &= dm^2 z_1 z_2 \int R_2^{\text{IPS}}(m, E, V_0, m_1, m_2) d_0^3(Q^2 - m^2) d^4Q \\ &= 2m dm z_1 z_2 \int R_2^{\text{IPS}}(m, E, V_0, m_1, m_2) \frac{d^3Q}{2E} \end{aligned} \quad (9)$$

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*) Dual models would give a Gaussian decrease.

Here z_1 and z_2 count internal degrees of freedom (spin) of m_1 and m_2 . In the rest frame of the box V_0 , Eq. (3.5) of Ref. 7) reads:

$$\mathcal{R}_2^{IPS}(m, E, V_0, m_1, m_2) = \frac{2\pi \sqrt{\lambda}}{m^2} \left[\frac{V_0}{(2\pi)^3} \right]^2 \left[E^2 \frac{3m^4 - 3(m_1^2 - m_2^2)^2 - \lambda}{12m^4} + \frac{\lambda}{12m^2} \right]$$

$$\lambda = m^4 + m_1^4 + m_2^4 - 2(m^2 m_1^2 + m^2 m_2^2 + m_1^2 m_2^2) \quad (10)$$

We introduce this in Eq. (9) and single out a factor $V_0/(2\pi)^3 d^3q$:

$$\int \{ \dots \} \frac{V_0}{(2\pi)^3} d^3q = \langle \{ \dots \} \rangle \int \frac{V_0}{(2\pi)^3} d^3q \quad (11)$$

The remaining integral counts the states arising from the movement of the "single mass m " inside the box V_0 ; these states are not internal of the mass m , therefore the integration has to be extended to yield just the value 1 (one state: m at rest in V_0); then $E=m$. Hence

$$Z(m)dm = dm z_1 z_2 \frac{V_0}{(2\pi)^3} \cdot 2\pi \sqrt{\lambda(m^2, m_1^2, m_2^2)} \cdot \frac{m^4 - (m_1^2 - m_2^2)^2}{4m^4} \quad (12)$$

Together with Eq. (6), we obtain

$$W(m)dm = Z(m)dm \cdot w(m) \quad (13a)$$

$$w(m) = V_0 \frac{m^2 T}{2\pi^2} K_2\left(\frac{m}{T}\right) \xrightarrow{m \gg T} V_0 \left(\frac{mT}{2\pi}\right)^{3/2} e^{-\frac{m}{T}} \quad (13b)$$

2. Correction factors arise from conservation of baryon number and strangeness: Eq. (13) is correct, if m carries no quantum numbers linked with heavy particles, otherwise one has to take into account that besides m one or more further particles must be created. Suppose m is the mass of the $\pi\bar{p}$ system. As $\bar{p}$ is not present in the initial state, its creation requires creation of another baryon (in any state). Let $\rho_B(m_B)$ be the non-strange baryon mass spectrum, then

$$N_B(T) = \int \rho_B(m_B) w(m_B) Z_B dm_B \quad (14)$$

is the total production rate for this additional baryon, and (13b) has to be multiplied by this factor. Inversely, if one of the particles making up m is already present in the initial state, for instance, in the πp system, then $w(m)$ takes into account the creation of a baryon, which need not be created; therefore in that case $w(m)$ has to be divided by $N_B(T)$. Similarly for K . For details see Ref. 6), pp. 97-106, last Ref. 5) App. I; a closed expression for $N_i(T)$ is given in Ref. 8), Eqs. (3.7) and (3.8).

We thus obtain the following total correction factors to $w(m)$:

pair	pp 1)	pp 2)	$p\pi$	pK^-	$p\bar{p}$	$\pi\pi$	πK^-	$\pi\bar{p}$	K^+K^-	K^-K^-	$K^-\bar{p}$	$\bar{p}\bar{p}$
factor	N_B^{-2}	1	N_B^{-1}	$N_K N_B^{-1}$	1	1	N_K	N_B	1	N_K^2	$N_K N_B$	N_B^2
1) in pp collisions } all others valid for πp collisions 2) in πp collisions } <u>and</u> pp collisions.												

Spin degrees of freedom of m_1 and m_2 are counted by z_1 and z_2 . A priori we have $z_p = 2$, $z_\pi = 1$, $z_K = 1$. However, if the two-body state had been for instance first $\rho\pi$ or $\rho\rho$ decaying into $(\pi\pi) + \pi$ or $(\pi\pi) + \pi + \pi$, respectively, then $z_\rho = 3$ would have to be taken. Allowing therefore intermediate resonant states, the z_i would differ from the a priori values. As the experimental normalization is uncertain by a factor ~ 2 , we fix by definition $z_p = 2$, but allow z_π and z_K to vary as free parameters (bound to be of order one). We have varied z_π , z_K , T and T_0 and found a good fit to the data of Ref. 1) with the following values: $z_\pi = 1.9$, $z_K = 0.85$, $T = 0.155$, $T_0 = 182$; for Eq. (14) we then obtain $N_B = 0.1234$, $N_K = 0.323$. The value of z_π reflects a substantial intermediate resonance contribution, while z_K combines the somewhat compensating effects of resonances and of fixing $z_p = 2$.

3. Results: The final mass distribution is then in absolute normalization

$$\frac{d\sigma}{dm} = \sigma_{\text{inel}} \cdot W(m) \cdot (\text{correction factors}) \quad (15)$$

We plot (Fig. 1) $\log_{10}(d\sigma/dm)$ and find absolute normalization, slope and band structure fairly reproduced; a straight line approximation to the πp curve in $2 \leq m \leq 4$ yields $\sim \exp(-5.18 m)$, while $1/T = 6.45 \text{ GeV}^{-1}$; the smaller slope is due to the phase space factor $Z(m)$. The band structure

comes from the correction factors z_1 and N_1 and is in our opinion the effect of a complicated interplay of conservation laws, coupling strengths, resonance contributions and spin degrees of freedom. In other words: it is rather accidental. However, having fixed the parameters, further predictions emerge, summarized in Fig. 2; we show $\log_{10}[d\sigma/dm(m=3 \text{ GeV})]$ for various two-body systems. Except for the K^+K^- case, we do not make predictions for reactions containing K^+ , because K^+ requires coproduction of either K^- or a hyperon and the simple factor N_K is then not sufficient. Note that our predictions differ somewhat from those of Ref. 3); in particular, except for the pair $\{pp\}$, we do not predict any changes if the incident beam carries pions instead of protons (same $\sqrt{s}$).

4. Further predictions: [Ref. 6), Eqs. (10.28), (10.29)] the transverse momentum distribution of m :

$$W(p_\perp) = \text{const} \cdot \sqrt{p_\perp^2 + m^2} \cdot K_1\left(\frac{\sqrt{p_\perp^2 + m^2}}{T}\right) \xrightarrow{m \gg T} \text{const} \cdot (p_\perp^2 + m^2)^{3/4} e^{-\frac{\sqrt{p_\perp^2 + m^2}}{T}} \quad (16)$$

where const. contains all the above correction factors, which we omit here. In the original formula, a factor $p_\perp$ arising from $d^2p_\perp = 2\pi p_\perp dp_\perp$ is present; here we omit it according to the experimental set-up. Figure 3 shows $\log_{10}[W(p_\perp)]$ for $m=1,2,3,4,5 \text{ GeV}$, all normalized to the same point at $p_\perp=0$. The figure agrees rather well with the empirical fit in Ref. 1).

The mean transverse momentum is given in Ref. 6) Eqs. (10.35), (10.36):

$$\langle p_\perp(m) \rangle = \sqrt{\frac{\pi m T}{2}} \cdot \frac{K_{5/2}\left(\frac{m}{T}\right)}{K_2\left(\frac{m}{T}\right)} \xrightarrow{m \gg T} \sqrt{\frac{\pi m T}{2}} \quad (17)$$

Figure 4 shows a plot of this function.

5. Conclusions

We believe that the observed behaviour has nothing to do with small distances, because everything follows qualitatively from single particle inclusive spectra and quantitatively from, e.g., the thermodynamical formulation of the statistical bootstrap model, where the scale is set by the Fermi volume V_0 . The quark counting model ³⁾ reproduces the band structure (relative weights) with two free parameters, of which the important one,

$\alpha \approx 0.5$, is the $q\bar{q}$ production probability. Such a value would give a too large $\mu^+\mu^-$ yield in pp collisions already from the Drell-Yan mechanism ⁹⁾ alone, without including further enhancements ^{10),11)} due to contributions of produced hadrons. If our picture is correct, it also suggests frequent production of heavy fireballs of mass $M \gtrsim m$ in the pionization region.

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Fig. 1: $\log_{10} (d\sigma/dm)$ for the reaction $p + p \rightarrow (h_1 + h_2) + X$ predicted by Eq. (17). Note the slight curvature. The average slope in the region $2 \leq m \leq 4$ GeV corresponds to $d\sigma/dm \propto \exp(-5.18 m)$.

Fig. 2: Band structure for various reactions at $m=3\text{GeV}$. Note the drop of pp by a factor ≈ 100 when the incident beam is changed from p to π . All other predictions should hold for $p+p \rightarrow h_1+h_2+X$ as well as $\pi+p \rightarrow h_1+h_2+X$, provided $\sqrt{s}$ is the same.

Fig. 3: The p_T distribution for $m = 1, 2, \dots, 5$ GeV, arbitrarily normalized to 1 at $p_T = 0$.

Fig. 4: The mean transverse momentum of the pair as function of its invariant mass.