On the generation and identification of optical Schrödinger cats

M.Genovese *, C.Novero
Istituto Elettrotecnico Nazionale Galileo Ferraris
Str. delle Cacce 91
I-10135 Torino, Italy

Abstract

We discuss the possibility of generating and detecting, by a tomographic reconstruction of the Wigner function, a macroscopic superposition of two coherent states. The superposition state is created using a conditioned measurement on the polarisation of a probe photon entangled to a coherent state. The entanglement is obtained using a Kerr cell inserted in a Mach-Zender interferometer. Some hint about generation of GHZ states is given as well.

PACS number: 03.65.Bz

Keywords: Schrödinger cats, decoherence, quantum computation

*genovese@ien.it, tel. 390113919234, fax 390113919259
Quantum Mechanics represents nowadays one of the pillars of modern physics: so far a huge amount of theoretical predictions deriving by this theory have been confirmed by very accurate experimental data. No doubts can be raised on the validity of this theory. Nevertheless, even at one century since its birth, many problems related to the interpretation of this theory persist, in particular concerning the concept of measurement in Quantum Mechanics and the related problem of the transition from a microscopic probabilistic world to a macroscopic deterministic world perfectly described by classical mechanics. The disappearance of superposition effects for macroscopic bodies is known as "decoherence". Besides the huge theoretical interest for understanding this phenomenon, a clear comprehension of decoherence is also mandatory for constructing quantum computers and thus it is of primary relevance in studies concerning quantum information.

Many different models have been proposed for describing decoherence [1], but none of them has reached a general acceptance.

However, it is now emerging the possibility of realising experiments concerning the transition region between quantum and classical mechanics. This is obtained studying mesoscopic states [2,3], i.e. systems composed by a number of particles for which one expects an intermediate behaviour between quantum and classical mechanics.

In particular a huge interest is devoted to the generation of linear superpositions of "macroscopically" distinguishable states, dubbed "Schrödinger cats", which would permit a direct study of decoherence. Many different schemes have been proposed for realising superpositions of many photons states using optical amplifiers [4] or cavities [5]. However, the practical realisation of these schemes is very difficult due to the necessity of creating a state with an average number of photons sufficiently high to be considered a mesoscopic system without that it decoheres too rapidly for being detected. Furthermore one has to find a smart scheme for detecting the superposition. Up to now only the beautiful experiment of Brune et al. [3] has succeeded in obtaining a superposition state involving an atom and
few photons in a cavity.

It remains therefore a large interest in proposing and studying new configurations for realising Schrödinger cats.

In a recent paper [6] we have proposed a scheme addressed to the creation of a superposition of two coherent states using a Kerr cell.

In this letter we would like to investigate in details the real possibility of creating and recognising such a state.

The idea is to use the set-up of fig.1, where a "signal" photon enters the Polarising-Beam-Splitter I (PBS I) from port 1. Let us suppose that this photon is in a superposition of vertical (V) and horizontal (H) polarisation, which will take different paths, for example let us assume the vertical one will follow path 2 and the horizontal path 3.

Furthermore, a probe laser crosses the Kerr cell on the arm 3 acquiring a phase or not according if the photon crosses or not the cell.

Thus the entangled state:

\[
|\Psi\rangle = \frac{|H\rangle|\nu\rangle + |V\rangle|\nu\rangle}{\sqrt{2}}
\]  

is generated, where the coherent state $|\nu\rangle$ differs in phase from $|\nu\rangle$.

The two signal photon paths are then recombined on a second beam splitter (BSII) and a polarisation measurement (P1) is performed on this photon on the base at 45° respect to the horizontal-vertical base. This is the conditional measurement producing the Schrödinger cat: if the signal photon is found to have a 45° polarisation, the coherent state is projected into the superposition

\[
|\psi_+\rangle = \frac{|\nu\rangle + |\nu\rangle}{\sqrt{2}}
\]  

\[3\]
on the other hand if the orthogonal polarisation is detected, the projection will be in

\[ |\psi_-\rangle = \frac{|\nu\rangle - |\nu\rangle}{\sqrt{2}} \]  

A superposition of two "many photons" states is thus obtained.

The one photon signal state can be easily produced, for example, using parametric down conversion (PDC) in a non-linear crystal. In this case the second photon of the down-converted pair can be used as trigger. Furthermore, if the same pulsed laser is used both for pumping the crystal and for the Kerr effect, one can easily obtain a good timing for the crossing of the Kerr cell for the signal photon and the coherent state.

Incidentally, the use as input of a photon from PDC in a Kerr cell allows also the creation of a three photons GHZ entangled state [9], which is one of the three elements necessary for realising an optical quantum computer [10], together with single qubit operations, which are easily implemented, and teleportation. For what concerns this last, a description of a scheme performing this operation using a Kerr cell appears in Ref. [11]. Using the same polarisation dependence of the Kerr interaction of Ref. [11], where there is no effect except when both the photons interacting in the Kerr cell have vertical polarization \((|V\rangle|V\rangle \rightarrow |V\rangle|V\rangle e^{i\phi})\), a GHZ state can be generated by the interaction of an entangled pair of photons with a third one. Let us assume, for example, of having generated, by PDC, an entangled state of the form [13]:

\[ |\Phi^+\rangle = \frac{|H\rangle|H\rangle + |V\rangle|V\rangle}{\sqrt{2}} \]  

A simple algebraic calculation shows that the interaction in the Kerr medium of the second photon of the pair with a third photon polarised at 45° (denoted by |45\rangle, whilst its orthogonal state will be |135\rangle), is, for a phase shift \(\phi = \pi/2\), the GHZ state:

\[ |\Psi_{GHZ}\rangle = \frac{|H\rangle|H\rangle|45\rangle + |V\rangle|V\rangle|135\rangle}{\sqrt{2}}. \]
Analogous results are easily derived for the other three Bell states.

Recently a GHZ state of three photons has been generated experimentally [12]. However, albeit very interesting for testing local realism, this scheme is not well suited for quantum computation, because of low efficiency. Our scheme could thus represent an alternative for generating GHZ states, which offers an interesting opportunity for applications to quantum computation.

The interest for these schemes derives by the fact that, although admittedly very difficult, the Quantum Non Demolition (QND) [14] detection of a single photon is at present possible [8,7]. QND measurements of welcher Weg (which path) have already been achieved using 100 meter long optical fibers (see Imoto et al. and Levenson et al. [15]). Of course, the implementation of the present schemes using such devices would be, even though not impossible in theory, almost impossible in practice. The recent discovery of new materials with very high Kerr coupling, could however allow an easier and more realistic implementation of this experiment. Two candidates as Kerr cell with ultra-high susceptibility to be used for this scheme are the Quantum Coherent Atomic Systems (QCAS) [17,8] and the Bose-Einstein condensate of ultracold (at nanoKelvin temperatures) atomic gas [18]. These are recent great technical improvements which could permit the realisation of small Kerr cells, capable of large phase shift, even with a low-intensity probe. In fact, both exhibit extremely high Kerr couplings compared to more traditional materials. In particular, the QCAS is rather a simple system to be realised (for a review see [19]) and thus represents an ideal candidate to this role. Incidentally, one can notice that Kerr coupling can be further enhanced by enclosing the medium in a cavity [20].

A main advantage of the present configuration respect to the Schrödinger cat generation using cavities is given by the fact that the coherent states superposition travels in air between the Kerr cell and the detection apparatus. Decoherence appears when at least one photon is lost permitting the identification of the phase of the state. In cavities this is related to the
damping factor of the cavity itself and decoherence becomes rather rapid when the number of photons is increased (being proportional to the average photon number). In the case under investigation the losses in air are rather small and could permit a much longer time before decoherence takes over.

In order to identify the macroscopic superposition we suggest to perform a homodyne measurement of the laser beam after the Kerr cell. Then the Wigner function can be reconstructed with tomographic techniques [21,22], the identification of a negative region of the Wigner function allows the identification of the cat (for a comprehensive study of quantum tomography see [23]).

More in details, one measures the probability distribution $p(x, \phi)$ for the field quadrature $x = 1/2(ae^{-i\phi} + a^\dagger e^{i\phi})$ at various phases $\phi$, then, using the Radon transform, the Wigner function is reconstructed. Experimentally, one obtains a table of $p(x, \phi)$ in terms of $x$ and $\phi$. In order not to have troubles with delicate cancellations of different elements in the summations, it is convenient to fit the $x$ dependence at different $\phi$ and then use this fit in integrals.

In the following we will study the real possibility of reconstructing the Wigner function and observing a negative region, when experimental errors are kept into account. For this purpose, we will use the reconstruction technique described in [22], appropriately deteriorating the calculated distribution $p(x, \phi)$ in order to simulate experimental errors. In practice, we introduce a random error, simulating the experimental error on each fit of $p(x, \phi)$ at fixed $\phi$ for each Montecarlo run.

We will consider different superpositions systematically in order to give a panorama of the different situations.

Probably, a good compromise between having a sufficiently high number of photons and keeping at the same time a sufficiently small decoherence time could be realised with a co-
herent state with an average number of photons about 5-10. This represents a "mesoscopic" system where intermediate properties between "classical" and quantum systems can be expected. Furthermore, one can hope to be able to study decoherence at work for such kind of systems.

Let us consider of having realised a superposition

$$|\Psi_{cat}\rangle = |\sqrt{5}e^{i\theta}\rangle + |\sqrt{5}e^{-i\theta}\rangle$$  \hspace{1cm} (6)

We begin with the case $\theta = \pi/2$. The Wigner function $W[Re(\alpha), Im(\alpha)]$ (plotted in fig.2) has an absolute minimum for $W[0.3346,0] = -3.16$. A tomographic reconstruction based on ten phase points for $p(x,\phi)$ (this function is shown for the sake of exemplification in fig.3) is sufficient for reproducing this value with an uncertainty less than 3%. We have then considered the situation where the distribution $p(x,\phi)$ is deteriorated simulating an experimental error around $\pm 25\%$, the result of an average on ten points calculated by a Montecarlo simulation is $W[0.3346,0] = -3.08 \pm 0.29$: then even with such large errors bars we are able to reconstruct the minimum negative value with only a 10\% uncertainty. For the sake of completeness, we have also repeated the evaluation with a simulated error of 50 \%, the result is $W[0.3346,0] = -2.84 \pm 0.84$. Even in this case the negative part of the Wigner function would be recognisable (with a 30 \% relative error).

Then we have investigated the case $\theta = 63^\circ$. The Wigner function $W[Re(\alpha), Im(\alpha)]$ (plotted in fig.4) has an absolute minimum for $W[0.8954,0] = -3.916$. Also in this case a tomographic reconstruction based on ten phase points for $p(x,\phi)$ is sufficient for reproducing this value with an uncertainty less than 3\%, albeit a larger calculation time is needed. Again, we have considered the situation where the distribution $p(x,\phi)$ is deteriorated simulating an experimental error around $\pm 25\%$, the result of the Montecarlo simulation (averaging 10 points) is $W[0.8954,0] = -3.83 \pm 0.48$: we are thus able to reconstruct the minimum negative value with only a 12.5\% uncertainty. For the sake of completeness, we have also
investigated the tomographic reconstruction of the local minimum $W[0.157,0] = -0.890$. An average of ten Montecarlo simulated points gives $W[0.157,0] = -0.910 \pm 0.193$, with a relative error of 21.3%.

When the relative phase between the two coherent states is further reduced the tomographic reconstruction of the Wigner function requires a very large calculation time; however, up to a phase difference of 0.1 radiants the negative region of the Wigner function can still be identified (e.g. for $\theta = 0.2$ radiants the minimum is $W[2.687,0] = -0.679$).

Finally, we have also investigated the case where the average number of photons is 10: a part the need of a longer calculation time, the same situation as the previous case is found.

In summary, the results of our numerical simulation show that the tomographic reconstruction of the Wigner function is such that even with 25%, or larger, errors on the homodyne reconstruction of $p(x,\phi)$ the cat can be easily identified. We think thus that this technique can be an interesting tool for recognising an optical Schrödinger cat realised as superposition of two coherent state with different phase.

In conclusion, we have shown that an optical Schrödinger cat (superposition of two coherent states) can be generated using a Mach-Zender interferometer crossed by a one photon state, where on one of the paths of the interferometer a Kerr cell is inserted. A laser crosses the cell as well and the coherent states superposition is generated by a conditional measurement on the one photon state. The produced state is robust against decoherence.

We have also discussed the experimental possibility of recognising the cat, once it has been created, using a tomographic technique for reconstructing its Wigner function. Our numerical simulations show that this reconstruction is such that the cat can be recognised even if rather large experimental errors affect the homodyne detection.

Altogether, our results show that the proposed scheme for generating and detecting an optical superposition of macroscopic states can be efficient, also thanks to the appearance
of new materials with very high Kerr couplings, as Bose condensate. We think that this scheme deserves attention for an experimental realisation of it.

Acknowledgements

We would like to acknowledge support of ASI under contract LONO 500172, of MURST via special programs ”giovani ricercatori” Dip. Fisica Teorica Univ. Torino and of Istituto Nazionale di Fisica Nucleare.

References


24. see e.g. R. L. Sutherland, ”Handbook of nonlinear optics”, ed. M. Dekker 1996.
**Figure captions**

fig.1) Scheme of the proposed experiment. The signal photon enters gate 1 of the polarising beam splitter PBS. A Kerr cell (K) is on one arm of the interferometer. A probe laser crosses the cell. The signal beam is measured by the photo-detectors D1 and D2 at the out gates of the interferometer. Finally a homodyne detection is performed on the laser after exiting the Kerr cell.

fig.2) Contour plot of the Wigner function $W[Re(\alpha), Im(\alpha)]$ of the state $|\sqrt{5}e^{i\pi/2}\rangle + |\sqrt{5}e^{-i\pi/2}\rangle$.

fig.3) Three dimensional plot of $p(x, \phi)$ for the state $|\sqrt{5}e^{i\pi/2}\rangle + |\sqrt{5}e^{-i\pi/2}\rangle$. The x-axis is the quadrature variable $x$, on the y-axis appear the points $n$, where $n$ varies between 0 and 10 and is related to $\phi$ through $\phi = n \times (\pi/2)/10$.

fig.4) Contour plot of the Wigner function $W[Re(\alpha), Im(\alpha)]$ of the state $|\sqrt{5}e^{i63^\circ}\rangle + |\sqrt{5}e^{-i63^\circ}\rangle$. 