Parity nonconservation in the deuteron photodisintegration

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Abstract

We calculate P-odd difference of the total cross-sections of the deuteron disintegration by left and right polarized photons. The relative magnitude of this difference varies in the threshold region from $10^{-7}$ to $10^{-8}$. Its experimental measurement would allow one to determine the value of the P-odd $\pi NN$ constant. This measurement would give also valuable information on the weak nucleon-nucleon interaction at short distances.
1 Introduction

The deuteron, being the simplest nuclear system, in many cases allows for a relatively reliable theoretical analysis. That is why the problem of parity nonconservation (PNC) in the deuteron for a long time attracts attention of both experimentalists and theorists. Unfortunately, PNC effects in the deuteron are tiny, so that up to now only upper limits on them have been obtained experimentally [1–3].

At present, however, new prospects have arisen here due to creating intense sources of polarized photons, electrons, and neutrons. On the other hand, now the experimental investigations of PNC effects in the deuteron have become of great interest. One may hope that they will resolve a contradiction which exists at present in the problem of P-odd nuclear forces. The point is that recently the nuclear anapole moment (AM) of $^{133}$Cs was discovered and measured with good accuracy in atomic experiment [4]. The result of this experiment is in a reasonable quantitative agreement with the theoretical predictions, starting with [5, 6], if the so-called “best values” [7] are chosen for the parameters of P-odd nuclear forces. However, the results of some nuclear experiments indicate that the P-odd $\pi$NN constant $\tilde{g}$ is much smaller than its “best value” (see, e.g., [8, 9]).

Since the deuteron is a relatively simple and loosely bound nucleus, some parity-nonconserving (PNC) effects in it yield to a reliable quantitative analysis. In particular, there is a range of photon energies where P-odd asymmetry in the deuteron disintegration by circularly polarized $\gamma$-quanta is dominated by the $\pi$-meson exchange, pion being the lightest possible mediator of the nucleon-nucleon weak interaction. This dominance takes place if the P-odd $\pi$NN constant $\tilde{g}$ is on the order of its “best value”. Our experience with calculating the deuteron anapole moment [10] (the deuteron AM is also dominated by the weak pion exchange), gives us good reasons to believe that the accuracy of our present calculation of the pion contribution to P-odd effects in the deuteron disintegration, can be estimated as 20% (for given value of the P-odd $\pi$NN constant), as well as in the case of the deuteron AM.

We wish to note here that while using the weak pion exchange potential for the description of the long-range P-odd interaction is reasonably legitimate, the situation with more short-range P-odd effects is quite different. The latter are commonly described by means of $\rho$- and $\omega$-exchanges. However, the range of these potentials, $1/m_{\rho,\omega} \sim 0.3$ fm, is much smaller than the proton mean-square radius, $\langle r_p^2 \rangle^{1/2} \sim 0.8$ fm. Therefore, all calculations of P-odd effects based on weak $\rho$, $\omega$-potentials (as well as using $\rho$, $\omega$-potentials for the description of strong interactions) have no sound theoretical grounds. Of course, the corresponding part of our calculations is no exception in this respect. Thus, our result for the P-odd asymmetry in the energy region very close to the threshold (the pion exchange does not dominate there) is more close to an estimate than to a real calculation.

Theoretical studies of PNC effects in the deuteron were started in [11–14]. Papers [11, 12] concentrated on the P-odd asymmetry in $d(\gamma, n)p$ reaction caused by linearly polarized photons. For the photon energies of several MeV this asymmetry is very small as compared to that due to circularly polarized $\gamma$-quanta, the latter being of interest to us. A phenomenological treatment of PNC effects in the deuteron was adopted in [13]. Later it was supplemented in [14] with quantitative estimates made in the dispersion approach and in the pion exchange model.
PNC effects in ed-scattering were considered in [15], but for a very special kinematics only. The general problem of parity nonconservation in ed-scattering was investigated in [16-19], with detailed numerical estimates in [16, 18]. However, in this process the effect of the nuclear parity violation is masked by the direct P-odd ed interaction due to weak neutral currents.

After [13, 14], P-odd effects in the deuteron disintegration by circularly polarized γ-quanta (and in the inverse reaction) were addressed in [16,20-24]. Though in the present paper we consider relatively wide range of the photon energies, our main interest refers to the region somewhat above the threshold where the P-odd interaction is dominated by the pion exchange. As distinct from [14,16,20-24], we treat the pion exchange contribution analytically and give a reliable (in our opinion) estimate of the accuracy of this our calculation. Though our results are in a qualitative agreement with most of previous ones (see below), we believe that the present independent investigation of the important and interesting problem is worth efforts.

2 Wave functions, transition matrix elements, and cross-sections

The deuteron ground state is $^3S_1$ (a small $^3D_1$ admixture to it will be neglected throughout the paper). In the zero-range approximation (ZRA) its wave function is

$$
\psi_d = \sqrt{\frac{\kappa}{2\pi}} \frac{e^{-\kappa r}}{r}.
$$

(1)

Here $\kappa = \sqrt{m^p\varepsilon}$, where $m^p$ is the proton mass, and $\varepsilon = 2.23$ MeV is the deuteron binding energy. To the same approximation, the $^3S_1$ and $^1S_0$ wave functions of the continuous spectrum, $\psi_{St}$ and $\psi_{Ss}$, respectively, are (see, for instance, [19])

$$
\psi_{St,Ss} = \frac{\sin pr}{pr} - a_{t,s} \frac{e^{ipr}}{r}, \quad a_{t,s} = \frac{\alpha_{t,s}}{1 + ip\alpha_{t,s}}.
$$

(2)

Here $\alpha_t = 5.42$ fm and $\alpha_s = -23.7$ fm are the triplet and singlet scattering lengths, respectively. At last, in the spirit of the zero-range approximation, for the $P$ state of the continuum we will use the free wave function

$$
\frac{1}{p} \frac{d}{dr} \frac{\sin pr}{pr}.
$$

Near the threshold the photodisintegration cross-section is dominated by $M1$ transition. Since the radial wave function of the deuteron is orthogonal to that of the $^3S_1$ state of the continuous spectrum, the $M1$ transition goes into the $^1S_0$ state. The expressions for the $M1$ matrix element and total cross-section, as calculated in the same zero-range approximation, are well-known (see, for instance, [25]):

$$
\langle ^1S_0 | (\mu_p \sigma_p + \mu_n \sigma_n) | ^3S_1 \rangle = \frac{2\sqrt{2\pi \kappa} (\mu_p - \mu_n) (1 - \alpha_s/\alpha_t)}{(\kappa^2 + p^2)(1 - ip\alpha_s)};
$$

(3)

$$
\sigma_{M1} = \frac{2\pi \alpha}{3} \frac{\kappa p (\mu_p - \mu_n)^2(1 - \alpha_s/\alpha_t)^2}{m^p_\pi (\kappa^2 + p^2)(1 + p^2\alpha_s^2)}.
$$

(4)

Here $p$ is the relative momentum of final nucleons, $\alpha = 1/137$, $\mu_p$ and $\mu_n$ are the proton and neutron magnetic moments, respectively.
Figure 1: Cross-sections. a) \( \sigma_{M1} \). b) \( \sigma_{E1} \). c) \( \sigma_{tot} = \sigma_{M1} + \sigma_{E1} \).

For higher energies the cross-section is dominated by \( E1 \) transitions into the \(^3P_{0,1,2}\) states. The corresponding matrix element and total cross-section are, respectively:

\[
\langle ^3P | r | ^3S \rangle = -4 \sqrt{\frac{2\pi \kappa}{1 - \kappa r_1}} \frac{P}{(\kappa^2 + p^2)^2};
\]

\[
\sigma_{E1} = \frac{8\pi \alpha}{3} \frac{\kappa p^3}{(1 - \kappa r_1)(\kappa^2 + p^2)^3}.
\]

The obtained cross-sections are presented in Fig. 1. The origin of the factors \((1 - \kappa r_1)^{-1/2}\) and \((1 - \kappa r_1)^{-1}\) in (5) and (6), respectively, is as follows. Large distances dominate in the matrix element \(\langle ^3P | r | ^3S \rangle\). In this asymptotic region the naive ZRA expression (1) for the deuteron wave function must be augmented by a correction factor \((1 - \kappa r_1)^{-1/2}\) (see [25, 26]), which obviously results in factor \((1 - \kappa r_1)^{-1}\) in \(\sigma_{E1}\). Here \(r_1 = 1.76(1)\) fm is the effective radius of the triplet state.

On the other hand, wave functions (1) and (2) have incorrect behaviour for \(r \to 0\). But it is of no importance for the above formulae, since short distances are inessential for \(\sigma_{E1}\), and even for \(\sigma_{M1}\). In general, the situation is different for matrix elements of the P-odd weak interaction (see below). The corresponding operators are singular at \(r \to 0\), so in this case it is dangerous to use blindly the naive wave functions (1) and (2). Here we will use model wave functions with somewhat more realistic properties. In particular, for the deuteron it is

\[
\psi_d = \sqrt{\frac{\kappa}{2 \pi (1 - \kappa r_1)}} e^{-\kappa r_1} \frac{e^{-\kappa r}}{r_1}, \quad r < r_1;
\]

\[
\psi_d = \sqrt{\frac{\kappa}{2 \pi (1 - \kappa r_1)}} e^{-\kappa r} \frac{e^{-\kappa r}}{r}, \quad r > r_1.
\]
This wave function has the correct asymptotics at \( r \to \infty \) (see above), tends to a constant at \( r \to 0 \), and at least is continuous everywhere. The numerical value \( r_1 = 1.60 \) fm is chosen in such a way that the wave function is normalized correctly. In fact, it is quite natural that this value is close to that of the triplet effective radius \( r_t \). Let us note that the unphysical cusp of \( \psi_d \) at \( r = r_1 \) is harmless for our problem since this wave function will enter integrands only (see formulae below).

As to the \(^1S_0\) wave of the continuous spectrum, the potential for the singlet state is rather shallow, and the effective radius \( r_s = 2.73(3) \) fm is larger than the triplet one. Thus, the variation of the wave function in the internal region is even more important here. We choose 

\[
\psi_{Ss} = A \frac{\sin \sqrt{p^2 + p_0^2} r}{\sqrt{p^2 + p_0^2} r}, \quad r < r_s; \\
\psi_{Ss} = \frac{\sin pr}{pr} - a_s e^{ipr} r, \quad r > r_s.
\]  

(8)

Requiring the continuity of the wave function and its first derivative at \( r = r_s \), we obtain

\[
p_0 r_s = 1.5; \\
A(p) = \frac{\sqrt{p^2 + p_0^2} r_s}{\sin \sqrt{p^2 + p_0^2} r_s} \frac{\sin pr - p0 \cos pr}{pr(1 + ip0)}.
\]

### 3 P-odd pion exchange

The Lagrangians of the strong \( \pi NN \) interaction and of the weak P-odd one, \( L_s \) and \( L_w \), respectively, are well-known:

\[
L_s = g \left[ \sqrt{2} (\pi i \gamma_5 n \pi^+ + \pi i \gamma_5 p \pi^-) + (\pi i \gamma_5 p - \pi i \gamma_5 n) \pi^0) \right]; \tag{9}
\]
\[
L_w = \bar{g} \sqrt{2} \, i (\bar{n} \pi^+ - \bar{p} \pi^-). \tag{10}
\]

Our convention for \( \gamma_5 \) is

\[
\gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}; \tag{11}
\]

the relation between our P-odd \( \pi NN \) constant \( \bar{g} \) and the commonly used one \( h_{\pi NN}^{(1)} \) is \( \bar{g} \sqrt{2} = h_{\pi NN}^{(1)} \). Our sign convention for the coupling constants is standard: \( g = 13.45 \), and \( \bar{g} > 0 \) for the range of values discussed in [7].

The resulting effective nonrelativistic Hamiltonian of the P-odd nucleon-nucleon interaction in the deuteron due to the pion exchange is in the coordinate representation (see, e.g., [10])

\[
V(\mathbf{r}) = \frac{g \bar{g}}{2\pi m_p} (-i \mathbf{I} \cdot \nabla) \frac{e^{-m_p r}}{r}; \tag{12}
\]

here \( \mathbf{r} = \mathbf{r}_p - \mathbf{r}_n \) is the relative coordinate. This P-odd interaction conserves the total spin

\[
\mathbf{I} = \frac{1}{2} (\sigma_p + \sigma_n)
\]

(and does not conserve the isotopic spin). Thus, in our problem it mixes only the \(^3P_1\) state of the continuous spectrum and the deuteron ground state \(^3S_1\). The corresponding imaginary
mixing matrix elements are related as follows: $\langle 3P_1 | V | 3S_1 \rangle = -\langle 3S_1 | V | 3P_1 \rangle$, the overall sign in (12) corresponds to $\langle 3P_1 | V | 3S_1 \rangle$.

It was mentioned already that $M1$ transition from the ground state proceeds only to the $^1S_0$ state of the continuous spectrum. Then, it can be easily seen that the P-odd pion exchange, which conserves the total spin $I$, operates in our problem as follows. In the regular $E1$ transition from the ground state $^3S_1$ into $^3P_{0,1,2}$, it admixes $^3P_1$ state of the continuous spectrum to the initial one, and $^3S_1$ state to the final one. To contribute to the admixed P-odd $M1$ amplitude, this last admixed $^3S_1$ state should be the ground state of the deuteron (recall the mentioned orthogonality of the $^3S_1$ radial wave functions of different energies).

We calculate the P-odd asymmetry due to the P-odd pion exchange with the ZRA wave functions. As previously, we augment the $E1$ amplitude here with a factor $(1 - \kappa r_t)^{-1/2}$. The ZRA $^3P_1$ wave function is conveniently presented as

$$\sqrt{\frac{3}{2}} (\text{In}) \frac{1}{p} \frac{d}{dr} \frac{\sin pr}{pr}, \quad \text{n} = \frac{\text{r}}{r}. \quad \text{(14)}$$

Then, one should recall that the regular $E1$ transition goes into all nine $^3P_{0,1,2}$ states, while the P-odd admixture is operative for three $^3P_1$ states only. The calculation is straightforward, though rather tedious. The result for this contribution to the asymmetry

$$A = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \quad \text{(13)}$$

is

$$A_\pi = -\frac{g\bar{g}}{2\pi} \frac{m_\pi}{m_p} \left( \mu_p + \mu_n + \frac{1}{6} \right) \sqrt{1 - \kappa r_t} \frac{p^2 + \kappa^2}{m_\pi p} f(p), \quad \text{(14)}$$

where

$$f(p) = \frac{1}{2} \arctg \frac{p}{m_\pi + \kappa} + \frac{m_\pi - \kappa}{2p} \left( 1 - \frac{m_\pi + \kappa}{\arctg \frac{p}{m_\pi + \kappa}} \right).$$

The dependence of $A_\pi$ on the photon energy is plotted in Fig. 2a. Here we assume for $\bar{g}$ its “best value” $3.3 \times 10^{-7}$. Under this assumption, the P-odd pion exchange dominates the asymmetry (13) starting from the photon energy $E_\gamma \simeq 3.2$ MeV. In this region we can neglect the $M1$ contribution to the denominator $\sigma_+ + \sigma_-$. 

Let us mention that the weak interaction (12) generates a contact current $j^c$. However, this current is directed along $r$ [10], and therefore does not contribute to the admixed $M1$ amplitude and to the asymmetry (14). This can be easily seen in the gauge where $A = 1/2 |B \times r|$: here the contact current interaction obviously vanishes, $-j^c A = 0$, if $j^c$ is parallel to $r$.

Numerical calculations with a more realistic ground state wave function (7) indicate that the error in the value (14) of the asymmetry does not exceed 20% (of course, for a given value of the P-odd $\pi NN$ constant $\bar{g}$). One can hardly expect that by itself the potential approach to the problem has better accuracy. This is why we confine here to an analytic calculation.

## 4 Contribution of the short-range P-odd interaction

Here we consider the close vicinity of the threshold where the deuteron disintegration is dominated by the regular $M1$ transition $^3S_1 \rightarrow ^1S_0$. In the admixed $E1$ transition the total spin is conserved. Therefore here we need the P-odd weak interaction which does not conserve the total spin (and conserves the isotopic spin). This interaction admixes $^1P_1$ state to the initial
Figure 2: Contributions to the asymmetry.  
  a) Pion exchange.  
  b) Short-distance effect, regular $M1$ transition.  
  c) Short-distance effect, regular $E1$ transition.
one $^3S_1$ and $^3P_0$ to the final state $^1S_0$. This interaction is of a short-range nature, and we will use its common description by a potential, corresponding to the $\rho$, $\omega$-exchange. Due to the shortcomings of this description, pointed out in Introduction, quantitative results obtained here should be considered as detailed estimates only. The short-range P-odd potential is [27]

$$W = -g_\rho \left[ h^0_\rho \tau_1 \tau_2 + \frac{1}{2} h^1_\rho (\tau^+_1 + \tau^+_2) + \frac{1}{2\sqrt{6}} h^2_\rho (3\tau^+_1 \tau^+_2 - \tau_1 \tau_2) \right]$$

$$\times \frac{1}{2m_\rho} (\{\sigma_1 - \sigma_2\} \{p_1 - p_2, f_\rho(r)\} + 2(1 + \chi_\rho [\sigma_1 \times \sigma_2] \nabla f_\rho(r)))$$

$$- g_\omega \left[ h^0_\omega \tau_1 \tau_2 + \frac{1}{2} h^1_\omega (\tau^+_1 + \tau^+_2) \right]$$

$$\times \frac{1}{2m_\rho} (\{\sigma_1 - \sigma_2\} \{p_1 - p_2, f_\omega(r)\} + 2(1 + \chi_\omega [\sigma_1 \times \sigma_2] \nabla f_\omega(r)))$$

$$- \frac{1}{2} (\tau^+_1 - \tau^+_2) (\sigma_1 + \sigma_2) \frac{1}{2m_\rho} \left\{ p_1 - p_2, g_\omega h^1_\omega f_\omega(r) - g_\rho h^1_\rho f_\rho(r) \right\}.$$  

Here

$$f_{\rho,\omega}(r) = \frac{e^{-m_{\rho,\omega} r}}{4\pi r}, \quad r = |r_1 - r_2|.$$ 

The numerical values of the parameters entering this expression are presented in Table 1. The values of the constants corresponding to strong vertices, $g_{\rho,\omega}, \chi_{\rho,\omega}$, are reasonably reliable. For the P-odd constants, $h^{0,1,2}_\rho, h^{0,1}_\omega$, we use the “best values” [7]. In particular, the vanishingly small values of the constants $h^1_\rho, h^1_\omega$ (together with the suppression of the $\rho$, $\omega$-exchanges as compared to the pion exchange in the weakly bound deuteron) allow us to disregard at all the last term in (15), which conserves the total spin.

Let us start with the correction to the deuteron wave function $\psi_d$, using the common stationary perturbation theory. In the ZRA approximation the admixed $P$ states of the continuous spectrum are free. Moreover, we can choose plane waves as the intermediate states since the perturbation (15) selects by itself the $P$ state from the plane wave. Thus obtained correction can be written as

$$\delta \psi_d(r) = \int \frac{d \mathbf{k}}{(2\pi)^3} \frac{e^{i\mathbf{k} \cdot \mathbf{r}}}{-\varepsilon - k^2/m_\rho} \int d \mathbf{r}' e^{-i\mathbf{k} \cdot \mathbf{r}'} W(r') \psi_d(r') \chi_t$$

$$= - \frac{m_\rho}{4\pi} \int d \mathbf{r}' e^{-\kappa |r - r'|} W(r') \psi_d(r') \chi_t$$

$$= \frac{m_\rho}{4\pi} \nabla \frac{e^{-\kappa r}}{r} \int d \mathbf{r}' W(r') \psi_d(r') \chi_t,$$  

Table 1: Numerical values of the constants in potential (15)
the last transformation being possible due to the short-range nature of \( W(\mathbf{r}) \) \((\kappa \ll m_{\rho,\omega})\). Here \( \chi_t \) is the triplet spin wave function of the deuteron (previously we omitted it for brevity).

Simple algebra transforms this expression into

\[
\delta \psi_d = -i\lambda_t (\Sigma \nabla) \sqrt{\frac{\kappa}{2\pi}} e^{-\kappa r} \chi_t,
\]

where \( \Sigma = \sigma_p - \sigma_n \). Though this P-odd admixture of the \( ^1P_1 \) state to \( ^3S_1 \) is expressed conveniently through the ZRA wave function, the constant \( \lambda_t \) introduced in (17), depends in fact on the true deuteron wave function \( \psi_d \) as follows:

\[
\sqrt{\frac{\kappa}{2\pi}} \lambda_t \chi_t = -i \frac{m_p}{16\pi} \int \! d\mathbf{r}' (\Sigma \mathbf{r}') W(\mathbf{r}') \psi_d(\mathbf{r}') \chi_t.
\]

In the same way we calculate the P-odd \( ^3P_0 \) admixture to the wave function of the \( ^1S_0 \) state of continuous spectrum:

\[
\delta \psi_{Ss} = i\lambda_s a_s (\Sigma \nabla) \frac{e^{ipr}}{r} \chi_s.
\]

Here \( \chi_s \) is the singlet spin wave function, and the constant \( \lambda_s \) is expressed via the wave function \( \psi_{Ss} \) of the \( ^1S_0 \) state (see, e.g., (2) or (8)) as follows:

\[
\lambda_s \chi_s = -i \frac{m_p}{48\pi} \int \! d\mathbf{r}' (\Sigma \mathbf{r}') W(\mathbf{r}') \psi_{Ss}(\mathbf{r}') \chi_t.
\]

We will need also the P-odd \( ^1S_0 \) admixture to the \( ^3P_0 \) state of continuous spectrum. It can be easily found from the requirement that the perturbed \( ^1S_0 \) and \( ^3P_0 \) wave functions should remain orthogonal. We obtain

\[
\delta \psi_p = -2i\lambda_s a_p \frac{e^{ipr}}{r} \chi_s.
\]

A general formula comprising all three cases, (17), (18), (19), was given previously in [13]. Straightforward calculations with wave functions (17) and (18) give

\[
\sigma_+ - \sigma_- = -\frac{16\pi \alpha}{9} \frac{\kappa \rho (\rho_\mu - \rho_n)(1 - \alpha_s/\alpha_t)(3\kappa^2 + p^2)}{m_\rho (\kappa^2 + p^2)^2(1 + p^2\alpha_s^2)} \times \left[ \lambda_s \frac{\alpha_s}{\alpha_t} \frac{\kappa^2 + 3p^2}{3\kappa^2 + p^2} + \lambda_t \left( 1 - \frac{\alpha_s}{\alpha_t} \frac{2\kappa^2}{3\kappa^2 + p^2} \right) \right].
\]

An analogous formula for the deuteron disintegration by longitudinally polarized electrons was derived in [19].

Wave functions (17) and (19) induce a P-odd asymmetry in one more way. Here the regular amplitude is \( E1: \ ^3S_1 \rightarrow \ ^3P_{0,1,2} \), as it was the case with the pion exchange, and the admixed amplitudes are \( M1: \ ^1P_1 \rightarrow \ ^3P_{0,1,2} \) and \( ^3S_1 \rightarrow \ ^1S_0 \). This contribution to the P-odd cross-section difference is

\[
\sigma_+ - \sigma_- = -\frac{32\pi \alpha}{9} \frac{\kappa \rho^3 (\rho_\mu - \rho_n)}{m_\rho (\kappa^2 + p^2)^2\sqrt{1 - \kappa \rho_t}} \left[ \lambda_s \frac{\alpha_s}{\alpha_t} \frac{1 + p^2\alpha_s \alpha_t}{1 + p^2\alpha_s^2} - \lambda_t \right].
\]

While the above derivation of expressions (20), (21) is a relatively simple procedure, the problem of calculating the constants \( \lambda_s \) and \( \lambda_t \) is quite different. In the ZRA the calculation of these constants is rather straightforward and results in

\[
\lambda_s^{(ZRA)} = (0.153h_\rho^0 - 0.124h_\rho^2 + 0.229h_\omega^0) \times 10^{-7} m_\pi^{-1} = -1.09 \times 10^{-7} m_\pi^{-1},
\]
\[
\lambda_t^{(zra)} = (0.281 h_\rho^0 + 0.116 h_\omega^0) \times 10^{-7} m^{-1} = -4.52 \times 10^{-7} m^{-1}.
\]

However, these naïve ZRA numbers for the effective constants \(\lambda_s,t\) certainly strongly overestimate true values of these constants. The first reason is that the ZRA wave functions of the S-states are singular at \(r \to 0\), while their correct wave functions are finite at the origin. This effect is much more essential here than for the pion exchange since \(m_\rho,\omega \gg m_\pi\) and, correspondingly, the range of the vector exchanges is much shorter. Therefore, here we will use instead of naïve ZRA wave functions (1) and (2), model functions (7) and (8) which are finite at the origin.

By the same reason of the short-range nature of vector exchanges, one more suppression factor is essential here (though it can be neglected for the pion exchange within the accuracy of 20% expected there). We mean the Jastrow repulsion between nucleons at small distances. Following [28-30], we will take it into account by a factor \(\phi^2(r)\) in the weak matrix elements, where

\[
\phi(r) = 1 - ce^{-dr^2}, \quad c = 0.6, \quad d = 3 \text{ fm}^{-2}.
\]

With these modifications, we arrive at more realistic (and much smaller!) estimates for the constants \(\lambda_s\) and \(\lambda_t\):

\[
\lambda_s = (0.028 h_\rho^0 - 0.023 h_\rho^2 + 0.028 h_\omega^0) \times 10^{-7} m^{-1} = -0.16 \times 10^{-7} m^{-1},
\]

\[
\lambda_t = (0.032 h_\rho^0 + 0.001 h_\omega^0) \times 10^{-7} m^{-1} = -0.37 \times 10^{-7} m^{-1}.
\]

The contributions to the P-odd asymmetry of the deuteron photodisintegration due to the cross-section differences (20) and (21), as calculated with the constants (24), are plotted in Figs. 2b,c, respectively (we have chosen different vertical scales in Figs. 2a,b,c to be able to reproduce details of the effects differing in order of magnitude). Let us mention that the energy dependence of the second of these contributions resembles that of the P-odd pion exchange (compare Figs. 2a and 2c). This is quite natural since both of them correspond to the situation when regular transition amplitude is \(E1\), and the admixed P-odd amplitude is \(M1\).

Our final result, the total asymmetry \(A\) comprising the pion and short-distance contributions, is plotted in Fig. 3 (the energy dependence of \(A\) in the threshold region see in Fig. 2b).

5 Discussion

We consider the reliable calculation of the asymmetry \(A\) for relatively large photon energies as the main result of our work, and therefore start the discussion from the right wing of the curve in Fig. 3. Here \(A\) is dominated by the interference between the regular \(E1\) and admixed \(M1\) amplitudes, and is due mainly to the P-odd pion exchange. The asymmetry is negative in this region with a typical value of \(A\) approaching \(-10^{-8}\). As mentioned already, numerical checks have demonstrated that the error of the analytical formula (14) for this contribution to \(A\) does not exceed 20%. An extra argument in favor of this estimate for the accuracy is good agreement, on the level of 10%, of formula (14) with the corresponding numerical results obtained previously in [24]. However, the short-distance P-odd interaction also contributes (with the same sign) to the interference between the regular \(E1\) amplitude and the admixed \(M1\) amplitude. Though this short-distance contribution is smaller than that of the pion exchange, at least perhaps by a factor of 3, it is known with much worse accuracy. Thus, we believe that 30% is a fair estimate for the accuracy of our prediction for the asymmetry beyond the threshold region (of course, for a given value of the P-odd \(\pi NN\) constant \(\bar{g}\)).
The P-odd asymmetry changes sign around $E_\gamma = 3.1$ MeV. Below this energy it is positive and is due mainly to the interference between the regular $M1$ amplitude and the admixed $E1$ amplitude, the latter caused by short-distance P-odd effects. The asymmetry is maximum, $\approx 10^{-7}$, at the threshold. Unfortunately, here the magnitude of this short-distance effect cannot be accurately predicted. Our result for this region is higher, by a factor of 2 to 5, than those of previous works, which also differ considerably among themselves. Different approaches and lack of details of calculations in those papers preclude elucidation of the exact origin of this disagreement. Anyway, there is no reliable theoretical treatment of the short-distance contribution, and all the results here are in fact estimates only. However, at least in the case of our discrepancy with [16] there is a plausible explanation for it. The cut-off adopted in [16] for the description of the short-range nucleon-nucleon repulsion (the short-range correlation factor therein turns to zero for $r < r_c$, $r_c = 0.43$ fm or 0.56 fm) is much more steep than the cut-off adopted by us (see (23)). Of course, nothing can be proven here rigorously, still there is an observation indicating that with our procedure the magnitude of nuclear P-odd effects is not overestimated. This procedure was used previously in [6] to derive the constant of the effective P-odd contact interaction of the valence nucleon with the nuclear core. Thus calculated value of the anapole moment of $^{133}$Cs in [6] is close to (in fact, even somewhat lower than) its experimental value obtained recently in [4].

Our last remark refers to the relation between the P-odd asymmetry $A$ and the degree of circular polarization $P$ of $\gamma$-quanta in the inverse reaction $np \rightarrow d\gamma$:

$$P = \frac{\tilde{\sigma}_+ - \tilde{\sigma}_-}{\tilde{\sigma}_+ + \tilde{\sigma}_-},$$

where $\tilde{\sigma}_\lambda$ is the production cross-section for a photon with circular polarization $\lambda(= \pm)$. In virtue of the principle of detailed balancing (which is valid here since the interactions considered
are T-even),

\[ A = P. \]  \tag{25}

If our threshold value for \( A \) is correct, i.e., if indeed

\[ A \approx 1.0 \times 10^{-7}, \]

then, according to (25), the experimental upper limit for \( P \) obtained in [1],

\[ P = (1.8 \pm 1.8) \times 10^{-7}, \]

is close to the real effect.

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