Soft SUSY Breaking, Dilaton Domination and Intermediate Scale String Models

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ABSTRACT: We present an analysis of the low-energy implications of an intermediate scale (∼ 10^{11} GeV) string theory. We mainly focus on the evolution of the physical parameters under the renormalisation group equations (RGEs) and find several interesting new features that differ from the standard GUT scale or Planck scale scenarios. We give a general discussion of soft supersymmetry breaking terms in type I theories and then investigate the renormalization group running. In the dilaton domination scenario, we present the sparticle spectra, analyzing constraints from charge and colour breaking, fine tuning and radiative electroweak symmetry breaking. We compare with the allowed regions of parameter space when the RGEs start running at the standard GUT or the intermediate scales, and find quite remarkably that the dilaton dominated supersymmetry breaking scenario, which is essentially ruled out from constraints on charge and colour breaking if the fundamental scale is close to the Planck mass, is allowed in a large region of parameter space if the fundamental scale is intermediate.

KEYWORDS: Supersymmetry Breaking, Beyond Standard Model, Supersymmetric Models.
1. Introduction

Recently we have seen a radical change in our understanding of the possible physical implications of a fundamental theory. If such a theory includes higher dimensional objects, such as D-branes, then the fundamental scale of the theory can be very small compared to the Planck scale [1, 2, 3, 4]. This is because if some or all of the observable matter and interactions is confined to a brane (our universe) which is embedded in a higher dimensional bulk world, then the fundamental scale becomes essentially a free parameter.

One could say that part of the recent progress in string theory is the realization that the fundamental scale of the theory is actually completely unknown, whereas it was previously thought that it had to be near the Planck scale if strings were to describe gravity. This new freedom arises mainly because the size of the extra dimensions is not fixed within the theory, a manifestation of the vacuum degeneracy problem in conjunction with the world as a brane scenario.

This ignorance has been turned into a virtue since now we may use physical arguments to motivate possible values of the fundamental scale. For instance we may entertain the idea that the fundamental scale can be as low as the current experimental limits allow, ie 1 TeV [2, 3, 4]. Two other, less radical proposals have been put forward as well. The first assumes that the size of the extra dimensions is such that the fundamental scale coincides with the GUT scale [1], automatically solving the problem of gauge coupling unification in string theory, which is so far the main indirect experimental information we have about physics at higher energies. The second proposal sets the string scale to the intermediate value $M_s \sim 10^{10-12}$ GeV [5, 6]. This choice is motivated by several indicators, most notably the scale of supersymmetry breaking in hidden sector or gravity mediated scenarios.

Lowering the string scale may have spectacular implications at low energies, many of which are currently being explored in great detail. The TeV scale scenario is clearly the most studied since it is closer to the experimental limits and has direct implications for LHC physics. Here we will consider in detail the intermediate scale scenario, and will show that it too can have important low-energy implications. First we will review the motivations for introducing such a scale, then we will study its phenomenological implications mainly in the context of type I string models although similar results may
be obtained e.g. in the type IIB non-orientifold orbifold models recently constructed in [7].

The most important issue that we will address is the running of the physical parameters with the renormalization group. In the standard approach, all of the physically relevant parameters such as the gauge couplings and the soft supersymmetry breaking terms start running from a very high scale, namely the GUT or Planck scale. An enormous amount of knowledge has been accumulated about the various implications of this running, particularly the constraints on the values of the soft supersymmetry breaking parameters. Our main goal in this article is to begin to see how this analysis changes when we start the running from the intermediate scale. This simple modification may have very interesting implications for the unification of gauge couplings, $b - \tau$ unification, the possible quasi-fixed points, and so on. We will focus mainly on the running of the soft supersymmetry breaking terms in order to study a wide set of standard constraints coming from fine tuning of parameters thus achieving electroweak symmetry breaking, imposing the absence of electric charge and $SU(3)$ colour breaking etc.

Our work ought to be considered as only a starting point on the phenomenological issues of the intermediate scale scenario motivated by strings. Recently there has been some progress in the construction of phenomenologically realistic brane models with supersymmetry explicitly broken [8]. The concrete models differ in several ways from the standard MSSM scenarios. Besides having unification at the intermediate scale, the hypercharge normalization is generically also different from the standard 5/3 GUT inspired value. In the present article we will not consider explicit models but rather will try to analyze the intermediate scale scenario in general. The only departure we make from the MSSM structure is that we assume gauge coupling unification takes place at the fundamental, intermediate, scale. To achieve this unification we will consider the simplest possibility of adding several lepton pairs to accelerate the $SU(2) \times U(1)$ running and cause precocious unification [6]. Other proposals [9], such as mirage unification [10], may achieve this unification without requiring extra matter fields. We will examine both cases and show that for most phenomenological purposes there is little difference between them. We leave for a future publication the consideration of further departures from the MSSM such as the changing of $U(1)$ normalization as suggested in ref.[8].

In the next section we review the arguments in favour of an intermediate scale. Then in section 3 we discuss general issues concerning the structure of type I models. In section 4 we discuss soft supersymmetry breaking terms mainly in the context of type I strings, which allow the possibility of lowering the string scale. Assuming that SUSY-breaking is transmitted by the closed string sector of the theory, we find general expressions for the soft breaking parameters in terms of the $F$ terms of the moduli fields, the dilaton $S$, compactification size moduli $T_i$ and twisted blowing-up fields $M$. In section 5 we discuss the implications of these modifications for the physically relevant questions mentioned above.
2. The Case for the Intermediate Scale Scenario

2.1. Supersymmetry Breaking

The origin of the intermediate scale may be traced back to the early 1980’s when studies of supersymmetric models showed that the most efficient way to break supersymmetry was in a hidden sector. In those days, the preferred and simplest transmission of the information of supersymmetry breaking to the observable world was via gravitational interactions, giving rise to the ‘hidden sector’ or ‘gravity mediated’ scenarios. In these scenarios, because the observable sector only knows about supersymmetry breaking through gravitational strength interactions, the splitting among supersymmetric multiplets is of order $M_{\text{SUSY}}^2/M_{\text{Planck}}$. If supersymmetry is to solve the hierarchy problem this splitting should be close to the electroweak scale thereby fixing the scale of supersymmetry breaking to be of order

$$M_{\text{SUSY}} \sim \sqrt{M_W M_{\text{Planck}}} \sim 10^{10-12} \text{ GeV.} \quad (2.1)$$

This is the most studied supersymmetry breaking scenario of the past 18 years. An alternative is the ‘gauge mediated’ scenario in which gauge interactions rather than gravity connect the hidden and observable sectors and the supersymmetry breaking scale is therefore close to the electroweak scale. Other possibilities have been introduced more recently, a particularly interesting one being the ‘anomaly mediation’ scenario. As is the case for the gravity mediated supersymmetry breaking, these contributions to the supersymmetry breaking in the observable sector are always present.

In perturbative heterotic strings the hidden sector scenario could be nicely realized since the gauge group was $E_8 \times E_8$. The standard model particles could be charged under only one of the two $E_8$ groups with the supersymmetry breaking hidden sector being charged under the second $E_8$. Gravity would then be the messenger of supersymmetry breaking to the observable sector. However, in this framework the scale of string theory was close to the Planck scale and the intermediate scale appeared only as a low energy phenomenon, being the scale at which the hidden gauge interactions become strong. In this class of models supersymmetry was broken via some non-perturbative field theoretical effect such as gaugino condensation, whereas the string theory was only treated at the perturbative level due to the lack of understanding of non-perturbative string effects.

Non-perturbative string effects are now beginning to be understood, thanks mainly to the discovery of D-branes (surfaces on which the endpoints of open strings are attached) in type I and type II strings, as well as the Horava-Witten formulation of the heterotic string which starts from 11 dimensional supergravity compactified on the interval $S_1/\mathbb{Z}_2$, with each of the $E_8$’s living at the endpoints of the interval.

D-branes participate in supersymmetry breaking. First, being BPS configurations, they partially break supersymmetry. Second, in the compactification process they may wrap around different topologically non-trivial cycles of the compact space. Depending
on the nature of these cycles, supersymmetry may or may not be further broken. Furthermore, there has been recent activity on non-BPS brane configurations that tend to break all supersymmetries. We can therefore imagine a situation with many different brane configurations, some of them breaking supersymmetry partially or completely, with a generic vacuum being non-supersymmetric. It is then natural for the fundamental string scale to be either the intermediate scale, if we live on a supersymmetric brane and supersymmetry is only broken by other distant branes, or the TeV scale if supersymmetry is broken in our brane. We naturally expect the fundamental scale to be of the same order or smaller than the intermediate scale since, if it were much larger, we would feel the supersymmetry breaking too strongly and would be faced with the hierarchy problem. In this case the supersymmetry breaking of the string physics can also play the role of low-energy supersymmetry breaking.

The brane/anti-brane models constructed recently realize these possibilities. The TeV scale scenario can be constructed with supersymmetry being either broken explicitly on our brane or communicated by gauge mediation. However, even though there are suggestions for addressing issues such as proton stability and gauge unification in this scheme, so far there are no convincing concrete models for the TeV scenario. On the other hand it is much simpler to realize the intermediate scale scenario whilst satisfying both of these requirements since, for instance, unification can still be achieved logarithmically, and generically baryon and lepton number violating operators are much more suppressed.

2.2. Strong CP Problem

The symmetries of the Standard Model allow for a term

$$\mathcal{L}_\Theta = \frac{g^2}{32\pi^2} \bar{\Theta} F^{\mu\nu} \tilde{F}_{\mu\nu},$$

(2.2)

where $F^{\mu\nu}$ is the QCD field strength and $\bar{\Theta}$ is an arbitrary parameter that essentially reflects the non-trivial nature of the QCD vacua and the fact that the Standard Model is chiral. This term explicitly violates CP and consequently $\bar{\Theta}$ is strongly bounded by the experimental limits on the neutron electric dipole moment,

$$\bar{\Theta} < 10^{-9}.$$ (2.3)

This unnaturally small bound is the strong CP problem. The most elegant solution to it is the Peccei-Quinn mechanism in which an additional chiral symmetry, $U(1)_{PQ}$, is added to the model and broken spontaneously. The corresponding Goldstone mode of this symmetry is the axion field and the static $\bar{\Theta}$ parameter is substituted by a dynamical one, $a(x)/f_a$, where $a(x)$ is the axion field and $f_a$ is a dimensionful constant known as the axion decay constant. The non-perturbative dynamics of QCD then fixes the axion field to be at the minimum of its potential, $a = 0$, thereby solving the strong CP problem (for a review see [11]).
The axion decay constant, $f_a$, is very strongly constrained, mostly by astrophysical and cosmological bounds. Requiring that axion emission does not over-cool stars gives lower bounds on $f_a$. These have been determined for many different systems, from the sun to red giants and globular clusters. The strongest constraint comes from supernova SN1987a which requires $f_a > 10^9$ GeV [12]. Since the axion couples so weakly to normal matter its lifetime exceeds the age of the universe by many orders of magnitude, and it also has a low mass ($m_a \sim \Lambda_{QCD}^2/f_a$). Consequently the axion also has interesting cosmological implications, especially as a cold dark matter candidate. Indeed coherent oscillations around the minimum of its potential may dominate the energy density of the universe if its potential is very flat. This puts a lower bound on the axion mass which leads to an upper bound for $f_a$ of order $f_a \lesssim 10^{12}$ GeV. Combining the astrophysical and cosmological bounds gives a very narrow window for the axion decay constant:

$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV.} \quad (2.4)$$

Therefore if we want the Peccei-Quinn mechanism to work, we must explain the value of $f_a$. The allowed range is remarkably consistent with the intermediate scale, thus providing a strong argument in favour of the intermediate string scale scenario. Alternative scenarios generally have great difficulty in explaining why $f_a$ falls precisely within its narrow allowed window.

In the mid 1980’s, one of the most interesting phenomenological arguments in favour of string theory was precisely the fact that these theories always predicted axion fields. There are two kinds of string axions in perturbative heterotic strings. The first is the model independent axion coming from the imaginary component of the complex dilaton field, $S (a = \text{Im}S)$, which is always present in string theory. The second kind of axion is model dependent and is associated with the moduli $T$ fields, which measure the size and shape of the extra 6D compact space. These clearly depend on the compactification. However none of these fields seems to be the required QCD axion. The main obstacle for the perturbative heterotic string is the value of the axion decay constant, which is constrained to be close to the Planck scale. Moreover, the model dependent axions do not have the right couplings to start with and their corresponding Peccei-Quinn symmetry is not preserved by world-sheet instanton corrections. On the other hand the model independent axion couples not only to QCD but also to the hidden sector gauge fields in a universal way:

$$\mathcal{L}_{\text{axion}} = \frac{a}{M_p} \left( [F^{\mu\nu} \tilde{F}_{\mu\nu}]_{QCD} + [F^{\mu\nu} \tilde{F}_{\mu\nu}]_{\text{hidden}} \right). \quad (2.5)$$

The non perturbative dynamics of the hidden sector is then almost certainly the main source of the axion potential and the QCD contributions are essentially irrelevant. Therefore none of the axions present in the heterotic string is able to solve the strong CP problem of QCD.

Recent studies of, for example, type I strings have changed the above picture in a radical way. First, as we mentioned above, the string scale does not have to be
similar to the Planck scale but can be as low as we want. Therefore type I realizations of the intermediate scale scenario have the right axion decay constant to satisfy the astrophysical and cosmological requirements. Furthermore, in these models there are many new candidates for axion fields which also couple to $F_{\mu\nu}\tilde{F}_{\mu\nu}$. These fields are Ramond-Ramond fields $M_\alpha$ associated with the blowing-up of orbifold singularities in orientifold constructions. Since the complex gauge coupling takes the form $f_i = S + s_i^\alpha M_\alpha$, where $i$ labels the different gauge groups, different combinations of these fields with the model independent axion couple to QCD and the hidden sector groups. This evades the second problem of the string axions mentioned above.

2.3. Other Arguments

In refs.[5, 6] several other arguments were given in favour of the intermediate scale scenario. First the realization of the see-saw mechanism for neutrino masses is consistent with a fundamental intermediate scale. Also cosmologically, several models of chaotic inflation prefer the intermediate scale [13]. Finally the observed ultra-high energy cosmic rays, which have energies of order $10^{20}$ eV, could be the products of string mode decays if the fundamental scale is intermediate. These string modes are also good candidates for non-thermal dark matter known as wimpzillas [14].

Concerning the arguments against any scale below $M_{GUT}$ the two leading arguments against are firstly that the only experimental indication we have for higher energies is the apparent joining of the strong and electroweak coupling constants at $M_{GUT}$ in the MSSM, and secondly that it is difficult to obtain proton stability with models of lower fundamental scales. However, as mentioned in the introduction, explicit type I string models have recently been constructed where unification and proton stability are achieved with an intermediate fundamental scale [8].

All the arguments given in this section make intermediate models serious alternatives to the MSSM GUT scale unification scenario, and in later sections we shall see that they have other phenomenological benefits as well.

3. Structure of Type I String Models

Most discussions of string phenomenology in the past focussed on the heterotic string, since this model appeared to have the best phenomenological properties. However following the discovery of D-branes and string dualities, we have come to appreciate the richness and phenomenological qualities of type I models. Let us briefly discuss their main features of relevance for the rest of the paper.

\[1\text{Notice in this respect that in string models with a large string scale } M_s \propto M_X = 2 \times 10^{16} \text{ GeV, proton stability is in general also a problem. Even if one manages to get a model with R-parity which forbids lepton and/or baryon number violating d=4 operators, generic dim=5 operators yield still too much proton decay unless they are forbidden by additional symmetries.}\]
Type I string models can be constructed by starting with the type IIB theory and performing a so-called orientifold twist $\Omega$ corresponding to the $\mathbb{Z}_2$ identification of the two different orientations of the closed type IIB string [15]. Open strings appear as kind of twisted sectors under this operation\(^2\) and are required in order to cancel all tadpoles of the twisted theory. Compactification to four dimensions can then be achieved by a standard orbifold twist in the six extra dimensions giving rise to models with $N = 1$ supersymmetry in four dimensions. Solitonic objects of the type IIB theory correspond to extended objects known as D-branes where the endpoints of the open strings are attached. In order to preserve supersymmetry there can be only D-branes of dimensions differing by a multiple of 4. So a generic compactification has for instance 3 and 7-branes, with different gauge groups on each. $T$-duality with respect to the 3 complex extra dimensions exchanges D3 branes with D9-branes and D7-branes with D5-branes. One can accommodate e.g., the standard model group inside D3-branes and different quarks and leptons will come from the exchange of open strings in between D3-branes or between D7-branes and D3-branes.

The $N = 0$ models have a similar construction but include the additional feature of anti-branes which break supersymmetry, with some of the branes and anti-branes being required to live at the orbifold fixed points in order to cancel tadpoles [17]. In these models supersymmetry breaking in the anti-branes is transmitted to the observable branes via gravitational interactions (for which we require that the fundamental scale be the intermediate scale as explained in the previous section) or directly for which the fundamental scale has to be close to the electroweak symmetry breaking energy scale.

In these models, as in the heterotic models, there is always a dilaton field and an antisymmetric tensor field, which in four-dimensions combine to make a single chiral superfield $S$ (after appropriate dualization of the antisymmetric tensor field to a scalar). There are also moduli fields $T$ associated with the size and shape of the extra six dimensions. The explicit expression for these fields in terms of the string scale $\alpha'$, the string coupling $\lambda = e^{-\phi}$ and the orbifold compactification radii $R_i$ depends on the particular brane configuration. For 3-branes, these fields can be written as [16]

\[
S = \frac{2}{\lambda} + i\theta \\
T_i = \frac{2R^2_i R^2_k}{\lambda \alpha'^2} + i\eta_i, \quad i \neq j \neq k
\]  

(3.1)

where $\theta$ and $\eta_i$ are untwisted Ramond-Ramond closed string states (dual to antisymmetric tensors). For other brane configurations the expressions for these fields can easily be obtained by using $T$-duality for each of the three complex dimensions. These transformations can for instance switch the role of $S$ and one of the $T_i$ fields, allowing

\(^2\)The form of the orientifold operation is related to the type of Dp-branes in the model. Thus e.g. for a $\mathbb{Z}_N$ orientifold with D3-branes, it is $\Omega(-1)^{F_L} R_1 R_2 R_3$, instead of just $\Omega$ for D9-branes. Here $R_i$ are reflection operators with respect to the three complex compact planes and $F_L$ is the left-handed fermion number of the Type IIB string.
for the possibility that $T_i$ rather than $S$ plays the role of the standard model gauge coupling constant. This new degree of freedom opens up several interesting possibilities as discussed in ref. [16] of allowing different gauge groups to live in different branes and so have different gauge coupling functions. However here we will restrict the discussion to the cases where all gauge fields live on a single brane and the effect of $T$ duality just amounts to a relabelling of the fields $S$ and $T_i$.

In orientifold models there are extra fields which also come from Ramond-Ramond antisymmetric tensors and combine with the blowing-up modes into full chiral multiplets. These fields, usually denoted as $M_\alpha$, play important roles in cancelling $U(1)$ anomalies and generating Fayet-Iliopoulos terms, in contrast with the heterotic case where only the dilaton plays that role. These are precisely the fields mentioned in the previous section which provide good stringy candidates for axion fields. The holomorphic gauge function takes the general form, for a $Z_N$ orientifold model at the disk level [18, 19],

$$f_a = S + \sum_\alpha s^a_\alpha M_\alpha,$$

(3.2)

where the $s^a_\alpha$ are computable model-dependent constants. The gauge coupling is given by $Re f_a = 4\pi/g^2_a$.

Let us briefly see how in type I models the string scale can be as small as is allowed by experiment. For a configuration with the standard model spectrum belonging to a Dp-brane, the low energy action takes the form

$$S = -\frac{1}{2\pi} \int d^4x \sqrt{-g} \left( \frac{R^6 M^8}{\lambda^2} R - \frac{(RM)^{p-3}}{4\lambda} F^{2}_{\mu\nu} + \cdots \right),$$

(3.3)

where $M = 1/\sqrt{\alpha'}$ is the type I string scale, $R$ is taken as the overall size of the compact 6D space and $\lambda$ is the dilaton. Comparing the coefficient of the Einstein term with the physical Planck mass $M_{Planck}$ and the coefficient of the gauge kinetic term with the physical gauge coupling constant $\alpha_p (\sim 1/24$ at the string scale), we find the relation

$$M^{7-p} = \frac{\alpha_p}{\sqrt{2}} M_{Planck} R^{p-6}.$$  

(3.4)

From this relation we can easily see that if the Standard Model fits inside D3-branes we may have $M \sim 10^{11}$ GeV as long as the radius of the internal space is as large as $R \sim 10^{-23}$ cm [6, 16]. Therefore the required radii are large compared with the Planck length but still extremely small compared with 1mm as required in some cases with $M \sim 1$ TeV. Notice that this analysis does not work for the perturbative heterotic string since in that case the relation between $M_{Planck}$ and $M$ is independent of the size of the extra dimensions $M = \sqrt{\alpha_8/8} M_{Planck}$.

The fields $S$ and $T_i$ are very familiar from previous studies, especially for heterotic strings, whereas the fields $M_\alpha$ are less well known. Even though these fields are attached to particular fixed points (twisted moduli) they may play an interesting role on breaking supersymmetry, therefore we will include them in our discussion of soft breaking terms.
In order to do that, we need to consider the expression for the Kahler potential as a function of $S, T_i$ and $M_\alpha$. To simplify matters we will concentrate on a single overall modulus $T$ and one blowing-up mode $M$.

General arguments and explicit calculations have been used to write the Kähler potential for these fields [20]. At tree level it takes the form

$$K = -\log(S + \bar{S}) - 3\log(T + \bar{T}) + \sum_\alpha \frac{C_\alpha \bar{C}_\alpha}{T + \bar{T}} + \hat{K}(M, \bar{M}; T, \bar{T}),$$  \hfill (3.5)

where $C_\alpha$ represent the matter fields of the theory. We have left the dependence on the field $M$ general, with the understanding that $\hat{K}$ mixes the fields $T$ and $M$.

In addition there are usually anomalous $U(1)$ symmetries which induce Fayet-Iliopoulos terms of the form $\xi \sim \hat{K}_M$. These (in contrast to the heterotic string case) may consistently be zero, giving mass to the $U(1)$ gauge field but allowing the possibility of that symmetry remaining as a global symmetry [21, 18, 22].

4. Soft Supersymmetry Breaking Terms

In this section we will present a general analysis of soft supersymmetry breaking terms in a class of models which admit an intermediate fundamental scale. As we mentioned above, we will concentrate on orientifold$^3$ compactifications of type IIB strings. These models have been subject to intense investigation recently and a number of interesting results have been obtained that allow us to study the structure of soft breaking terms. They share some similarities with the better known soft terms derived from heterotic string compactifications [23] under the simple assumption of dilaton/moduli dominance of the origin of SUSY-breaking, but there are also some important differences.

These compactifications have generically three different classes of moduli-like fields. As in the perturbative heterotic string, we have the standard dilaton field $S$ and the moduli fields of which we will consider, for simplicity, only an overall modulus $T$ that measures the overall size of the compact space. As discussed in the previous section, there are also Ramond-Ramond fields which are not present in perturbative heterotic models. Again for simplicity we will consider only a single one of these which we call $M$. The $S$ and $T$ fields propagate in the whole of space-time, i.e. the bulk, whereas the $M$ fields are localized in a particular brane$^4$. However, the $M$ fields can still play an important role in supersymmetry breaking, in the sense that its $F$-term may get a non-vanishing vacuum expectation value (VEV) breaking supersymmetry, if the $M$ field lives in the observable brane this will induce supersymmetry breaking on the brane, otherwise the breaking may be communicated by the couplings to the $S$ and $T$ fields.

We will then analyze the structure of soft breaking terms when all three fields $S, T$
and $M$ can have non-vanishing $F$-terms and therefore contribute to supersymmetry breaking. We will also consider matter fields $C_{\alpha}$, and will assume that the full gauge group of the standard model comes from one single brane, which is the most generic case in the explicit models studied so far and is the natural scenario for gauge coupling unification.

We now proceed to compute the soft breaking terms using the tree-level Kähler potential (3.5). As usual, for $F$-term breaking we need to consider the $F$ part of the scalar potential

$$V = e^G \left(G_\alpha (G^{-1})_\beta^a G^b - 3\right), \quad (4.1)$$

where $G = K + \log |W|^2$ and $W$ is the, unspecified, superpotential. We will closely follow the analysis of ref.[23] defining different goldstino angles for the mixing between the $F$ terms of the fields $S, T$ and $M$ and assume that SUSY-breaking effects are dominated by them. The feature here that differs from the previous approaches is the inclusion of the field $M$ as another source for supersymmetry breaking. Also, in contrast to the heterotic string, there is no self $T$-duality in these models and therefore there are no $T$-dependent holomorphic threshold corrections to the gauge couplings.

In this class of models there is a mixing between the $T$ and $M$ fields due to the presence of a generalized Green-Schwarz mechanism and the function $\hat{K}$ of (3.5) is of the form

$$\hat{K} = \hat{K}(M + \bar{M} - \delta_{\text{GS}} \log (T + \bar{T})). \quad (4.2)$$

Thus one has the Kähler metric for $S, T, M$ ($m, n = S, T, M; i, j = T, M$):

$$(K_{mn}) = \begin{pmatrix} \frac{1}{(S + \bar{S})^2} & 0 \\ 0 & (K_{ij}) \end{pmatrix}, \quad \text{with} \quad (K_{ij}) = \begin{pmatrix} K_{TT} & -\delta_{\text{GS}} \hat{K}'' \\ -\delta_{\text{GS}} \hat{K}'' & \frac{T + \bar{T}}{T + \bar{T}} \end{pmatrix}, \quad (4.3)$$

where

$$K_{TT} = \frac{1}{(T + \bar{T})^2} \left(3 + 2 \sum_{\alpha} \frac{C_\alpha \bar{C}_\alpha}{T + \bar{T}} + \delta_{\text{GS}}^2 \hat{K}'' + \delta_{\text{GS}} \hat{K}'\right)$$

and the primes denote derivatives with respect to the argument of $\hat{K}$.

Since there is mixing between the $T$ and $M$ fields, we would like to normalize the kinetic terms, i.e. multiply by a matrix $P$ such that $P^\dagger (K_{ij}) P = I$. Defining

$$k = (T + \bar{T})^2 K_{TT}/\hat{K}'' - \delta_{\text{GS}}^2$$

and expanding in powers of $1/(T + \bar{T})$ we obtain

$$P = \frac{1}{\sqrt{\hat{K}''}} \left(\frac{\delta_{\text{GS}}^{\text{GS}}}{\sqrt{k}} + \frac{\delta_{\text{GS}}^{\text{GS}} T}{(T + \bar{T})^2} + \mathcal{O}(1/(T + \bar{T})^3)\right) + \mathcal{O}\left(\frac{1}{(T + \bar{T})^3}\right). \quad (4.4)$$

In this expansion we assumed that $\hat{K}''$ goes to a constant in the limit $T + \bar{T} \to \infty$.

Note that in limit where all fields are at the minimum of their potential, i.e. $C_\alpha = 0 = \hat{K}'$, one has $k = 3/\hat{K}''$.\footnote{Note that in limit where all fields are at the minimum of their potential, i.e. $C_\alpha = 0 = \hat{K}'$, one has $k = 3/\hat{K}''$.}
After having diagonalised the Kähler metric we can now define the mixing between the $F$-terms of the fields $T$ and $M$ by using a vector $\Theta$ of modulus one:

$$\Theta = \begin{pmatrix} \Theta_T \\ \Theta_M \end{pmatrix} = \begin{pmatrix} \sin \phi \\ \cos \phi \end{pmatrix}.$$  \hfill (4.5)

The $F$-terms $F^T$ and $F^M$ are then defined by

$$\begin{pmatrix} F^T \\ F^M \end{pmatrix} = \sqrt{3} C m_{3/2} \cos \theta P \Theta$$  \hfill (4.6)

where $C = \sqrt{1 + \frac{V_0}{3 m_{3/2}^2}}$ and $V_0$ is the vacuum energy.

We can now write an explicit expression for the corresponding $F$-terms in the large $T$ limit:

$$F^T \approx \sqrt{3} C m_{3/2} \sin \theta \left( \frac{P_M \bar{T} \sin \phi - \frac{\delta_{GS}}{T + \bar{T}} \cos \phi}{\sqrt{k}} \right),$$

$$F^M \approx \sqrt{3} C m_{3/2} \sin \theta \left( \left( \frac{\delta_{GS}}{\sqrt{k}} + \frac{\delta_{GS} \sqrt{k}}{T + \bar{T}} \right) \sin \phi + \left( 1 - \frac{\delta_{GS}^2}{(T + \bar{T})^2} \right) \cos \phi \right).$$  \hfill (4.7)

For the dilaton $S$ there is no mixing and its $F$-term is simply given by

$$F^S = \sqrt{3} C m_{3/2} \sin \theta (S + \bar{S}).$$  \hfill (4.8)

### 4.1. Gaugino masses

In general the gaugino masses for gauge group $G_a$ are given by

$$M_a = \frac{1}{2} (\text{Re} f_a)^{-1} F^m \partial_m f_a.$$  \hfill (4.9)

Here $f_a = S + \frac{s_a}{4 \pi} M$ denotes the gauge kinetic function normalised as $\text{Re} f_a = 4\pi / g_a^2$. We then find

$$M_a = \frac{\sqrt{3} C m_{3/2}}{S + \frac{s_a}{4 \pi} (M + \bar{M})} \left( \sin \theta (S + \bar{S}) + \frac{s_a}{4 \pi} \cos \theta (P_M \bar{T} \sin \phi + P_M \bar{M} \cos \phi) \right).$$  \hfill (4.10)

In the limit $T + \bar{T} \to \infty$ and using $2\text{Re} f_a = 2 \alpha_{a}^{-1}$ and $S + \bar{S} = 2 \alpha_{GUT}^{-1}$ we then obtain

$$M_a = \sqrt{3} C m_{3/2} \frac{\alpha_a}{\alpha_{GUT}} \left( \sin \theta - \frac{s_a}{8 \pi} \alpha_{GUT} \cos \theta \left( \frac{\delta_{GS}}{\sqrt{k}} \sin \phi + \cos \phi \right) \right) + O \left( \frac{1}{(T + \bar{T})^2} \right).$$  \hfill (4.11)
4.2. Scalar masses

Writing the Kähler potential (3.5) as
\[ K = \bar{K}(S, S, T, T, M, \bar{M}) + \sum_{\alpha} \bar{K}_{\alpha}(S, S, T, T, M) C_{\alpha} \bar{C}_{\alpha}, \] (4.12)
the scalar mass squared of the field \( C_{\alpha} \) is given by
\[ m_{\alpha}^2 = (m_{3/2}^2 + V_0) - F^{m} F^{n} \partial_{m} \partial_{n} \log \bar{K}_{\alpha}. \] (4.13)

In our case we find
\[ m_{\alpha}^2 = m_{3/2}^2 \left( 1 - 3C^2 \cos^2 \theta \left( \frac{P_{TT} \sin \phi + P_{TM} \cos \phi}{(T + \bar{T})^2} \right) \right) + V_0. \] (4.14)

In the limit \( T + \bar{T} \to \infty \) we obtain
\[ m_{\alpha}^2 = V_0 + m_{3/2}^2 \left( 1 - \frac{3}{\kappa} C^2 \cos^2 \theta \sin^2 \phi \right) + \mathcal{O} \left( \frac{1}{(T + \bar{T})^2} \right). \] (4.15)

4.3. A-terms

The A-terms are derived from the formula
\[ A_{\alpha\beta\gamma} = F^{m} \left( \bar{K}_{m} + \partial_{m} \log Y_{\alpha\beta\gamma} - \partial_{m} \log (\bar{K}_{\alpha} \bar{K}_{\beta} \bar{K}_{\gamma}) \right). \] (4.16)

From the structure of the Kähler potential (4.12) and assuming that the Yukawa couplings do not depend on the moduli, i.e. \( \partial_{m} \log Y_{\alpha\beta\gamma} = 0 \), we obtain the following in the large-volume limit:
\[ A_{\alpha\beta\gamma} = -\sqrt{3} C m_{3/2} \left( \sin \theta + \cos \theta \cos \phi \bar{K}' \right) + \mathcal{O} \left( \frac{1}{(T + \bar{T})^2} \right). \] (4.17)

4.4. Dilaton Domination Scenario

Rather than discussing details of the general scenario for the soft breaking terms, it is more useful to concentrate on particular limiting cases which tend to have very different physical implications. Let us begin therefore by considering the well studied dilaton domination scenario which can be obtained by setting \( \cos \theta = 0 \). We find
\[ M_a = \sqrt{3} m_{3/2} (1 + \kappa_a)^{-1}, \quad \kappa_a = \frac{s_a M + \bar{M}}{4\pi S + \bar{S}} \]
\[ m_{\alpha}^2 = m_{3/2}^2 \]
\[ A = -\sqrt{3} m_{3/2}. \] (4.18)
where we have set $C = 1$ (vanishing cosmological constant). Notice that the $\kappa_a$ dependence makes the gaugino masses non universal, in contrast to the heterotic case. This, however, is a one-loop effect and to first approximation, for very small values of $\kappa_a$, the dilaton dominated type I string has the same supersymmetry breaking terms as the standard dilaton dominated perturbative heterotic string. As we shall see in the next section, the big difference arises when we start the RG running at the intermediate scale, something which is allowed for type I strings but it not for perturbative heterotic string.

4.5. $M$ Domination Scenario

A completely new scenario which is allowed in these models is the limit in which only the $M$ field is the main source of supersymmetry breaking. We obtain this scenario by setting $\sin \theta = \sin \phi = 0$ in the above equations, and we find the following expressions for the soft breaking terms:

$$M_a = -\frac{\sqrt{3}}{8\pi} \alpha_a s_a \frac{m_{3/2}}{m_2}$$

$$m^2_{\alpha} = m_{3/2}^2$$

$$A = -\sqrt{3} \frac{m_{3/2}}{k} \hat{K}' .$$

This scenario applies only if the $M$ field lives in the same brane as the standard model fields. $M$ domination could also occur with the $M$ fields and standard model fields living on different branes, however in that case the induced soft breaking terms will all be vanishingly small since $M$ does not couple directly to the visible sector.

4.6. Moduli Domination Scenario

The $T$ dominated scenario corresponds to the limit $\sin \theta = \cos \phi = 0$, for which we obtain

$$M_a = -\frac{\sqrt{3} \delta_{\text{GS}}}{\sqrt{k}} \frac{s_a \alpha_a}{8\pi} \frac{m_{3/2}}{k - 3}$$

$$m^2_{\alpha} = m_{3/2}^2 \frac{k - 3}{k}$$

$$A = 0 .$$

Notice that the structure of the soft breaking terms is different from that found in ref.[23] for the moduli domination scenario in heterotic models. The main reason for this difference is the fact that in the heterotic case there are $T$ dependent threshold corrections for the gauge couplings which are required by $T$-duality. Type I models are not self dual under $T$ duality and there are no $T$ dependent thresholds [18].

Notice that the moduli and $M$ domination soft breaking terms simplify substantially if the minimum of the $M$ field potential is at vanishing Fayet-Iliopoulos term for
which \( \dot{K}' = 0 \). In this case one has \( k = 3 \) if the VEVs of the charged fields vanish. Independent of this, in both scenarios the gaugino masses are one-loop suppressed, unlike the dilaton dominated case. Therefore these soft terms are of the same order as or dominated by those induced by anomaly mediated supersymmetry breaking [25]. Hence anomaly mediation should also be included in the analysis and cannot be neglected in the renormalization group analysis. Furthermore, once one-loop effects become relevant, we would have to consider loop corrections to the Kähler potential which are not yet well understood in type I models. A complete analysis including the effects of anomaly mediated supersymmetry breaking is beyond the scope of the present article and it is left for a future publication. In the following we will therefore concentrate on the dilaton dominated scenario in which the effects of anomaly mediation are negligible.

5. Phenomenological Considerations

The phenomenological study of SUSY breaking in various string models has in the past been almost exclusively focussed upon high (GUT or higher) scale stringy models [26, 27]. Here, we turn to pertinent phenomenological consequences of the dilaton domination scenario discussed above. We perform the analysis for an intermediate string scale \( M_X = 10^{11} \) GeV and the conventional GUT scale \( M_{GUT} = 2 \times 10^{16} \) GeV. This allows us to contrast the intermediate scale models with their GUT-scale counterparts. We will focus upon the case where the effective theory below the string scale is the MSSM. The effective theory below the string scale affects the results by altering the renormalization of the soft SUSY breaking and supersymmetric parameters down to the weak scale. In fact, as we demonstrate, going beyond the MSSM approximation by adding extra leptons in order to achieve gauge unification does not significantly alter many of the results. Where there is a large change, we display the results when we take new leptons into account in the gauge beta functions.

First, we show that adding just a few (in an explicit example, 5) extra vector-like representations of leptons can achieve gauge unification at the intermediate scale. We then perform an analysis of Yukawa unification. The implications and viability of bottom-tau and top-bottom-tau Yukawa unification are investigated briefly. These results are approximately independent of the assumed form of SUSY breaking, so we ignore sparticle splittings in order to make the analysis more model independent. For the other phenomenological analyses, we take the full non-degenerate sparticle spectrum into account. Next, the assumed limit of SUSY breaking (dilaton dominated) is employed in order to set the boundary conditions of the soft SUSY breaking parameters at the string scale. We will focus upon the spectra, charge and colour breaking (CCB) bounds and fine-tuning measure in each case. We then combine these quantities with the experimental bounds upon sparticle masses to identify the allowed parameter space, how fine-tuned it is, and predict the sparticle and Higgs masses. This will allow a useful comparison of the two different string-scale scenarios.
5.1. Gauge and Yukawa Unification

We now turn to the constraints and fits from gauge and Yukawa unification, both of which may be successful in SUSY GUTs [28]. In this subsection only, we will use the two-loop MSSM RGEs [29] above \( m_t \), thus assuming a degenerate MSSM spectrum (with the extra states) at \( m_t \). Later, when we consider SUSY breaking phenomenology, we will therefore go beyond this approximation, taking sparticle splittings into account. Although loop corrections involving sparticles to the weak-scale Yukawa couplings are expected, we will ignore them. This approximation allows us to make broad statements that do not depend upon the details of SUSY breaking and are therefore more model independent.

The issue of gauge unification at the intermediate scale has already been studied to one-loop order [6]. It was suggested that extra leptons may be added to the MSSM in order to achieve it. The simplest model found involved adding \( 4(L_L + E_R) \) supermultiplets in vector-like copies to the MSSM. We now examine this statement to two loop order, in the hope of finding the minimal addition of leptons to the MSSM which achieves gauge unification at the intermediate scale. To two-loop order and using central experimental inputs for the gauge couplings and masses as in section 5.5, we find that \( 2 \times L_L + 3 \times E_R \) extra vector-like representations\(^6\) are enough to achieve approximate gauge unification at \( M_X \sim 10^{11} \) GeV. With this spectrum, we obtain

\[
\begin{align*}
g_1(M_X) &= 0.81, \\
g_2(M_X) &= 0.82, \\
g_3(M_X) &= 0.81.
\end{align*}
\]

(5.1)

As a case study, we will consider the MSSM augmented by \( 2 \times L_L + 3 \times E_R \) vector-like representations when we want to examine the possible effect of adding extra states to the MSSM spectrum in order to achieve gauge unification. The default analysis will however be valid for the MSSM, where we assume that either the corrections from the extra states are small, or that mirage unification (where stringy corrections change the boundary conditions at the string scale in just the correct way to agree with the measured gauge couplings) occurs at \( M_X \).

Yukawa unification in the third family can be a prediction of SUSY GUTs, and has successfully passed empirical constraints [30]. It may also be predicted in some particular string models. This prediction may appear in either its weaker form

\[
R_{b/\tau} \equiv \frac{h_b(M_X)}{h_{\tau}(M_X)} = 1
\]

(5.2)

or in a stronger form

\[
h_t(M_X) = h_b(M_X) = h_{\tau}(M_X).
\]

(5.3)

For a given \( \tan \beta \), we determine the third family Yukawa couplings at \( m_t \) as in section 5.5 by running the low energy empirical inputs up. We run the Yukawa and gauge

\(^6\)Note that we assume the extra states do not contribute to electroweak symmetry breaking, or mix with the MSSM leptons.
Figure 1: Bottom-tau Yukawa unification in the GUT and intermediate scenarios, where $M_X = 2 \times 10^{16}$ GeV and $10^{11}$ GeV respectively.

couplings (using the MSSM RGEs) from $m_t$ to $M_X$ for $M_X = 2 \times 10^{16}, 10^{11}$ GeV, i.e. the GUT-scale and intermediate scale unification hypotheses respectively.

$R_{b/\tau}$, as defined in eq.(5.2), is displayed for the intermediate and GUT-scale unification scenarios for various $\tan \beta$ in fig. 1. For each scenario, the range of $R_{b/\tau}$ predicted by varying $\alpha_s = 0.119 \pm 0.002$, $m_b(m_b) = 4.25 \pm 0.15$ GeV and $m_t(m_t) = 165 \pm 5$ GeV within their 1σ errors is depicted as the region between two lines. In the MSSM SUSY GUT scenario (e.g. SUSY minimal $SU(5)$), this region (between the two solid lines) constrains $\tan \beta$ to be either $2 - 3$, or $\gtrsim 55$, since that is where it crosses $R_{b/\tau} = 1$. In the intermediate scale scenario, we see that there is no such constraint upon $\tan \beta$ because there is a point between the dashed lines consistent with $R_{b/\tau} = 1$ for any $\tan \beta$.

In the case of top-bottom-tau Yukawa unification, there are enough constraints to perform a $\chi^2$ fit. The errors upon $m_\tau (m_\tau) = 1.77705 \pm 0.00027$ GeV are so small that we use the central value as a constraint. For a given $\tan \beta$ value, it is possible to find the unified Yukawa coupling $\lambda(M_X)$ consistent with this value. We then predict $m_b(m_b)$ and $m_t(m_t)$ consistent with $\lambda(M_X)$. In totality, the parameters are $\alpha_s(M_Z)$ and $\tan \beta$, with which we fit $\alpha_s(M_Z)$, $m_b(m_b)$ and $m_t(m_t)$, therefore the number of degrees of freedom is $3 - 2 = 1$. Fig. 2 displays the 68% and 90% C.L. contours for the GUT, intermediate MSSM and intermediate MSSM augmented by $2L_L + 3E_R$ vector-like representations at $m_t$ (INT MSSM+X). Table 1 displays the best-fit points for each of the three cases considered. It is clear that the intermediate scale top-bottom-tau unified scenario is
Figure 2: Top bottom-tau Yukawa unification in the GUT and intermediate (INT MSSM) scenarios, where $M_X = 2 \times 10^{16}$ GeV and $10^{11}$ GeV respectively. 68% and 90% C.L. contours are shown. The INT MSSM+X case refers to intermediate-scale unification, with additional leptons (explained in the text). The crosses display best-fit points.

![Graph showing top bottom-tau Yukawa unification in GUT and INT MSSM scenarios](image)

<table>
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<tr>
<th>Model</th>
<th>$M_X$/GeV</th>
<th>$\tan \beta$</th>
<th>$\alpha_s(M_Z)$</th>
<th>$m_b(m_b)$/GeV</th>
<th>$m_t(m_t)$/GeV</th>
<th>$\chi^2$</th>
</tr>
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<tbody>
<tr>
<td>MSSM</td>
<td>$2 \times 10^{16}$</td>
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<td>0.118</td>
<td>4.42</td>
<td>170</td>
<td>2.62</td>
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<tr>
<td>MSSM+X</td>
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<td>58.2</td>
<td>0.119</td>
<td>4.20</td>
<td>165</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 1: Best-fit points of top-bottom-tau Yukawa unification. The field content is displayed in the first column. MSSM+X stands for the MSSM with additional vector-like lepton representations (as explained in the text).

preferred over the GUT scale one, with a $\chi^2$ of 0.02 and 2.62 respectively. Adding the extra leptons in order to achieve gauge unification makes very little difference to the fits, as can be seen from table 1 and fig. 2.

5.2. CCB bounds

Supersymmetric models have many directions in field space that are $F$ and/or $D$ flat, and supersymmetry breaking can cause these directions to develop global minima which break charge and colour (CCB minima) [31, 32, 33, 34, 35, 36, 37, 38], and we shall include the constraints that arise from avoiding such minima in our later analysis. Of particular relevance is the fact there is always such a minimum in dilaton dominated
models if unification occurs at the GUT scale [33]. (These minima also afflict M theory models [35, 36].) This is a pity since dilaton domination has many phenomenological advantages in addition to the purely aesthetic ones of simplicity and predictability. For example, dilaton dominance guarantees universal supersymmetry breaking terms thereby solving the problem of large FCNCs.

Before continuing we should mention a general ‘solution’ to the problem of CCBs which is really a cosmological observation; the rate of tunneling from a false vacuum (i.e. the physical vacuum in which we are living) to the global CCB minimum is usually many orders of magnitude longer than the age of the universe in standard cosmology. Thus provided that cosmology can also place the universe in the relatively small physical vacuum initially (for example during a period of heating to temperature higher than the supersymmetry breaking scale) the existence of a CCB minimum is allowed. On evidence, since dilaton domination has fallen out of favour, this solution does not seem to be very appealing.

In our numerical results we shall see that one of the most significant effects of reducing the string scale is a complete change in the behaviour of CCB minima. In fact at lower string scales the most restrictive CCB minima disappear and dilaton domination is allowed once again. We find this solution to CCB minima an appealing feature of a lower string scale. This change in the CCB bounds was anticipated near the low tan $\beta$ quasi-fixed point (where the Yukawa coupling blows up at the string scale) in ref.[38] by using approximate analytic solutions for the renormalization group equations. Since that type of analysis is necessarily rather technical we briefly summarize the main findings.

There are two important kinds of bounds; those corresponding to $D$-flat directions which develop a minimum due to large trilinear supersymmetry breaking terms; those corresponding to $D$ and $F$ flat directions which correspond to a combination of gauge invariants involving $H_2$. The first kind of flat directions give a familiar set of constraints on the trilinear couplings which is typically of the form

$$A_i^2 \lesssim 3(m_{H_2}^2 + m_{t_R}^2 + m_{t_L}^2), \quad (5.4)$$

where the notation is conventional. These constraints turn out to be very weak. Much more severe bounds come from the directions which are $F$ and $D$ flat. (The bound can be optimized as in ref.[32] and indeed as has been done in our numerical analysis, but the optimal direction is very close to the $F$ and $D$ flat direction and the bounds do not change significantly.)

$F$ and $D$ flat directions can be constructed from conjunctions of $LH_2$ plus any one of the following gauge invariants [35],

$$LLE, LQD, QULE, QUQD, QQQLLLE. \quad (5.5)$$

Absence of CCB minima along the first two directions is usually enough to guarantee their absence along the rest [35]. As an example consider the $L_iL_3E_3$, $L_iH_2$ direction,
which corresponds to the choice of VEVs,

\[ h_2^0 = -a^2 \mu / h_{E33} \]
\[ \tilde{e}_{L3} = \tilde{e}_{R3} = a \mu / h_{E33} \]
\[ \tilde{\nu}_i = a \sqrt{1 + a^2 \mu / h_{E33}}, \]  
(5.6)

where \( a \) parameterizes the distance along the flat direction. The potential along this direction depends only on the soft supersymmetry breaking terms;

\[ V = \frac{\mu^2}{h_{D33}^2} a^2 (a^2 (m_{H_2}^2 + m_{L_{ii}}^2) + m_{L_{ii}}^2 + m_{E_{33}}^2 + m_{L_{33}}^2). \]  
(5.7)

In order to minimize the one-loop corrections, the mass squared parameters in eq.(5.7) are evaluated at a renormalization scale of \( Q = \max(h_t h_2^0, M_{susy}) \). The first term in the potential dominates at large VEVs when \( a \gg 1 \) and, because \( m_{H_2}^2 < 0 \) in order to give electroweak symmetry breaking, this radiatively generates a dangerous CCB minimum with a VEV which is typically a few orders of magnitude larger than the weak scale.

The general weakening of the CCB bounds as we lower the string scale is caused by the interplay of the first \( (a^4) \) and second \( (a^2) \) terms in eq.(5.7). Both terms are assumed to be positive at the string scale but only the first can become negative due to the strong scale dependence of \( m_{H_2}^2 \). However when \( a \ll 1 \) the second term dominates since the mass squared terms are of the same order. Hence if a CCB minimum forms at all along this direction it can only do so for \( a \gg 1 \) or from eq.(5.6)

\[ h_2^0 \gg \left| \frac{\mu}{h_{E33}} \right|. \]  
(5.8)

This implies that if we choose a string scale which is less than \( \left| \frac{\mu}{h_{E33}} \right| \) the possibility of a CCB minimum along this direction is excluded entirely. The behaviour of the bounds as the unification scale is increased towards the usual GUT scale was examined in ref.[38]. This entailed a detailed treatment of the renormalization group equations, but the net result is that the CCB bounds increase rather smoothly towards their usual GUT scale values.

The \( F \) and \( D \) flat direction discussed here provides the severest bounds on the parameter space and for the usual constrained MSSM with \( M_X = 2 \times 10^{16} \) it can be expressed as a bound on the degenerate scalar mass at the GUT scale,

\[ m_0 < \lambda (\tan \beta) M_a, \]  
(5.9)

where the GUT scale gaugino masses are of course degenerate, and where \( \lambda (\tan \beta) \approx 1 \) at the quasi-fixed point and falls off to \( \approx 0.4 \) and larger values of \( \tan \beta \). The \( m_0 \) and \( M_a \) parameters are related by the dilaton and moduli VEVs, and we can immediately see that the dilaton dominated scenario is ruled out by the above. We show this numerically later using the techniques summarized in ref.[32]
5.3. Fine Tuning

At tree-level, the $Z$ boson mass is determined to be

$$\frac{1}{2} M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$  \hspace{1cm} (5.10)$$

by minimizing the Higgs potential. $\tan \beta$ refers to the ratio of Higgs vacuum expectation values (VEVs) $v_1/v_2$ and $\mu$ to the Higgs mass parameter in the MSSM superpotential. In the universal models discussed here, $m_{H_2}$ has the same origin as the super-partner masses ($m_0$). Thus as search limits put lower bounds upon super-partners’ masses, the lower bound upon $m_0$ rises, and consequently so does $|m_{H_2}|$. A cancellation is then required between the first and second terms of eq.(5.10) in order to provide the measured value of $M_Z < |m_{H_2}|$. Various measures have been proposed in order to quantify this cancellation [39].

The definition of naturalness $c_a$ of a ‘fundamental’ parameter $a$ employed in ref.[40] is

$$c_a \equiv \left| \frac{\partial \ln M_Z^2}{\partial \ln a} \right|.$$  \hspace{1cm} (5.11)$$

From a choice of a set of fundamental parameters $\{a_i\}$, the fine-tuning of a particular model is defined to be $c \equiv \max(c_a)$. Our choice of free, continuously valued, independent and fundamental parameters are

$$a_i \in \{\mu, B, m_{3/2}\},$$  \hspace{1cm} (5.12)$$

i.e. the one on the right hand side of eq.(4.18), but augmented by the superpotential Higgs mass $\mu$ and the soft SUSY breaking Higgs bilinear parameter $B$. Notice that, following ref.[40], we have not explicitly considered variations of $M_Z$ with respect to the top Yukawa coupling $h_t$. The origin of soft terms and Yukawa couplings are so different that putting them on equal footing seems, in our opinion, inadequate. Other authors have considered including $h_t(M_X)$ [41], in scenarios where it makes a large difference ($m_0 \gg M_{1/2}$). We have checked that here, the exclusion or inclusion of $h_t$ makes no difference since the fine-tuning measure comes predominantly from $\mu$.

5.4. Gaugino masses

It is possible to make some predictions of the MSSM gaugino masses $M_{1,2,3}$ at low energy scales depending upon their boundary conditions at the unification scale $M_X$. We are able to do this because to one-loop order and for the MSSM (with any additional extra $N = 1$ supermultiplets), we have [29]

$$16\pi^2 \frac{d(M_i/\alpha_i)}{d \ln \mu} = 0 \Rightarrow \frac{M_i(\mu)}{\alpha_i(\mu)} = \frac{M_i(M_X)}{\alpha_i(M_X)}.$$  \hspace{1cm} (5.13)$$

In the standard unification scenario where gauge couplings and gaugino masses are unified at $M_X$, this leads to the familiar prediction

$$M_1(\mu)\alpha_1(\mu)^{-1} = M_2(\mu)\alpha_2(\mu)^{-1} = M_3(\mu)\alpha_3(\mu)^{-1},$$  \hspace{1cm} (5.14)$$
valid at any renormalisation scale $\mu$. In particular, we may take $\mu \approx M_Z$ in order to estimate $M_{1,2,3}$. From experiment, we have $\alpha_{1,2,3}^{-1}(M_Z) \sim \{90, 30, 8\}$ roughly (for $\alpha_1$ in the GUT normalisation). Thus

$$9M_1 \sim 3M_2 \sim M_3$$  \hspace{1cm} (5.15)

predicts the gaugino mass ratios in the canonical unification scenario, $M_X \sim 10^{16}$ GeV. If $M_{1,2,3}$ are deduced from experiments, the relation eq.(5.15) would therefore provide an immediate test of this scenario. We note here that the same relation results in the case where the gauge couplings meet at the intermediate scale (for example by adding the vector-like copies of $2 \times E_R + 3 \times L$ to the MSSM).

5.5. SUSY Breaking Numerical Analysis

The softly broken MSSM RGEs used are contained within ref.[29]. We follow here a fairly standard numerical algorithm to calculate the MSSM sparticle spectrum, e.g. see ref.[42], so here we merely briefly review our approximations. The soft breaking parameters’ RGEs are calculated to one-loop order with full family dependence, but all supersymmetric parameters are evolved to two-loop order. The Yukawa matrices are approximated to be diagonal. To obtain their $\overline{MS}$ values (as well as that of $\alpha_s(\mu)$) at $\mu = m_t(m_t)$, we use 3 loop QCD$\otimes$1 loop QED as an effective theory to run the quark and lepton masses up (with step-function decoupling of quarks and the tau).

Near the weak scale, thresholds from sparticles are modeled by the step function and all finite corrections are neglected. Thus the sparticles are decoupled below their running mass as in ref.[43]. The tree-level MSSM Higgs potential supplemented by the largest (top and stop) one-loop corrections [42] to the tadpoles is used to calculate $\mu$ and $B$ from the radiative electroweak symmetry breaking constraints at the scale where one-loop corrections are small [44], $\hat{Q} = \sqrt{(m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2)/2}$. Tree-level mass matrices are used to calculate the sparticle masses from the weak scale MSSM parameters, except for the Higgs masses, where state-of-the-art expressions including finite terms and some large two-loop contributions are utilized [45].

The most important empirically-derived inputs used [46] are shown in table 2. $m_x(\Lambda)$ are running masses in the $\overline{MS}$ scheme at scale $\Lambda$. The weak-boson masses are extracted in the on-shell scheme. All masses are measured in GeV. Note that $m_t(m_t)$ is kept constant over SUSY breaking parameter space. A more accurate calculation would use a constant value of the empirically derived pole mass, and derive the $\overline{MS}$ running mass from it by taking QCD and gluino radiative corrections into effect. Our procedure corresponds to taking a slightly different value of the top pole mass at different points of parameter space, but should still be mostly within

\begin{align*}
G_F &= 1.6639 & M_Z &= 91.1867 \\
M_W &= 80.405 & \alpha(M_Z)^{-1} &= 127.9 \\
m_t(m_t) &= 160 & m_b(m_b) &= 4.25 \\
m_\tau(m_\tau) &= 1.777 & \alpha_s(M_Z) &= 0.119
\end{align*}

**Table 2:** Inputs used in the numerical analysis [46].
\[ m_t = 175 \pm 5 \text{ GeV} \] Similarly, \( m_h(m_h) \) receives contributions from sparticles for \( \tan \beta > 40 \) that we have neglected.

We will constrain the models to respect empirical lower bounds upon MSSM particle masses \([27]\), as shown in table 3. The most restrictive bounds turn out to be those upon \( h^0 \), the lightest CP even Higgs, \( \tilde{g} \) the gluino, and \( \chi_1^\pm \), the lightest charginos.

\[
\begin{align*}
m_{\tilde{g}} &> 300 \\
m_{\tilde{t}_1} &> 83 \\
m_{\chi_1^0} &> 31.6 \\
m_h &> 89.3 \\
m_{\chi_1^\pm} &> 84 \\
m_{\tilde{e}, \tilde{\mu}} &> 80 \\
m_{\tilde{u}, \tilde{d}} &> 250 \\
m_{h^0} &> 83
\end{align*}
\]

Table 3: Empirical lower bounds upon MSSM sparticle masses (in GeV) \([27]\)

5.6. Spectra

We now turn to the spectrum of the dilaton dominated SUSY-breaking scenario, \textit{i.e.} using universal boundary conditions as in eq.(4.18). Fig. 3 displays the fine-tuning over the whole dilaton-dominated parameter space, assuming the usual GUT scale \( M_X = M_{GUT} = 2 \times 10^{16} \text{ GeV} \) (and \( \text{sgn}(\mu)=1 \)). This value of \( M_X \) approximates the true perturbative heterotic string-scale \( M_H \sim 5 \times 10^{17} \text{ GeV} \). We have neglected the renormalization between \( M_H \) and \( M_{GUT} \), which should not be a bad approximation because the running depends logarithmically upon the renormalization scale, and we have only neglected one order of magnitude compared with 14 between the GUT scale and the weak scale. The whole parameter space is \textit{ruled out} by the CCB constraint, as explained in the previous section. In order to compare the rest of the parameter space with a lower value of \( M_X \), we do not display this constraint in the figure, but instead show the spectra of the lightest Higgs \( h \), the gluino \( \tilde{g} \), the lightest neutralino \( m_{\chi_1^0} \) and the lightest stop \( m_{\tilde{t}_1} \). The experimental limits derive from the gluino and lightest MSSM Higgs constraints in table 3. The region denoted ‘charged LSP’ has the stau as the LSP and is therefore ruled out (if R-parity is conserved)\(^7\)

The fine-tuning parameter (as defined above) is displayed in the background and by the bar to the right of each figure. The fine-tuning increases sharply for the region of low \( \tan \beta \) and high \( M_{3/2} \). We have extended the region of parameter space for larger values of \( M_{3/2} \) than those shown, and have found that \( m_h < 116 \text{ GeV} \).

Fig. 4 shows the equivalent spectra and fine-tuning for an intermediate scale \( M_X = 10^{11} \text{ GeV} \) and \( \text{sgn}(\mu)=1 \). It is important to note that the CCB bound, shown by the shaded region, is now even less restrictive than the empirical lower bounds upon the gluino and lightest MSSM Higgs. We also see that \( \tan \beta < 28 \) is required by the charged LSP constraint, thus ruling out top-bottom-tau Yukawa unification in this scenario. The fine-tuning is roughly half-that of the canonical GUT scale scenario, for a given point in parameter space. Extending the region of parameter space covered yields \( m_h < 117 \text{ GeV} \).

\(^7\)In this region, \( m_{\chi_1^0} \) is almost degenerate with \( m_{\tilde{\tau}} \). Thus, higher order radiative corrections could potentially raise the stau mass above the lightest neutralino. We therefore counsel care in the interpretation of this bound.
Figure 3: Spectra and fine tuning of GUT scale dilaton dominated scenario. $M_X = 2 \times 10^{16}$ GeV and $\text{sgn}(\mu) = +$. Fine tuning is displayed by the bar to the right. Note that the whole plane is ruled out by CCB constraints, but these have not been displayed. Regions of flat shading are ruled out by the labelled constraint. Contours of spectra (in GeV) are shown for (a) $m_h$, (b) $m_{\tilde{g}}$, (c) $m_{\chi^0_1}$, (d) $m_{\tilde{t}_1}$.

We now examine the effect of adding extra states in order to achieve gauge unification at the intermediate scale. As a case study, we pick the example of extra leptons: $3 \times E_R + 2 \times L$ (and vector-like partners). We assume these extra states have negligible Yukawa couplings so that their effect upon the spectra may be encapsulated by changes in the beta functions of the gauge couplings only. We take these changes into account to one-loop order. There is negligible difference to any of the spectra except for the weak gauginos, and so we display the lightest chargino and neutralino masses in fig. 5. We note that, due to small corrections to the stau and lightest neutralino masses, the charged LSP bound has significantly relaxed, allowing top-bottom-tau Yukawa unification in the model.
Figure 4: Spectra and fine tuning of the intermediate scale dilaton dominated scenario. $M_X = 10^{11}$ GeV and $\text{sgn}(\mu) = 1$. Fine tuning is displayed by the bar to the right. Regions of flat shading are ruled out by the labelled constraint. Contours of spectra (in GeV) are shown for (a) $m_h$, (b) $m_{\tilde{g}}$, (c) $m_{\chi^0_1}$, (d) $m_{\tilde{t}_1}$.

5.7. FCNCs and CP

Flavour Changing Neutral Current processes (FCNCs) such as $b \rightarrow s\gamma$ and Electric Dipole Moments (EDMs) are important experimental tests for supersymmetry. Since a generic supersymmetric model fails these tests, they offer important insight into the structure of supersymmetry breaking. The dilaton dominated models in the present paper represent a significant improvement in this direction, as we now discuss.

First let us review the current status. It is generally believed that the observed absence of FCNCs and large EDMs implies that one or more of the following is an integral feature of the supersymmetry breaking (see ref.[47] for a review):

- Universality: This proposal uses the fact that the masses of squarks and quarks can be simultaneously diagonalised if the supersymmetry breaking is degenerate
Figure 5: Spectra and fine tuning of the extra sleptons intermediate scale dilaton dominated scenario, $M_X = 10^{11}$ GeV and $\text{sgn}(\mu) = 1$. The effects of $3 \times E_R + 2 \times L$ on the gauge couplings have been added at $m_t$ (to one-loop order) in order to display the possible effect of extra matter introduced to provide gauge unification. Fine tuning is displayed on the bar to the right. Flat shaded regions are excluded by the labelled constraints on the figures. Contours of spectra (in GeV) are shown for (a) $m_{\chi^\pm_1}$, (b) $m_{\chi^0_1}$. All other spectra are identical to the case without extra sleptons.

For particles with the same hypercharge. The resulting suppression of FCNC is similar to the GIM mechanism.

- Heavy 1st and 2nd generations [48]: really a variant of the first proposal which relies on the fact that the most severe constraints tend to involve the 1st and 2nd generations, whereas electroweak symmetry breaking involves the 3rd generation. It is therefore possible to make the squarks of the 1st and 2nd generation simultaneously heavy without paying too high a price in fine tuning. These models can, for example, be motivated by horizontal flavour symmetries.

- Specific flavour structure [49]: a relaxation of the first proposal which relies on the fact that EDMs and FCNCs depend on certain elements in the supersymmetry breaking. For example, EDMs can be acceptably small if the CP violation occurs only in flavour off-diagonal elements of the supersymmetry breaking. This type of situation can arise in heterotic string models with a suitable choice of modular weights.

Concerning FCNC, the first of the above suggestions is the oldest and arises in the simpler 4 dimensional $N = 1$ supergravity models. In phenomenological studies the model with degenerate $A$-terms and mass squareds (the Constrained MSSM) has

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8Note that as well as these proposals there is of course the possibility that contributions to various processes happen to cancel [50]. This in general will involve a certain amount of fine tuning.
becomes something of a benchmark. However it is important to realise that such
degeneracy only arises naturally in string theory in the dilaton dominated scenario
and as we have seen this scenario is excluded for theories in which the soft parameters
are set at the conventional GUT scale because the physical vacuum is unstable. The
remaining two proposals can therefore be seen as attempts to find a natural (in the
sense of t’Hooft) solution to the SUSY flavour and CP problems that is consistent with
cosmological constraints.

However in the previous sections we have seen that one of the predictions of in-
termediate scale string scenarios is that the physical vacuum is stable in the dilaton
dominated scenario and that fine-tuning is relatively mild. They are therefore the first
realistic (string derived) models that automatically solve the SUSY flavour problem,
and are consistent with cosmological constraints.

It is easy to see that in models with a lowered string scale the supersymmetric
contributions to these FCNC processes are qualitatively the same as in the CMSSM.
Running the RGEs in models with universal supersymmetry breaking at the string
scale results in squark mass squareds that have small flavour off-diagonal components,
\( \Delta_{ij} \). The phenomenological constraints are conveniently expressed as bounds on

\[ \delta = \frac{\Delta}{\tilde{m}^2}, \]

where \( \tilde{m}^2 \) is a measure of the average squark mass in the offending diagram. One
may use the mass insertion approximation to calculate bounds on these parameters
(see e.g. ref.[51]). For example \( b \to s\gamma \) implies that \( \left| (\delta_{23}^d)_{LR} \right| \lesssim 10^{-2} \) in order to satisfy
the experimental bounds. Now, a typical contribution to the off-diagonal piece is
almost linear, i.e. of the form \( \Delta \sim \frac{A_{D23}}{M^3} \) (this of course is necessary for the mass
insertion approximation to be valid at all). Renormalization contributions to \( A_{D23} \)
are proportional to \( \log(M_X/M_W) \) and this implies that the FCNC effects are relatively
independent of the string scale and qualitatively the same as those for standard SUGRA
models. The quantitative differences will be discussed in detail in a future work [52].

For CP violation, the dilaton domination scenario does not solve the EDM problem,
even though it has been argued that it can ameliorate it. In the dilaton domination
scenario, almost all the CP violation that we observe in the Kaon system, for instance,
must be a result of CP violation in the Yukawa couplings. One remaining question for
the dilaton breaking scenario is how this CP violation arises. In string theory CP is a
discrete gauge symmetry and consequently must be broken spontaneously. A natural
assumption in the dilaton dominated models is that this breaking is caused by the
VEVs of moduli fields, even though they do not enter the supersymmetry breaking.

This idea has been examined for orbifold models in ref.[53] and we briefly re-cap
how CP violation appears. Ref.[53] makes use of the PSL(2,Z), \( T \)-duality present in
heterotic string models. In that case the scalar potential for the \( T \) field always has
supersymmetric extrema at the points \( T = 1 \) (real) and \( T = e^{i\pi/6} \) which breaks CP. If
the minimum of the potential is at any of these points \( T \) will not break supersymmetry
Table 4: Highlights of phenomenological results in the various models. Shown are: (a) $c_{\text{min}}$, the minimal fine-tuning parameter allowed by the experiment and theoretical constraints, (b) the maximum lightest CP even Higgs mass possible, (c) the possibility of top-bottom-tau Yukawa unification being in accordance with the bounds (denoted by $\sqrt{\cdot}$), (d) whether the global minimum of the scalar potential is CCB (denoted by $\times$). ‘Plus leptons’ denotes the case where the MSSM spectrum, plus $3 \times E_R + 2 L_L$ vector-like representations is used.

<table>
<thead>
<tr>
<th>SUSY Model</th>
<th>$M_X$/GeV</th>
<th>$c_{\text{min}}$</th>
<th>$m_{h_\text{max}}$/GeV</th>
<th>$t - b - \tau$</th>
<th>CCB</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSSM</td>
<td>$2 \times 10^{16}$ GeV</td>
<td>17.4</td>
<td>116</td>
<td>$\sqrt{\cdot}$</td>
<td>$\times$</td>
</tr>
<tr>
<td>MSSM</td>
<td>$10^{11}$ GeV</td>
<td>8.8</td>
<td>117</td>
<td>$\times$</td>
<td>$\sqrt{\cdot}$</td>
</tr>
<tr>
<td>MSSM plus leptons</td>
<td>$10^{11}$ GeV</td>
<td>8.6</td>
<td>116</td>
<td>$\sqrt{\cdot}$</td>
<td>$\sqrt{\cdot}$</td>
</tr>
</tbody>
</table>

and we have dilaton dominance. Therefore if the minimum is at the CP violating fixed point $T = e^{i\pi/6}$, this will not induce CP violation on soft supersymmetry breaking terms, however CP violation enters the Yukawa couplings in a non-trivial way.

As we have said, for heterotic strings, the drawback is that dilaton dominance implies the existence of vacuum instability and CCBs. However for intermediate scale string models this scenario of CP violation becomes extremely attractive. In particular it automatically excludes CP violation from all the supersymmetry breaking terms. There is an interesting additional difference between the heterotic case and the present one. In the models we have been discussing there is no modular symmetry. Thus the non-perturbative contributions to the superpotential coming from gaugino condensation are not restricted by modular invariance and consequently dilaton dominance plus spontaneous CP violation do not require us to be at the special fixed point $T = e^{i\pi/6}$. In the models being considered here, the only requirement is dilaton dominance. The expectation value of the modulus may be at any point consistent with this requirement and generically this point will have a phase that spontaneously breaks CP.

The reason why the dilaton domination scenario, through a mechanism like this, does not really solve the EDM problem is that it does not say anything about the phase of the $\mu$ parameter (see for instance [54]) which can then contribute to CP violation. Therefore we can say that reviving the dilaton domination scenario, by having an intermediate string scale, has the spin off of solving the FCNC problem and may allow the possibility of ameliorating the EDM problem but does not solve it.

6. Conclusions

Lowering the value of the fundamental scale clearly has very important consequences for supersymmetric models. It makes available a new degree of freedom that radically changes the nature and analysis of the low-energy implications of particular models through the running of the different physical parameters under the renormalization group. Our work is only the beginning of this exploration. The most striking implication of our results is that the dilaton domination scenario which was ruled out by the
CCB constraint for high-scale strings becomes viable when the string scale is lowered to the intermediate scale. We summarise some highlights from the phenomenological results in table 4. Dilaton domination has reasonable fine-tuning, independent of whether or not leptons are added in order to achieve field-theoretic gauge unification at the intermediate scale.

Here we have performed the analysis for an intermediate fundamental scale and compared the results with the standard GUT scale scenario. These two particular scales are motivated by very different physical pictures. However, it may also be interesting to repeat our analysis for fundamental scales in other ranges, even if they are not physically motivated at present. An interesting possibility might be to start the running at a scale significantly closer to the TeV scale that, although leaving some room for the running of the parameters, may still be relevant at lower energies.

We have not explored the phenomenological prospects of all possible scenarios discussed in the text. In particular the new possibility allowed in type I string models of having the blowing-up modes be the dominant source of supersymmetry breaking may be worth exploring in the future, as well as other different combinations.

Furthermore, recently a new class of D-brane models has been constructed in which supersymmetry is explicitly broken and transmitted to the observable sector via gravitational interactions, for which the intermediate scale is naturally selected [8, 7]. The most interesting models in this class happen to be versions of the left-right symmetric models, with $SU(3) \times SU(2)_L \times SU(2)_R$ gauge symmetry surviving at very low energies. They have spectra that give unification at the intermediate scale and other interesting properties, such as automatic R-parity symmetry and a stable proton. It would be very interesting to extend our analysis to include such models.

In conclusion we believe that our analysis opens up new avenues of exploration for supersymmetric models. Our predictions allow the possibility of direct experimental verification in the near future.

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