A Likelihood Based Scheme for Coincidence Analysis

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Abstract. A fresh approach to coincidence analysis is initiated. The coincidence window size and the rates of source and background that best explain a given list of event times from two detectors is found using Maximum Likelihood Estimation. This approach is transparent and provides a unified scheme for a wide variety of situations.

I MOTIVATION

A statistically significant excess in the number of coincidences in arrival times of events in two detectors indicates a common source. Very often, however, either the rate of background events or the correct coincidence window size or both are not known *a priori*. For instance, what coincidence window size to use for the gravitational wave strain channel and a magnetometer? How do gaps in the data and environmental variability affect the significance of the results? Usually, such issues are addressed in a very data specific manner with the data itself influencing some of the assumptions made in the analysis. This makes it very difficult to understand the result and its statistical significance. These problems will become more serious for the large datasets that will be produced by the upcoming interferometers.

Maximum Likelihood Estimation (MLE) [1] appears to be a promising avenue for addressing the concerns raised above. As a demonstration, we consider here the simplest problem : two detectors producing data that consists of only event arrival times. Both background and source events are assumed to occur as Poisson processes (constant rates and no gaps in data collection). A procedure that can handle both gaps and variable rates is under development [2].

II AN OUTLINE OF THE METHOD

The data consists of two lists of event times, $L_1 = \{t_1, \dots, t_{n_1}\}$ and $L_2 = \{\tau_1, \dots, \tau_{n_2}\}$, from detectors 1 and 2 respectively. The likelihood, $P(L_1, L_2 | r_s, r_1, r_2, \Delta)$, of the data is its probability *given* source rate r_s , background

rates r_1 and r_2 in detector 1 and 2 and the coincidence window size Δ . The values of r_s , r_1 , r_2 and Δ at which $P(L_1, L_2 | r_s, r_1, r_2, \Delta)$ is maximized is the MLE estimate for the best fit values of these parameters.

Note that no prior estimation of source or background rates is obtained from the data. By allowing Δ to be a free parameter we have added an extra flexibility which will be useful when comparing, for instance, two environmental monitors.

Monte Carlo simulation: Several realizations of L_1 and L_2 were generated with $r_1 = 20$ events/day, $r_2 = 30$ events/day, $r_s = 50$ events/year and $\Delta = 1$ min and the MLE values of r_s , r_1 and r_2 were obtained for each. Fig. 1 shows the histograms for r_s , r_1 and r_2 . We did not search over Δ here because computational requirements multiply with every additional parameter in the Monte Carlo simulation.

FIGURE 1. Histograms of best fit values. Left panel : r_1 . Middle panel : r_2 . Right panel : r_s (events/day).

Future work — Our final aim is to construct a code that implements MLE taking into account variable event rates, gaps and also extends coincidence to quantities such as event amplitude. Methods for handling gaps (such as Bayesian Blocks [3]) and variable rates are under exhaustive study. The detection problem, as opposed to estimation, is also being addressed.

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