Two ways of biasing galaxy formation

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ABSTRACT
We calculate the galaxy bispectrum in both real and redshift space adopting the most common prescriptions for local Eulerian biasing and Lagrangian evolving-bias model. We show that the two biasing schemes make measurably different predictions for these clustering statistics. The Eulerian prescription implies that the galaxy distribution depends only on the present-day local mass distribution, while its Lagrangian counterpart relates the current galaxy distribution to the mass distribution at an earlier epoch when galaxies first formed. Thus, detailed measurement of the bispectrum can help establish whether galaxy positions are determined by the current mass distribution or an earlier mass distribution.

Key words: galaxies: statistics – large-scale structure of Universe.

1 INTRODUCTION
Galaxy clustering in the nearby Universe has been mapped through a variety of surveys, including different populations of luminous objects. These run from optical galaxies in the APM, CfA and LCRS surveys to the sources of the IRAS catalogue at 60 µm. Reconstructing the overall mass power spectrum from these data represents one of the main goals of modern cosmology. It is already known, however, that different tracer populations show different clustering amplitudes even after redshift-space and small-scale corrections are applied. Thus, their clustering patterns are not unambiguously related to any one given mass power spectrum (see Peacock 1999 for a review).

The simplest and most common description of biasing adopted in the literature is that, at any spatial position \( \mathbf{x} \), the fluctuation in the number density of galaxies \( \delta_g(\mathbf{x}) \) responds linearly and locally to the underlying mass fluctuation \( \delta(\mathbf{x}) \), namely \( \delta_g(\mathbf{x}) = b^E \delta(\mathbf{x}) \), where \( b^E \) is a space-independent bias factor (e.g. Dekel & Rees 1987). As discussed below, higher-order bias factors can be introduced, but the point is that such a bias prescription is inherently Eulerian: it relates the present-day galaxy and mass clustering properties, ignoring their past evolution. However, if gravity is the main force acting in the Universe, there is no doubt that galaxy biasing evolves in time, as collapsing mass fluctuations keep accreting luminous matter onto them, the galaxy distribution eventually relaxing to the mass one (Fry 1996; Tegmark and Peebles 1998). So, the biasing in the present-day galaxy distribution might well be rooted into the deep past of the history of the Universe: the strong Lyman break galaxy clustering seems to suggest that this might be the case (Steidel et al. 1997). Any primordial biasing, arising at the epoch of galaxy formation, cannot be described by an Eulerian model. Instead, a Lagrangian one has to be adopted: it is the primordial fluctuation in galaxies that is proportional to the mass fluctuation, \( \delta_g(\mathbf{q}) = b^L \delta(\mathbf{q}) \), where \( \mathbf{q} \) denotes the Lagrangian position; in general \( b^L \) differs from \( b^E \) and, in principle, higher order factors can be defined.

In this Letter we show that the local Eulerian and Lagrangian bias models are inconsistent. In fact, the clustering patterns predicted by the two bias models are different. Specifically we study the galaxy bispectrum and skewness, both in real and redshift space, on scales where the mildly non-linear approximation suffices, starting from Gaussian initial conditions. In Section 2 we review the general Eulerian and Lagrangian bias models in terms of infinite hierarchies of bias factors \( \{ b^E_j \} \) and \( \{ b^L_j \} \). In the whole discussion, these must be considered as free, position-independent, parameters. In Section 3 we discuss the galaxy bispectrum and skewness in real space, for both bias models. In Section 4 we carry out the same analysis, but taking into account the effect of redshift distortions. Section 5 contains our conclusions.

2 THE TWOFOLD BIASING PRESCRIPTION
Let us start by fixing the notation of basic quantities. If \( \varphi_g(\mathbf{q}) \) is the primordial gravitational potential (growing mode only, smoothed on some scale \( R_0 \), and linearly extrapolated to the present time), then \( \delta^{(1)}(\mathbf{q}, z) = D(z) \nabla^2 \varphi_g(\mathbf{q}) \) is the linear density field, and \( D(z) \) its growth factor with \( z \) the cosmological redshift [we put \( D(0) = 1 \)]. The linear peculiar velocity is given by \( \mathbf{u}^{(1)}(\mathbf{q}) = -\nabla \varphi_g(\mathbf{q}) \), and it is constant in time. The Eulerian density field will be indi-
cated by $\delta(x,z)$ and the $n$-th order perturbative solutions $\delta^{(n)}$ are such that $\delta = \sum_n \delta^{(n)}$ (Goroff et al. 1986). The Fourier transform is, e.g., $\delta(k) = \int d\mathbf{x} \delta(x) \exp(i \mathbf{k} \cdot \mathbf{x})$.

2.1 Local Eulerian bias

In this approach, the galaxy number density field at a given position $\mathbf{x}$ and time $z$ (e.g. ‘here’ and ‘now’) is assumed to be a local function of the underlying mass density field at the same location and instant, $\delta_g(\mathbf{x}, z) \equiv E[\delta(\mathbf{x}, z; R)]$, where the smoothing scale $R$ is much larger than the typical size of the selected objects. Usually, assuming that $E[\delta]$ can be expanded about $\delta = 0$ as a power series, an infinite series of “Eulerian bias factors” $b_j^E$ can be defined (Fry & Gaztañaga 1993):

$$\delta_g = \sum_{j=0}^{\infty} \frac{b_j^E}{j!} \delta^{(j)}. \tag{1}$$

This series is such that $\langle \delta_g \rangle = 0$ and $\delta_g(\delta = -1) = -1$. The linear coefficient $b_1^E$ corresponds to the usual bias factor. The origin of this local Eulerian prescription is essentially phenomenological, and it is an a priori devoid of any insight about the dynamics of the clustering. Galaxy clustering is analyzed for instance in terms of $N$-point correlation functions $\langle \prod_{n=0}^{N} \delta_g(x_n, z) \rangle$, and the bias factors are tuned to fit the observational data. This is the approach that has been implicitly adopted in most of the published literature on biasing, at least in its leading approximation.

2.2 Local Lagrangian bias

According to this alternative prescription, the sites of galaxy formation are identified with specific regions of the primordial density field, $\delta_g(q)$, being $\delta_g(q)$ the evolved galaxy density field at the Eulerian position $\mathbf{x}$ and instant $z$ is related to the primordial galaxy field and evolved density field by the relation (Catelan et al. 1998)

$$1 + \delta_g(x, z) = [1 + \delta_g(q)] [1 + \delta(x, z)]. \tag{3}$$

We stress the fact that eq.(3) is inherently non-local. Smoothed regions in Lagrangian space can be mapped to Eulerian space through the transformation $x = q + S(q, z)$, where $S$ is the displacement vector. In the Zel’dovich (1970) approximation, the simplest transformation, $S(q, z) = D(z) q(1)(q)$, where $\delta_g(q, z)$ and $\delta(x, z)$ are not deterministically related. In fact, for any given $\delta$ the galaxy field $\delta_g$ can assume different values (see Dekel & Lahav 1999). This stochastic behaviour is inherent to the gravitational instability dynamics.

The question now is the following: are the predictions about the clustering (in terms of standard statistics as the correlation functions, for example) as deduced from the local Eulerian and Lagrangian bias equivalent? In other terms, do there exist two sets of non-trivial Eulerian and Lagrangian bias factors, $\{b_j^E\}$ and $\{b_j^L\}$, such that the predictions for galaxy clustering are identical? In order to answer to these questions, let us analyze the galaxy bispectrum from Gaussian initial conditions as induced by mildly non-linear density evolution.

3 GALAXY BISPECTRUM

3.1 Eulerian bias case

The lowest order contribution to the galaxy bispectrum $\langle 2\pi \rangle^3 \delta_g(k_1 + k_2 + k_3) B_g(k_1, k_2, k_3; z) = \langle \delta_g(k_1, z) \delta_g(k_2, z) \delta_g(k_3, z) \rangle$ comes from the appearance of non-negligible second-order fluctuations $\delta^{(2)}$. From eq. (1), this is $\delta_g^{(2)}(x, z) = b_1^E \delta^{(1)}(x, z) + \frac{1}{2} b_2^E \delta^{(2)}(x, z)$. (Note that this expression does not have zero mean, so an offset term should be introduced; however, since we are interested in the spectral properties of the galaxy clustering, we will ignore it since it contributes only to $k = 0$.) Defining $\nu_{12} \equiv k_1 \cdot k_2 / k_1 k_2$, and the second-order growth factor $E \approx \frac{3}{2} \Omega^{-1/2} D^2$ in an open Universe with no cosmological constant (Bouchet et al. 1992) or $E \approx \frac{3}{2} \Omega^{-1/140} D^2$ in a Universe with a cosmological constant or quintessence (Kamionkowski & Buchalter 1999), we introduce the symmetric kernel

$$J_S^{(2)} \equiv \frac{1}{2} \left( 1 - \frac{E}{D^2} \right) + \frac{1}{2} \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \nu_{12} + \frac{1}{2} \left( 1 + \frac{E}{D^2} \right) \nu_{12} \tag{4}$$

and the second-order convolution integral operator $T^{(2)} \equiv \delta^{(1)} * \delta^{(1)}$ (Fry 1984). Finally, we can simply write $\tilde{\delta}^{(2)} = T^{(2)} J_S^{(2)}$ and

$$\tilde{\delta}^{(2)}(x) = T^{(2)} \left( b_1^E J_S^{(2)} + \frac{1}{2} b_2^E \right). \tag{5}$$

Thus, the galaxy bispectrum is (Matarrese, Verde & Heavens 1997)

$$B_{gB} = 2 D^4 b_1^E \left[ b_1^E J_S^{(2)} + \frac{1}{2} b_2^E \right] P(k_1) P(k_2) + \text{c. t.}, \tag{6}$$

where $P(k, z) = D(z)^2 P(k)$ is the mass linear power spectrum. The skewness of the galaxy density field smoothed on scale $R$ is therefore,
There will be higher-order corrections to the bispectra from
els may be emphasized simply by calculating the difference
The different clustering predictions of the two biasing mod-
3.3 Disentangling the two biasing schemes
Let us now repeat the previous calculations assuming the
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Intriguingly, the dependence on the filtering scale cancels out,
Correspondingly, the skewness difference is
More sophisticated and predictive relations may be proposed if one assumes that the set of Lagrangian bias factors \( \{b^L_j\} \) are not free parameters, as in the present discussion, but rigorously computed within the framework of a given theoretical model. The ‘excursion set’ formalism (Peacock & Heavens 1990; Bond et al. 1991), for example, where dark-matter halos are identified by first-upcrossings of a collapse threshold, predicts that \( b^L_1 \) and \( b^L_2 \) are functions of both halo size and redshift (Mo & White 1996; Porciani et al. 1998).

Clustering of Lyman-break galaxies suggests that biases at redshifts \( z \approx 3 \) could be as large as 3 (for \( \Omega_m \approx 0.3 \)). Matarrese, Verde, & Heavens (1997) have estimated that the Eulerian bias factors could be determined with the 2dF and/or SDS surveys to \( O(\text{few}) \% \). Although not directly applicable, their results suggest that presumably the two biasing prescriptions will be distinguishable with these new surveys.

4 Redshift Distortion Effects
Given that the two biasing schemes are in principle distinguishable, we proceed to calculate the Eulerian and Lagrangian bispectra in redshift space, which is where they are most likely to be measured. Peculiar motions associated with structures on any scale systematically distort the clustering pattern in redshift space (Kaiser 1987). So, in order to reconstruct the actual distribution of galaxies from redshift catalogues, we must be able to invert the distortion process.

This can be easily done if we consider a distant region of the Universe so that the distortions essentially occur along the line-of-sight, and we restrict to large scales for which the mildly non-linear approximation suffices. If \( u \) is the physical coordinate, and \( u = \mathbf{v} \cdot r/r \) is the line-of-sight component of the peculiar velocity \( \mathbf{v} \), assuming that the observer’s peculiar velocity is zero, the apparent galaxy fluctuation \( \delta^G_\nu(s) \)<

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at the apparent position \( s = (1 + u/r)r \) is related to the actual one \( \delta_s \) computed at the same apparent position by the relation

\[
\delta_s^b(s) = \delta_s(s) - u'(s) - \left[ u(s) \left( \delta_s(s) - u'(s) \right) \right].
\] (16)

Here \( u' \) indicates the first radial derivative of \( u \). Since in this section we will compute the effects of redshift distortions on the galaxy bispectrum, both for a local Eulerian and Lagrangian bias, only corrections up-to second order are considered. We remind the reader that in the distant-observer limit, the Fourier transform of \( d^4r \to i\mu \eta w \) where \( \mu = k \cdot r/kr \) and \( \eta = \mu f(\Omega)\eta/k \), where \( f(\Omega) \approx \Omega^{1/6}, \) \( a \) is the universal scale factor, and \( \eta \) is the divergence of the velocity field.

### 4.1 EB galaxy bispectrum in redshift space

In this case, inserting eq. (5) into eq. (16), the Fourier transform of \( \delta_s^b(s) \) is given by

\[
\tilde{\delta}_b^2 = \tilde{b}_s^2 + \mu^2 f(1 + \mu^2) \tilde{\delta}^{(1)} + \tilde{\mathcal{I}}^{(2)} \mathcal{S}_{EB}^{(2)},
\] (17)

where the redshift-distorted symmetric kernel is

\[
\mathcal{S}_{EB}^{(2)} \equiv \tilde{b}_s^2 f J_s^{(2)} + \frac{1}{2} \tilde{b}_s^2
+ \frac{1}{2} \tilde{b}_s^2 f \left[ \mu_1^2 + \mu_2^2 + \mu_1 \mu_2 \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \right]
+ f^2 \left[ \mu_1^2 \mu_2^2 + \frac{1}{2} \mu_1 \mu_2 \left( \mu_1^2 + \mu_2^2 \right) \right].
\] (18)

The quantity \( K^{(2)}_S \) describes the second-order contribution to \( \eta \) (Goroff et al. 1986). The distorted galaxy bispectrum is (Heavens, Matarrese & Verde 1998)

\[
B^{(EB)}_S = 2D(z) \left( b_s^2 + \mu^2 f(1 + \mu^2) \right) \left( \tilde{\mathcal{S}}^{(2)}_{EB} \right)
\]

where

\[
\tilde{\mathcal{S}}^{(2)}_{EB} = \left[ \mathcal{S}_{EB}^{(2)} P(k_1) P(k_2) + c. \ t. \right].
\] (19)

### 4.2 LB galaxy bispectrum in redshift space

We adopt in this case the expression in eq. (9), obtaining, after analogous calculations,

\[
\tilde{\delta}_s^2 = (1 + b_s^2 + b_r^2 + \mu^2 f(1 + b_s^2 + b_r^2 + \mu^2)) \tilde{\delta}^{(1)} + \tilde{\mathcal{I}}^{(2)} \mathcal{S}_{LB}^{(2)},
\] (20)

Thus, the galaxy bispectrum is

\[
B^{(LB)}_S = 2D(z) \left( 1 + b_s^2 + b_r^2 + \mu^2 f(1 + b_s^2 + b_r^2 + \mu^2) \right)
\]

\[
\times \left[ \mathcal{S}_{LB}^{(2)} P(k_1) P(k_2) + c. \ t. \right],
\] (21)

where, in this bias prescription, the redshift-distorted second-order kernel \( \mathcal{S}_{LB}^{(2)} \) is

\[
\mathcal{S}_{LB}^{(2)} \equiv \left( 1 + \tilde{b}_s^2 \right) \mathcal{S}_{LB}^{(2)} + \mathcal{S}_{LB}^{(2)} + \mu^2 f K^{(2)}_S
+ \frac{1}{2} \left( 1 + \tilde{b}_s^2 + \tilde{b}_r^2 \right) f \left[ \mu_1^2 + \mu_2^2 + \mu_1 \mu_2 \left( \frac{k_1}{k_2} + \frac{k_2}{k_1} \right) \right]
+ f^2 \left[ \mu_1^2 \mu_2^2 + \frac{1}{2} \mu_1 \mu_2 \left( \mu_1^2 + \mu_2^2 \right) \right].
\] (22)

### 4.3 Comparing bias in redshift space

If we assume, once again, the validity of the algebraic relation \( b_s^2 = 1 + b_r^2 + b_s^2 \), it follows that between the redshift-distorted kernels holds the relation \( \mathcal{S}_{EB}^{(2)} = \mathcal{S}_{LB}^{(2)} + b_s^2 J_s^{(2)} + \frac{1}{2} b_s^2 - \mathcal{B}_S^{(2)} \). This relation should be immediately compared with the one in eq. (12), to understand that we obtain the concise expression between the quantities \( \Delta B_s^{(2)} \) and \( \Delta B_s^{(2)} \) which emphasize the inconsistency between the two biasing prescriptions,

\[
\Delta B_s^2 = (1 + \mu_1^2 \beta) \left( 1 + \mu_2^2 \beta \right) \Delta B_s,
\] (23)

where \( \beta \equiv f/(1 + b_r^2 + b_s^2) \). Thus, the only redshift effect on the quantity \( \Delta B_s^2 \) comes from the first-order distortion of the galaxy number density field, \( \delta_s^{(1)} = (1 + \mu^2 \beta) \delta_s^{(1)} \). It has to be like that, if one thinks that the distortion effects due to peculiar motions—the terms proportional to \( f \) or to \( f^2 \) in the redshift-distorted kernels \( \mathcal{S}^{(2)} \)—are either independent of the bias factors or proportional to the first-order bias factors, then they cancel out. This feature implies that the capability of the observable \( \Delta B_s^2 \) to discriminate between the two bias schemes we analyzed here is well preserved when transformed to the real-space counterpart \( \Delta B_s^2 \): despite being related to the galaxy bispectrum, it transforms like the lower-order power spectrum. Similarly for the galaxy skewness.

### 5 DISCUSSION AND CONCLUSIONS

We compared the galaxy clustering predictions of the local Eulerian bias scheme versus those of the Lagrangian one. We showed that the two bias models are inconsistent, since the predicted three-point galaxy correlations are different. A similar inconsistency characterizes correlations of higher order, or of lower order but higher perturbative corrections. Qualitatively, these results are independent on whether the Lagrangian zero-order bias factor \( b_r^0 \) is zero, as for Press-Schechter dark matter halos, or not, as in the most general case we have considered here. The galaxy bispectrum is possibly better suited to distinguish between the two bias models than the corresponding skewness, since the latter is spatially averaged: the bispectrum depends on the shape of the triangle \( k_1 + k_2 + k_3 = 0 \), thus two shapes can disentangle the two bias factors \( b_1 \) and \( b_2 \) (Matarrese, Verde & Heavens 1997; Scoccimarro 2000) and the two bias models. The next generation redshift catalogues, as the ongoing Two Degree Field Survey and the Sloan Digital Sky Survey, will contain enough galaxies to establish whether \( B^{(EB)}_S \) in eq. (19) or \( B^{(EB)}_S \) in eq. (21) better fits the observational data, but they cannot be both correct, whatever the assumed cosmology.

Both bias schemes represent rather extreme and idealized approaches. Lagrangian models imply a sort of infinite-memory process, since the sites for galaxy formation are known from the beginning, and dynamical evolution changes their spatial distribution. On the other hand, in local Eulerian schemes galaxies are simply ‘painted’ on a snapshot of the density field, without a record of the past. However, even though real galaxy formation is probably a process with intermediate characteristics with respect to the biasing schemes discussed here, recent models based on a Lagrangian selection of the sites for object formation were shown to be very successful in reproducing the clustering of dark-matter halos found in numerical simulations (e.g. Catelan, Matarrese & Porciani 1998; Porciani, Catelan & Lacey...
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The issue discussed in this Letter surely deserves further investigation. It would be of interest to test which biasing scheme better describes galaxy power spectrum and bispectrum from a combination of numerical simulations and semianalytic models (Porciani et al., in preparation).

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