Nonlocality at the quantum level manifests itself in various kinds of phenomena. The study of this so far, has been predominantly confined to the study of interactions amongst similar kinds of particles, for example, photon-photon interactions or the interaction of subatomic particles among themselves. With the advance of technology over the last several years, it has now become conceivable to investigate a new kind of nonlocality in a controllable fashion, viz., the nonlocality generated through the interaction of distinct entities, like atoms and photons inside a high quality cavity.

The mathematical framework for demonstrating the violation of local realism in quantum mechanics was first provided by Bell [1] through his famous inequalities. This work was subsequently generalized and also extended to consider the interaction of more than two particles [2,3]. A different kind of proof of nonlocality without the use of inequalities, also exists [4]. Furthermore, it has been shown that quantum nonlocality continues to persist even for the case of a large number of particles, or large quantum numbers [5]. This has raised certain questions regarding the issue of the macroscopic or classical limit of quantum mechanics in examples where both the number of particles, and their quantum number is made arbitrarily large [6]. The phenomenon of environment induced decoherence is of direct relevance here. It would be interesting if decoherence could be experimentally controlled and its effect on nonlocality be quantitatively monitored in particular examples of study.

Experimental proposals of demonstrating nonlocality have mostly been concerned with spin-1/2 particles [7], photons [8,9], or mesons [10]. In recent times, several schemes involving two-level Rydberg atoms have been proposed [11,12]. The primary advantages of the experiments using atoms, compared to those with photons or spin half particles are two-fold. First, the realization of spacelike separation for Rydberg atoms is easier because of their smaller velocities than photons or electrons. Secondly, the efficiencies of detectors used for the former is much larger in general in comparison with the detectors used for elementary particles. In addition, the interaction of large-sized atoms with the environment can be significant enough to be monitored in certain cases. In fact, in the experimental schemes which involve the interaction of Rydberg atoms with photons in a microwave cavity, dissipation through the loss of cavity photons always occurs. The effect of this is manifested in the form of loss of coherence in the atom-photon interactions. Thus, this is a natural arena to study the effects of decoherence on quantum nonlocality in a quantitative manner.

In particular we consider the following experimental scenario. A two-level atom initially in its upper excited state $|e\rangle$ traverses a high-Q single mode cavity. The cavity is in a steady state and tuned to a single mode resonant with the transition $|e\rangle \rightarrow |g\rangle$. The emerging single-atom wavefunction is a superposition of the upper $|e\rangle$ and lower $|g\rangle$ state, and it leaves an imprint on the photonic wavefunction in the cavity. After leaving the cavity, the atom passes through an electromagnetic field which gives it a $\pi/2$ pulse the phase of which can be varied for different atoms. The atom then reaches the detector, placed at a sufficient distance, capable of detecting the atom only in the upper or lower state. Thus, the role of the $\pi/2$ pulse may be considered as a component of the detection mechanism in the experiment [13]. During the whole process, dissipation takes place, and is taken into account. Next, this process is repeated for a similar second atom. The important difference is that the second atom interacts with a photonic wavefunction which has been modified due to the passage of the first atom. There is no direct interaction between the two atoms, although secondary correlations develop between them. In other words, though there is no spatial overlap between the two atoms, the entanglement of their wavefunctions with the cavity photons can be used to formulate local-realist bounds on the detection probabilities for the two atoms [11,12].

The interplay of the atomic statistics with the photonic statistics plays a crucial role in the investigation of nonlocality here. The initial state of the cavity is built up by the passage of a large number of atoms, but only one at a time, through it. The pump parameter and the atom-photon interaction time are key inputs for the profile of the resultant photonic wavefunction which in turn governs the nature of entanglement between two
successive experimental atoms detected in their upper or lower states by the detector. As stated earlier, dissipation due to the interaction of the pumping atoms with their reservoir, as well as the loss of cavity photons can be controlled, and their effects on the statistics of detected atoms can be studied. The formalism used by us has another generic feature. The effects of decoherence on nonlocality can be studied in context of the micromaser, as well as the microcavity, its optical counterpart.

It is easy to obtain a Bell-type inequality suitable for the scenario considered by us in analogy to Bell’s original reasoning. Two level Rydberg atoms are analogous to spin-1/2 systems, and the phase of the electromagnetic field plays the role of the polarization axis of the Stern-Gerlach apparatus used for spin-1/2 systems. In fact, several local realist bounds have earlier been derived to tailor such a situation [11,12]. Let us very briefly describe one such derivation [12] which we shall use in the present analysis. Assigning the value +1 for the atom detected in the upper state |e>, and −1 for the lower state |g>, one can in any local realist theory define functions \( f(\phi) = \pm 1; \ g(\phi) = \pm 1 \) describing the outcome of measurement on the atom 1 and 2 when the phase of the electromagnetic field giving \( \pi/2 \) pulse to the atoms is set to be \( \phi_1 \) and \( \phi_2 \) for the respective atoms. The ensemble average for double click events is therefore defined as

\[
E^{\lambda}(\phi_1, \phi_2) = \int d\lambda f(\phi_1)g(\phi_2)
\]  

(1)

where \( \lambda \) is a suitable probability measure on the space of all possible states.

The quantum mechanical expectation value for double click events is calculated from the probabilities of all possible double-click sequences. This is given by

\[
E(\phi_1, \phi_2) = P_{ee}(\phi_1, \phi_2) + P_{gg}(\phi_1, \phi_2)
- P_{eg}(\phi_1, \phi_2) - P_{ge}(\phi_1, \phi_2)
\]  

(2)

where \( P_{eg}(\phi_1, \phi_2) \) stands for the probability that the first atom is found to be in state |e> after traversing the \( \pi/2 \) pulse with phase \( \phi_1 \), and the second atom is found to be in state |g> with the phase of the \( \pi/2 \) pulse being \( \phi_2 \) for its case. Defining \( E_0 = E^{\lambda}(\phi_1 = \phi_2) \) and \( M_0 = P_{ee}(\phi_1 = \phi_2) + P_{gg}(\phi_1 = \phi_2) \), and assuming perfect detections, it follows that \( E_0 = 2M_0 - 1 \). Further, it is easy to see that \( f(\phi) = +g(\phi) \) with probability \( M_0 \), and \( f(\phi) = -g(\phi) \) with probability \( (1 - M_0) \). Hence, \( E^{\lambda}(\phi_1, \phi_2) \) can be written as

\[
E^{\lambda}(\phi_1, \phi_2) = E_0 \int dpf(\phi_1)f(\phi_2)
\]  

(3)

Now, one can define a Bell sum

\[
B \equiv |E^{\lambda}(\phi_1, \phi_2) - E^{\lambda}(\phi_1, \phi_3)|
+ \text{sign}(E_0)[E^{\lambda}(\phi_2, \phi_3) - E_0]
\]  

(4)

It follows immediately from (2-4) that \( B \leq 0 \). Below we shall calculate this Bell sum \( B \) and see how it evolves for various values of the cavity parameters. It is convenient to set the values of the phases \( \phi_1 = 0, \phi_2 = \pi/3, \) and \( \phi_3 = 2\pi/3 \), as for these values the Bell-type inequality is always violated, i.e., \( B > 0 \) for the case of an idealised micromaser [12].

In realistic situations, one must consider the interacting systems (atoms as well as cavity field) coupled to their respective reservoirs. The couplings are governed by their equations of motion [14],

\[
\dot{\rho}_{\text{atom-reservoir}} = -\gamma(1 + \bar{n}_{th})(s^+s^-\rho - 2s^-\rho s^+ + \rho s^+s^-)
- \gamma\bar{n}_{th}(s^-s^+\rho - 2s^+\rho s^- + \rho s^-s^+)
\]  

(5)

for the atom and

\[
\dot{\rho}_{\text{field-reservoir}} = -\kappa(1 + \bar{n}_{th})(a\dagger a\rho - 2a\rho a\dagger + \rho a\dagger a)
- \kappa\bar{n}_{th}(aa\dagger \rho - 2a\dagger a\rho + \rho aa\dagger)
\]  

(6)

for the field. \( \rho \) is the reduced density operator obtained after tracing over the reservoir. \( \gamma \) and \( \kappa \) are the decay constants for the atom and the field respectively. \( \bar{n}_{th} \) is the average photon number representing the reservoir. \( s^+ \) and \( s^- \) are the usual Pauli operators for the pseudospin representation of the two-level model of the atom. \( a(a\dagger) \) is the photon annihilation (creation) operator.

The dynamics we are interested in, involves two-level atoms steamed into a single mode cavity in such a way that there is at most one atom in the cavity at any time. Thus we have sequences of events (atom-field interactions) taking place randomly, but with each event of a fixed duration \( \tau \). This interaction is governed by

\[
\dot{\rho} = [H, \rho]
\]  

(7)

where \( H = g(s^+a + s^-a\dagger) \) is the well known Jaynes-Cummings [15] Hamiltonian with \( g \) being the coupling constant.

Thus, we have to solve the equation of motion

\[
\dot{\rho} = \dot{\rho}_{\text{atom-reservoir}} + \dot{\rho}_{\text{field-reservoir}} + \dot{\rho}_{\text{atom-field}}
\]  

(8)

where the terms on the r.h.s. are the r.h.s’ of (5), (6) and (7) respectively. For the duration between two events, we have to solve the equation of motion (6) only.

The steady-state photon statistics of the cavity field undergoing such dynamics has been presented in detail in [16]. The experimental atoms on which we plan to test the Bell’s inequality (BI), encounter this steady state radiation field. It will be appropriate to mention here that although decoherence effects are inherent in the build up of the cavity field to its steady state since it encounters a large number of atoms over the time required for
the steady state to be reached, the dynamics of a single experimental atom interacting with this field for a short duration will be unaffected by the decoherence effects, as has been observed in the cavity-QED of Jaynes-Cummings interaction [17]. It was shown there that typical durations of atom-field interactions in realistic environments can be as large as \( t \sim 10/g \) up to which decoherence effects are insignificant. We again stress here that even though the individual atom-field interaction time in the dynamics of the cavity field, pumped repeatedly with atoms, is uniformly of this order, the dissipative forces there play a crucial role due to the number of atoms \( (\gg 1) \) and the time \( (\gg 10/g) \) involved in reaching the steady state. Indeed, we keep the above argument in mind in our choice of parameters while calculating the probabilities in the Bell sum (4).

![Diagram](image1.png)

**FIG. 1.** Violation of Bell’s inequality in a micromaser [20]. Atoms in the upper of the two Rydberg levels are streamed into a cavity, one at a time, in such a way that the flight time of an atom \( \tau \) is much shorter than the lifetime of the long-lived Rydberg levels. Hence, we set the atomic decay constant \( \gamma = 0 \). The average thermal photons \( \bar{n}_{th} = 0.15 \) in the cavity represent its temperature at 0.5 K. The leakage of the cavity photons is represented by \( \kappa/g = 0.1 \times 10^{-6} \). The pump rate \( N \), the number of individual atoms that pass through the cavity in a photon lifetime, = 20 (full), 50 (broken), and 100 (dotted). \( D = \sqrt{Ng}\tau \). The parameters are very close to the experimental data in [20].

Our results are shown in Figure 1 (micromaser) and Figure 2 (microlaser). In general, BI is violated in such dynamics involving atom-photon interactions. Our study shows clearly that the Bell sum reflecting the degree of nonlocality exhibited in the atom-atom secondary correlations depends heavily on decoherence effects. In particular, it is seen that the value of Bell sum \( B \) decreases with the increase of pump rate \( N \) for a large range of interaction times \( \tau \). This can be understood from the way decoherence effects creep into the dynamics through two parameters \( N \) and \( \tau \) [16]. The genesis of atom-photon and the resultant atom-atom entanglement competes with decoherence in an interesting fashion over time. For shorter values of single atom interaction times we find that the correlations build up sharply with \( \tau \), and the peak value of \( B \) signifying maximum violation of BI is larger for higher values of \( N \), as a magnification of Figure 1 reveals (similar trends are also observed for the micromaser).

The latter feature is a curious example of multiparticle induced nonlocality. This is analogous to the enhancement of nonlocality for multiparticle systems, and is in conformity with the mathematical demonstration of larger violation of BI with increase in the number of particles involved [5,6]. For short interaction times, naturally the effects of decoherence are too small to affect the correlations. One noticeable feature in Figure 1 is the structures for low values of \( N \) (full line). This originates from “trapped state” dynamics of photonic statistics [16,18] where it has been shown that dissipative effects gradually wash out such states, as can be seen from the broken and dotted lines. In case of the microlaser, atomic damping \( \gamma \) is a dominating factor, in contrast to the cavity photon loss in the micromaser. This makes the Bell sum fall off rapidly for large values of \( \tau \). However, the second peak in \( B \), a consequence of the Jaynes-Cummings dynamics, survives such dissipation [19].

![Diagram](image2.png)

**FIG. 2.** Demonstration of nonlocality in a microlaser, the optical counterpart of the micromaser. The results of our numerical simulations can be tested in a microlaser of the type in [21]. Atomic levels having transition frequency in the optical regime have shorter lifetimes compared to Rydberg levels, and hence, we set \( \gamma/g = 0.1 \). At optical frequencies, thermal photons are very close to zero, and thus we take \( \bar{n}_{th} = 0 \). \( N = 100 \). The cavity leakage rate is \( \kappa/g = 0.01 \) (full), 0.001 (broken) and 0.0001 (dotted).

To summarize, we have shown that a demonstration of nonlocality, encompassing several of its varied aspects in atom-photon interactions in cavities and the effects of de-
coherence on it, can be possible in experimental set-ups already in operating conditions for the micromaser [20], as well as for the microlaser [21]. Certain notable features, such as enhancement of nonlocality for increased number of atoms, when decoherence effects are small, can be observed. We have seen how such features can be quantitatively monitored by control of decoherence. In an actual experiment, certain points have to be borne in mind. A few atoms may go undetected between two detector clicks. However, the steady state nature of the cavity field contributes to making the effect of this on the Bell sum insignificant compared to the effect of decoherence which we have probed in detail. Finally, the observed magnitude of violation of BI would be brought down by finite detector efficiency. Nevertheless, our selection of the particular type of BI [12], and the phases of $\phi$, ensure that this BI is always violated for the range of parameters chosen irrespective of the efficiency factor of the detector, which can in any case be accounted for easily by the introduction of appropriate scaling factors in the expressions of the various probabilities appearing in the Bell sum.

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