High Energy Neutrinos from Gamma Ray Bursts:
Event Rates in Neutrino Telescopes

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Abstract

Following Waxman and Bahcall [1] we calculate the event rate, energy and zenith
angle dependence of neutrinos produced in the fireball model of gamma ray bursts
(GRB). We emphasize the primary importance of i) burst-to-burst fluctuations and
ii) absorption of the neutrinos in the Earth. From the astronomical point of view,
we draw attention to the sensitivity of neutrino measurements to the boost Lorentz
factor of the fireball $\Gamma$, which is central to the fireball model, and only indirectly
determined by follow-up observations. Fluctuations result in single bursts emitting
multiple neutrinos, making it possible to determine the flavor composition of a beam
observed after a baseline of thousands of Megaparsecs.

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1 Introduction

The origin of GRB is one of the most fascinating outstanding problems in astronomy. Their
observed energy injection in the Universe is sufficiently large to possibly resolve another
long-standing puzzle: the origin of the highest energy cosmic rays [2, 3]. Mounting evidence
suggests that GRB emission is produced by a relativistically expanding fireball, energized
by a process involving neutron stars or black holes\cite{4}. In the early stages the fireball, its radiation trapped by the very large optical depth, cannot emit photons efficiently. The fireball’s kinetic energy is therefore dissipated until it becomes optically thin — a scenario that can explain the observed energy and time scales of GRBs, provided the bulk Lorentz factor of the expanding flow, $\Gamma$, is $\geq 10^2 - 10^3$.

Protons accelerated in shocks in the expanding fireball interact with photons to produce charged pions, the parents of high-energy neutrinos \cite{1, 5}. Assuming that particles accelerated in the GRB sources produce the observed cosmic rays above the “ankle” of the energy spectrum near $3 \times 10^{18}$ eV, one derives that the average single burst produces only $\sim 10^{-2}$ neutrino events in a high energy neutrino telescope with 1 km$^2$ effective area. Although the expected rate is therefore low, the neutrino signal should be relatively easy to observe provided the detector is large enough: GRB neutrinos will have a hard spectrum extending well beyond the background from atmospheric neutrinos and, even more important, the high-energy GRB neutrino events should coincide with observed GRB photon events within a narrow time window.

In this paper we calculate the experimental signatures of GRB in a kilometer-scale neutrino detector such as the proposed IceCube \cite{6}. We emphasize the importance of taking into account burst-to-burst fluctuations \cite{7} as well as absorption of the neutrino signal in the Earth for both event rates and experimental signatures. Both effects produce additional and striking signatures with discriminating sensitivity to the value of the bulk Lorentz factor whose value is only indirectly inferred from other astronomical observations \cite{4, 8}.

The observation of GRB neutrinos over a cosmological baseline has scientific potential beyond testing the “best-buy” fireball model: the observations can test with unmatched precision special relativity and the equivalence principle, and study oscillating neutrino flavors over the ultimate baseline of $z \simeq 1$ \cite{1}.

2 Calculation of GRB Neutrino Rates and Signatures

In calculating the event rates and experimental signatures of GRB neutrinos in a high energy neutrino telescope we follow the model of Waxman and Bahcall \cite{1} as implemented by Halzen
and Hooper [7]. The neutrino flux is given by:

$$\frac{dN_\nu}{dE_\nu} = \begin{cases} \frac{A}{E_B} \frac{1}{E_\nu}; & E_\nu < E_B \\ \frac{A}{E_\nu^2}; & E_\nu > E_B \end{cases}$$  (1)

where $A$ is a normalization constant that is determined from energy considerations. The observed neutrino rates are calculated by folding this generic flux over the distributions of individual bursts in distance, $f(z)$, energy, $f'(E_{GRB})$ and boost factor, $f''(\Gamma)$:

$$N_\nu \propto \int \int \int \frac{dN_\nu}{dE_\nu}(E_{GRB}, z, \Gamma, E_\nu) P(E_\nu) f(z) f'(E_{GRB}) f''(\Gamma) dE_{GRB} dz d\Gamma dE_\nu.$$  (2)

where $E_{GRB}$ is the energy emitted by a particular GRB, $z$ its redshift and $\Gamma$ the boost factor. $P(E_\nu)$ represents the efficiency of detecting a neutrino of energy $E_\nu$. The first two distributions can be modelled after observations: a cosmological distribution in distance, and an energy distribution which assumes that ten percent of GRB produce more energy than average by a factor of ten, and one percent by a factor of 100 [4].

Most important however are the fluctuations in the $\Gamma$ factor around its average phenomenological value of $10^2$–$10^3$. The fluctuations in $\Gamma$ affect the efficiency for producing pions in the $p - \gamma$ collisions in the fireball as $\Gamma^{-4}$ [1], as well as the break energy, $E_B$, which varies as $\Gamma^2$. Unfortunately the distribution in $\Gamma$ cannot even be guessed at. Nevertheless, it is critical in making quantitative predictions[7]. The physics is clear. In GRBs, high luminosities are emitted over short times, therefore the large photon density would render GRB opaque unless $\Gamma$ is very large. Only transparent sources with large boost factors emit photons. They are however relatively weak neutrino sources because the actual photon target density in the fireball is diluted by the large Lorentz factor. An even moderately reduced value of $\Gamma$ will produce a prolific neutrino source.

We remind the reader that the results obtained from Eq. 2 are at variance with the neutrino rate obtained by multiplying the average rate per burst by the number of bursts. Even neglecting the all important fluctuations in $\Gamma$, there is no such concept as an average GRB. E.g. for $\Gamma = 300$, the correct computation of Eq. 2 yields a rate of $\sim 75$ events per km$^2$ and year, roughly an order of magnitude larger than the prediction obtained by neglecting the observed burst-to-burst fluctuations in distance and energy. Another consequence of fluctu-
ations is that the signal is dominated by a few very bright bursts, which greatly simplifies their detection.

As pointed out above, the average neutrino energy varies with the square of boost factor and therefore the calculated event rates, especially their dependence on $\Gamma$, is strongly affected by the fact that higher energy neutrinos are preferentially absorbed in the Earth before reaching the detector [9]. Also this effect has been neglected in all previous calculations. For instance, the 75 events just mentioned are reduced by a factor 3 by absorption. One should on the other hand remember that by oscillations, a large fraction of $\nu_\mu$ can oscillate into $\nu_\tau$ which penetrate the Earth [10]. It is in this context important to realize that a kilometer-scale detector such as IceCube can measure the energy of the neutrinos. Therefore, signal events can be separated from the low energy atmospheric background by energy measurement, which allows the identifications of neutrinos from all directions and not just in the hemisphere where they pass through the Earth.

3 Results

We calculate the flux of neutrinos from GRB in the fireball model following reference [7]. The number of protons in the fireball is fixed by the assumption that they are the source of the ultra high energy cosmic rays above $10^{19}$ eV. The results are shown in Figure 1 which shows the $\nu_\mu + \bar{\nu}_\mu$ flux from GRBs for different values of $\Gamma$. The fluxes have been multiplied by $E_\nu^2$ so that they represent the energy emitted in the form of neutrinos. Notice the variation of the break in the spectrum $E_B$ as $\Gamma^2$. Also, for values of $\Gamma$ below $\sim 100$ the fireball becomes opaque to protons and at this point the total amount of energy available for neutrino production is converted. The neutrino flux, which roughly scales as $\Gamma^{-4}$, saturates and no longer grows with decreasing boost factor; see Figure 1.

As usual [9], we calculate the event rate per GRB by folding the neutrino flux with the probability of detecting a muon produced in a $\nu_\mu$ or $\bar{\nu}_\mu$ interaction. Fluctuations in distance $z$ and total GRB energy, or fluency, $E_{GRB}$ are accounted for by a Monte Carlo simulation we have developed for this purpose. We simulate a large number of GRBs at different zenith angles assuming an isotropic distribution and subsequently obtain the event rate per year by
Figure 1: Left: $\nu_\mu + \bar{\nu}_\mu$ fluxes from GRB for different values of the Lorentz factor $\Gamma$ assuming GRBs are responsible for the observed cosmic ray spectrum above $10^{19}$ eV. Right: Energy injected in GRB neutrinos as a function of the boost factor $\Gamma$ of the fireball.

assuming 1000 GRBs/year. Absorption of the neutrinos in the Earth prior to reaching the detector is taken into account using the density profile of reference [11]. It is also important to implement the fact that above the detector there is a limited column density of atmosphere and ice available for neutrino detection.

In Table 1 we separately show, for different $\Gamma$’s, the event rate of upgoing, i.e. neutrinos that cross the Earth before interacting near the detector, and downgoing neutrinos. The first two columns show for comparison the number of events when neither absorption nor the limited amount of target above the detector are taken into account. It is clear that both effects play an important role in obtaining the correct event rate. This is not surprising: the Earth becomes opaque to neutrinos of energy around 100 TeV, and the muon range exceeds the $\sim 2$ km vertical depth of the IceCube detector at energies around 1 TeV.
<table>
<thead>
<tr>
<th>Events/(km$^2$ yr) in 2π sr</th>
<th>No absorption</th>
<th>Absorption</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Downgoing</td>
<td>Upgoing</td>
</tr>
<tr>
<td>$\Gamma = 100$</td>
<td>1133</td>
<td>1112</td>
</tr>
<tr>
<td>$\Gamma = 300$</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>$\Gamma = 1000$</td>
<td>0.14</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Table 1: $\nu_\mu + \bar{\nu}_\mu$ events (km$^{-2}$ yr$^{-1}$). Only fluctuations in distance and energy are taken into account.

The dependence of the number of events on the $\Gamma$ factor is shown in Figure 2. Two competing effects determine the shape of the curve. The event rate decreases with increasing $\Gamma$ following the dependence of the neutrino flux which varies as $\Gamma^{-4}$. This decrease is partially offset because higher energy neutrinos resulting from larger boost factors are more efficiently detected. For low values of $\Gamma$, below about 100, the saturation of the total energy available for neutrino production is seen. On the other side of the $\Gamma$ range, for large values of $\Gamma$, the spectrum is very flat ($\sim E^{-1}$) up to $\sim 70$ PeV where the absorption by the Earth dominates. This reduces the event rate of upgoing events for values of $\Gamma$ above 1000 as can be seen in Figure 2. In the end the downgoing and upgoing event rates are similar except for large values of $\Gamma$. It is important to keep in mind that although downgoing neutrinos are not affected by absorption, their detection is limited by the column density of matter available for neutrino interaction.

In Figure 3, we show the energy dependence of upgoing and downgoing $\nu_\mu + \bar{\nu}_\mu$ events for three representative values of $\Gamma$. The zenith angle distribution of upgoing neutrinos is shown in Figure 4, i.e. $-1 < \cos(\theta_{\text{zenith}}) < 0$. As $\Gamma$ increases, the higher energy neutrinos are attenuated by the Earth. This explains why, as $\Gamma$ increases, the distributions increasingly resemble an exponential attenuation function.
Figure 2: $\nu_{\mu} + \bar{\nu}_{\mu}$ event rate per km$^2$ yr as a function of the boost factor $\Gamma$ taking into account fluctuations in distance and the GRB fluency. Absorption in the Earth and the limited target above the detector are taken into account.

### 3.1 $\nu_{\tau} + \bar{\nu}_{\tau}$ events

$\nu_{\tau}$ production is expected to be very small in GRBs and in general in any astrophysical environment where $\nu$’s are produced in $p - p$ or $p - \gamma$ collisions. Several calculations suggest a ratio [12]:

$$
F_{\nu_{\tau}/\nu_{\mu}} = \frac{\Phi(\nu_{\tau} + \bar{\nu}_{\tau})}{\Phi(\nu_{\mu} + \bar{\nu}_{\mu})} \sim 10^{-5}, \quad (3)
$$

Oscillation scenarios in which $\nu_{\mu}$’s convert into $\nu_{\tau}$’s can however provide abundant sources of $\nu_{\tau}$’s. Assuming a typical value $F_{\nu_{\tau}/\nu_{\mu}} = 0.5$ suggested by SuperKamiokande measurements [13], we obtain the double bang [14] $^1$ event rates shown in Table 2. The probability of detecting a $\nu_{\tau}$ induced double bang event in IceCube is typically two orders of magnitude

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$^1$Events in which two separated showers can be identified, one initiated by the struck nucleon and the other by the decay of the $\tau$ produced in the $\nu_{\tau}$ charged current interaction.
Figure 3: Energy distribution of $\nu_\mu + \bar{\nu}_\mu$ events for $\Gamma =100$, 300 and 1000. Solid (dashed) lines correspond to upgoing (downgoing) neutrinos. Only fluctuations in distance and energy are accounted for. Absorption in the Earth and the limited target above the detector are taken into account.

smaller than the probability of observing a $\nu_\mu$ at energies around 10 PeV [12, 15]. This explains the small values of the event rates in Table 2. In the event rate calculation we have accounted for the energy loss of the $\nu_\tau$'s when they propagate along the Earth’s interior. This produces a pileup of events around 100 TeV, as pointed out in [10], reducing the number of upgoing double bang events with respect to the downgoing ones. This is due to the probability of $\nu_\tau$-induced double bang detection which is limited to a broad peak between $\sim 1$ PeV and $\sim 100$ PeV (outside of which it is negligible). This also explains the fact that the energy distribution of the event rate peaks in the vicinity of 10 PeV (see Figure 5).

We also calculated the $\nu_\tau + \bar{\nu}_\tau$ events that would be detected by the appearance of a $\tau$ which decays to $\mu$ just below the detector. The event rates are also shown in Table 2. In this
Figure 4: \( \cos(\theta_{\text{zenith}}) \) distribution of \( \nu_\mu + \bar{\nu}_\mu \) events for \( \Gamma =100, 300 \) and 1000. \( \cos(\theta_{\text{zenith}}) = -1 \) corresponds to upgoing neutrinos and \( \cos(\theta_{\text{zenith}}) = 0 \) to horizontal. Notice that for \( \Gamma = 1000 \) we have multiplied the event rate by a factor 1000.

The energy distribution of the events peaks around the energy at which the events pileup (100 TeV); see Figure 5. The probability of detecting the \( \mu \) is \( \sim 17 \% \) of the probability of detecting it in a \( \nu_\mu \) interaction due to the branching ratio of the \( \tau \) to \( \mu \) decay channel. This accounts for a factor \( \sim 6 \) difference between \( \nu_\tau \to \tau \to \mu \) in Table 2 and the \( \nu_\mu + \bar{\nu}_\mu \) rate in Table 1.

The expected event rates generated by both mechanisms (double bang and \( \tau \to \mu \) decay) have very different and characteristic zenith angle distributions. These are shown in Figure 6. For large column depths inside the Earth (i.e. \( \cos(\theta_{\text{zenith}}) \sim -1 \)), the events pileup around 100 TeV and the small probability of detecting a double bang event at this energy reduces the number of double bangs with respect to events in which a \( \mu \) is detected. When \( \cos(\theta_{\text{zenith}}) \sim 0 \), because the amount of matter neutrinos have to cross is very small, there...
Double Bang

<table>
<thead>
<tr>
<th>Events/(km$^2$ yr) in 2π sr</th>
<th>Double Bang</th>
<th>ν$_\tau$ → τ → µ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Downgoing</td>
<td>Upgoing</td>
</tr>
<tr>
<td>Γ = 100</td>
<td>0.54</td>
<td>0.13</td>
</tr>
<tr>
<td>Γ = 300</td>
<td>3.6 × 10$^{-2}$</td>
<td>8.8 × 10$^{-3}$</td>
</tr>
<tr>
<td>Γ = 1000</td>
<td>2.9 × 10$^{-4}$</td>
<td>6.6 × 10$^{-5}$</td>
</tr>
</tbody>
</table>

Table 2: ν$_\tau$ + ¯ν$_\tau$ double bang events and events in which the τ decays to µ (km$^{-2}$ yr$^{-1}$). Only fluctuations in distance and energy are considered. Absorption in the Earth and the limited column of matter above the detector are taken into account.

is no pileup of events and the angular distributions have roughly the same shape. Figure 6 also shows the zenith angle distribution of events produced by ν$_\mu$ + ¯ν$_\mu$ for comparison, making it clear that despite of the flatter distribution of the µ’s from ν$_\tau$ + ¯ν$_\tau$ they are still outnumbered by µ’s from ν$_\mu$ interactions.

### 3.2 Fluctuations in Γ

Finally, we discuss burst to burst fluctuations in the boost factor Γ. So far we, quite unrealistically, assumed that Γ is single valued. We have, at present, no information on the distribution of Γ factors. It has been shown [7] that the neutrino rates can be significantly enhanced by fluctuations around the average value. It may be, of course, that the fluctuations in energy and distance which we took into account following the experimental evidence, already reflect some or all of the fluctuations in the boost factor. Following reference [7] we will illustrate the effect by assuming Gaussian distributions with half width σ. The results are shown in Table 3 for different values of ⟨Γ⟩. It is interesting to note that for ⟨Γ⟩ = 100 the event rates are almost independent of the value of σ. For Γ around 100 the event rate is weakly dependent on Γ due to the saturation of the amount of energy that goes into neutrino production (see Figures 1 and 2) and hence fluctuations in the value of the boost factor do not affect the event rate. This is not the case when ⟨Γ⟩ = 300 or 1000 since the event rate behaves roughly as Γ$^{-4}$ in that Γ range. Figure 7 shows more clearly the dependence of the
Figure 5: Energy distribution of upgoing $\nu_\tau + \bar{\nu}_\tau$ double bang events (dashed line) and events in which the $\tau$ decays to $\mu$ (solid line) for $\Gamma = 100$, 300 and 1000. The double bang event rate has been multiplied by a factor 100 in all the plots. Absorption in the Earth and the limited target above the detector are taken into account.

event rate on $\sigma$ for different values of $\langle \Gamma \rangle$ illustrating these points. It can be easily shown that when $\sigma \ll \langle \Gamma \rangle$ the following relation holds:

$$ \frac{\text{Rate}(\langle \Gamma \rangle \pm \sigma)}{\text{Rate}(\langle \Gamma \rangle)} \propto \frac{\sigma}{\langle \Gamma \rangle}, $$

explaining why for $\langle \Gamma \rangle = 1000$ the variation of the event rate with $\sigma$ is not as strong as when $\langle \Gamma \rangle = 300$, even though the rate scales with $\Gamma^{-4}$ in both cases. The same comments apply to double bang events produced by $\nu_\tau$'s and events in which the produced $\tau$ decays to $\mu$ (both are shown in Table 4).
Figure 6: $\cos(\theta_{\text{zenith}})$ distribution of $\nu_{\tau} + \bar{\nu}_{\tau}$ double bang events (dashed line) as well as events in which the $\tau$ decays to $\mu$ (solid line), for $\Gamma = 100$, 300 and 1000. $\cos(\theta_{\text{zenith}}) = -1$ corresponds to upgoing neutrinos and $\cos(\theta_{\text{zenith}}) = 0$ to horizontal. Also shown for comparison are the $\nu_{\mu} + \bar{\nu}_{\mu}$ event rates (dotted line).

4 Summary and conclusions

We have investigated potential signatures in km$^3$ telescopes of high energy neutrino fluxes produced in $p - \gamma$ interactions in GRB environments. We stress the fact that the rate is dominated by fluctuations in distance, GRB energy and in the bulk Lorentz factor $\Gamma$ of the expanding GRB fireball. We have used an euclidean distribution to account for fluctuations in distance. Using a distribution in which GRB’s follow the star formation rate, as suggested in [18], the event rates are reduced only by $\sim 20\%$. On the other hand, a cosmological distribution following galaxies [16, 17] allows for more close GRB’s so that the event rate increases by a factor of 20 (see Table 5). Neutrino telescopes may help to constrain the
Table 3: \( \nu_\mu + \bar{\nu}_\mu \) events (km\(^{-2}\) yr\(^{-1}\)), taking into account fluctuations in boost factor \( \Gamma \), distance and energy. The distribution of boost factors is assumed to be a Gaussian of half width \( \sigma \) centered in \( \langle \Gamma \rangle \). Absorption in the Earth and the limited column of matter available for neutrino interaction above the detector are taken into account.
Table 4: $\nu_\tau + \bar{\nu}_\tau$ double bang events and events in which the $\tau$ decays producing a $\mu$ (km$^{-2}$ yr$^{-1}$). We take into account fluctuations in boost factor $\Gamma$, distance and energy. The distribution of boost factors is assumed to be a Gaussian of half width $\sigma$ centered in $\langle \Gamma \rangle$. Absorption in the Earth and the limited column of matter available for neutrino interaction above the detector are taken into account.
Figure 7: Upgoing $\nu_\mu + \bar{\nu}_\mu$ event rate as a function of $\sigma$, the half width of the boost distribution around $\langle \Gamma \rangle$, for $\langle \Gamma \rangle = 100$, 300 and 1000. The rate for $\langle \Gamma \rangle = 1000$ has been scaled up by a factor 100. Absorption in the Earth is taken into account.

distribution in distance of GRBs. The most relevant parameter is $\Gamma$, which determines the rate of $p-\gamma$ interactions and hence the amount of energy that goes into neutrino production. For small values of $\Gamma$ ($\Gamma \sim 50$ or less) the expansion of the GRB fireball is not sufficiently fast and the large photon density makes it opaque to $p-\gamma$, efficiently producing pions — the parents of the neutrinos — and saturating the amount of energy available for neutrino production. Due to the scaling of the energy break in the spectrum with $\Gamma^2$, mostly low energy neutrinos are produced whose detection efficiency is smaller due to the small range of the muon at low energies and the relatively high energy threshold of the neutrino telescopes ($\sim 100$ GeV). However the large neutrino flux compensates for the small detection efficiency. For large values of $\Gamma$ ($\Gamma > 300$), an increasingly larger fraction of the neutrino energy goes into the high energy part of the spectrum, however the overall amount of energy is very
small, producing small event rates. Values of $\Gamma < 100$ give rise to large number of events that would even be observable in existing smaller neutrino telescopes such as AMANDA [19] (one has to scale the results down by roughly two orders of magnitude to account for the smaller effective area of the detector).

We have shown that absorption of the upgoing $\nu_\mu$'s inside the Earth, as well as the limited column of matter available for downgoing neutrino interactions, play a relevant role, making upgoing and downgoing event rates roughly equal. The change of the energy break of the spectrum with $\Gamma^2$ combined with the absorption of the Earth is reflected in the zenith angle distributions of the event rates which may give some complementary information about $\Gamma$.

GRB neutrino detection with km$^3$ neutrino telescopes also has the potential to investigate $\nu_\mu \rightarrow \nu_\tau$ oscillations over a baseline of 1000 Mpc. Double bang $\nu_\tau$ induced events offer an unmistakeable signature which allows downgoing $\nu_\tau$ detection. A km$^3$ telescope operating for 10 years may detect $\sim 10$ downgoing double bang events if $\Gamma = 100$ without any potential background. Upgoing double bang events are not going to be detected since the $\nu_\tau$'s pileup around 100 TeV, where the probability of a double bang detection is negligible. Muons produced in $\tau$ decays are outnumbered by $\mu$'s from $\nu_\mu$ interactions at least for the type of fluxes expected from GRBs [15]; besides, their energy distribution does not show a clear and characteristic signature so they are difficult to identify.

In summary, neutrino telescopes open up the possibility of determining the value of $\Gamma$ and its fluctuations, as well as the possibility of identifying $\nu_\tau$'s. They are powerful instruments to reveal important astrophysical information about the most energetic objects ever observed in the universe and about neutrino oscillation scenarios over cosmological baselines.

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Table 5: $\nu_\mu + \bar{\nu}_\mu$ events (km$^{-2}$ yr$^{-1}$). Only fluctuations in distance and energy are taken into account. Different distributions in distance are used: the first column corresponds to an euclidean distribution, the second to a cosmological distribution following galaxies [16] and the third one assumes that GRB's follow the distribution of star formation regions [18]. Absorption in the Earth as well as the limited column of matter above the detector are taken into account.

### References


