Vortices in $SU(2)$ lattice gauge theory

Srinath Cheluvaraja

Dept. of Physics and Astronomy, Louisiana State University, Baton Rouge, LA, 70803

ABSTRACT
We investigate some properties of thick vortices and thick monopoles in the $SU(2)$ lattice gauge theory by inserting operators which create these excitations. Some quantities associated with the dynamical behaviour of thick vortices and thick monopoles are studied. We measure the derivative of the free energy of the vortex with respect to the coupling and we find that it falls exponentially with the cross sectional area of the vortex size. We also study the monopole-monopole potential energy for thick and thin monopoles. Our results suggest that vortices and monopoles of increasing thickness will play an important role in the large $\beta$ limit.

PACS numbers:12.38Gc,11.15Ha,05.70Fh,02.70g
been under discussion for a long time. Vortices are topological excitations which give rise to non-removable phase factors in the Wilson loop. Vortices in non-abelian gauge theories [1] can be defined using the center of the group \( Z(N) \) for the group \( SU(N) \) – thus defined, they are the natural generalizations of the Peierls contours in the two-dimensional Ising model. Though these vortices are not the most general vortices that can be defined in a non-abelian theory, they can play an important role in the theory and their properties need to be carefully studied. The analogy between vortices and Peierls contours was used in [2] to arrive at certain rigorous bounds for the Wilson loops in \( SU(2) \) lattice gauge theories. Vortices in lattice gauge theories were also studied in [3]. The role of vortices was further investigated in [4] by rewriting \( SU(2) \) LGTs in terms of \( SO(3) \) and \( Z(2) \) variables. It was noted in [2] that the continuous and non-abelian nature of the group \( SU(2) \) can give rise to a different kind of vortex – a thick vortex – whose core has a thickness of more than one lattice spacing. It was proposed [2, 4] that these thick vortices can have an important bearing on the confining properties of the theory. The thick vortices have their analogues in the domain walls of ferromagnets with a continuous symmetry – they are the generalizations of thick Peierls contours. The free energy of these vortices can be made arbitrarily small by spreading them over large regions. The thick vortices should be distinguished from the thin vortices – thin vortices are like the vortices in a pure \( Z(2) \) gauge theory – their core has a thickness of only one lattice spacing. It must be noted that these vortices are only one of the many possible excitations of the gauge theory. The vortex theory of confinement aims to show that these vortices are sufficient, and perhaps necessary, to produce the disordering of Wilson loops and causes them to decay as the area of the minimal surface tiling the loop. In this sense the vortices are expected to produce the same kind of disordering of the Wilson loops as the Peierls contours do for the spin-spin correlation function of the Ising model. In order to quantify the vortex theory of confinement we have to first identify the vortices, study their physical effects, and then show that the minima of the free energy
quite complicated and in this paper we will present some measurements which probe some properties of the thick vortices.

We should mention that there are different approaches to studying vortices in non-abelian gauge theories which are being pursued at present. One of them is the center projection approach. In this approach [6], vortices are defined after the theory is fixed to a particular gauge which still allows the vortex degrees of freedom. This gauge, referred to as the maximal center gauge, gets rid of many other degrees of freedom but retains the vortex excitations. Vortices are defined in the gauge fixed theory just as in a $Z(2)$ LGT. Studies of vortex excitations in this gauge have led to the observation of center dominance—the ungauged degrees (the center degrees of freedom) are sufficient to reproduce the string tension and some other non-perturbative quantities. More discussion of this approach can be found in [7, 8, 9]. Another approach developed further in [5] is to look at monopoles and vortices using the $Z(2)XSO(3)$ decomposition of the $SU(2)$ LGT. In this approach the role of the Wilson loop as a vortex counter is quite transparent and it is capable of addressing the issues of thin and thick monopoles and vortices. Studies of vortices in this approach were recently carried out in [10].

We will first recapitulate what we mean by center vortices and how they arise in lattice gauge theories. Much of this is well known and we repeat it merely to set the definitions and to make our discussion self-contained. We will talk about vortices first in three, and then in four space-time (Euclidean) dimensions. In three dimensions a vortex is said to pierce a two dimensional region (simply connected) $R$ if every Wilson loop surrounding this region picks up a phase ($Z(N)$ for the group $SU(N)$). By definition, a vortex is a global disturbance which cannot be confined into any finite region. The effect of this vortex can also be understood as the action of a pure gauge transformation which is not single valued on every closed loop surrounding $R$. Because of topological considerations the region $R$ cannot be shrunk to a point by regular gauge transformations and this region is associated with
A vortex can extend in the dimensions orthogonal to $R$ and can either stretch indefinitely, form closed loops, or end in objects (monopoles) which absorb the vortex flux. In four dimensions the above picture gets repeated on every slice in the extra dimension and the vortex line becomes a vortex sheet whose area is the product of the length of the vortex line and the duration in time for which the vortex propagates. These vortex sheets can either form closed two-dimensional surfaces or they can end in monopole loops. The well-known example of a vortex solution in a gauge theory is the Nielsen-Olesen vortex which occurs in three-dimensional scalar QED [11] with a Higgs potential. This vortex is the relativistic generalization of the Abrikosov vortex seen in a type II superconductor. Apart from the Nielsen-Olesen vortex there are not many vortex solutions known, especially in unbroken non-abelian gauge theories. However, the Nielsen-Olesen vortex carries an integral vortex charge and is not a center vortex (which carries a $Z(N)$ charge). Next we come to the definition of vortices on the lattice. First we will consider a $Z(2)$ gauge theory whose action is given by

$$ S = \beta \sum_p \sigma(p) , \quad (1) $$

with the $\sigma(p)$s given by $\sigma(p) = \prod_{l \in p} \sigma(l)$. In a $Z(2)$ gauge theory a vortex configuration is a set of co-closed plaquettes with $\sigma(p) = -1$. It follows from their definition that Wilson loops linked by these configurations will have a negative sign. In three (four) space-time dimensions these vortices cost an action which is proportional to their length (area), and they can condense in a phase in which the entropy overwhelms their loop (surface) energy (i.e. when the free energy density of a vortex becomes negative). However, there is also a phase in which the vortices are energetically suppressed (the free energy density of a vortex becomes positive). The two phases of the $Z(2)$ LGT (in three and four dimensions) can be distinguished by the behaviour of these vortex excitations. It is the aim of the vortex

---

A set of plaquettes in a three (four) dimensional lattice is said to be co-closed if the links (plaquettes) dual to it form a closed loop (surface).
The action for the $SU(2)$ LGT can be chosen to be the Wilson action

$$S = \frac{\beta}{2} \sum_p tr_f U(p) \ ,$$

where $U(p)$ are the usual plaquette variables. It is possible to define a vortex in the $SU(2)$ LGT just as in the $Z(2)$ LGT by using the sign of the trace of the plaquette variable. A thin vortex is defined to be a co-closed set of plaquettes for which $\text{sign}(trU(p))=-1$. This definition of the vortex can always be made because just by restricting the $SU(2)$ elements to their center values we should regain the vortex of the $Z(2)$ theory. However, the non-abelian and continuous nature of the group $SU(2)$ allows us to define another kind of vortex in the $SU(2)$ theory. This vortex, to be called a thick vortex, has the property that $\text{sign}(trU(p))=+1$ for all the plaquettes, but nevertheless Wilson loops pick up a center element whenever they surround the vortex. It is obvious that such configurations can never appear in the $Z(2)$ theory or, in fact, in any abelian theory. In any abelian theory the Wilson loop surrounding a region can always be decomposed into the plaquettes that are tiling its minimal surface, and the Stokes theorem ensures that if all the plaquettes are positive, every Wilson loop is also positive. Classical vortex solutions with these properties have also been recently studied by the author [17]. An important property of these vortices is that, because they come with $\text{sign}(trU(p))=+1$, they need not be suppressed in the $\beta \to \infty$ limit, unlike the thin vortices, and they can play an important role in the zero lattice spacing limit. Apart from these excitations, there are other excitations that can absorb the flux of thin and thick vortices. These excitations will be referred to as $Z(2)$ monopoles following the terminology in [2]. A thin vortex can end in a thin $Z(2)$ monopole which (in three space-time dimensions) is defined on an elementary 3 dimensional cube $c$ violating the Bianchi identity. The thin $Z(2)$ monopole density on a cube is given by

$$\rho_1(c) = (1/2)(1 - \text{sign}(\prod_{p \in \partial c} trU(p))) \ .$$

Analogously, a thick vortex can end in a thick $Z(2)$ monopole and this configuration violates
density (in three space-time dimensions) is given by

\[ \rho_d(c) = (1/2)(1 - \text{sign}(\prod_{d \in \partial c} tr U(d))) \]  

where the product is taken over all \( dXd \) plaquettes bordering a cube of side \( d \). The subscript \( d \) indicates that the density can be defined on any 3 dimensional cube of side \( d \) lattice spacings. In four space-time dimensions the monopoles become loops on the dual lattice. It is evident that both, thick vortices and thick monopoles, are possible only in a non-abelian theory. Though the above excitations have only been defined on a lattice, it is a subtle question whether they will go over to physical vortices in the continuum limit. According to the arguments in [1], a scalar field in the adjoint representation can dynamically arise in the gauge theory and create these vortex excitations. If this scalar field field develops a non-zero vacuum expectation value, these vortices can condense and can cause quark confinement. The very possibility that the thick vortices on the lattice can go over to the dynamically generated vortices in the continuum merits a further investigation of their properties.

In this paper we take a step towards studying the properties of these excitations. We introduce a term in the partition function which creates a stress in the system and we study the effect of this stress on the system. More specifically, we add a term to the Wilson action

\[ S' = \frac{\beta}{2} \sum_p tr U(p) - \frac{\beta}{2} \sum_{p'} tr U(d) \]  

The extra term is a Wilson loop variable defined over a square of side \( d \) and it comes with a negative sign. This term is introduced at a point in the (say) (12) plane, and it extends in the 3 direction, and the term repeats itself in the 4 direction. The effect of the extra term is to push the system such that \( tr U(d) \) is negative, the effect of the usual Wilson action being to push the system such that \( tr U(p) \) is positive. The above term can also be understood as a way of implementing certain boundary conditions for the system. By
In an abelian theory, the Stokes theorem enables us to reduce any trace over a large loop to a product of the traces over subloops, for instance

$$tr U(C) = \prod_{p \in C} tr U(p) . \quad (6)$$

The term that we have introduced in the action explicitly violates the above constraint. We mention here that such stresses in gauge theories were introduced in [12] and have been studied before but all the studies carried out so far consider the special case in which only the plaquettes appear with an opposite sign. Some recent works which discuss the case of the flipped plaquette are [13]. This corresponds to the case $d = 1$ but we will be interested in the case $d > 1$ and how the stress behaves as a function of $d$. We would like to measure the change in the free energy of the system before and after applying this stress.

From the previous discussion, the stress that we are introducing into the system is nothing but a thick vortex of thickness $d$ piercing a region in the (12) plane and wrapping around the lattice in 3 direction. On the other hand, if the stress began at a point, say $z = z_1$, and stopped at, say $z = z_2$, we would have a thick monopole-antimonopole pair bound together by a thick vortex line. Since a $Z(2)$ antimonopole is the same as a $Z(2)$ monopole we will always refer to the antimonopole as a monopole. By introducing these stresses we can get some information about the properties of the monopoles and vortices that we have discussed earlier. Since direct free energy measurements are quite forbidding we appeal to a simpler method in order to determine the affect of this stress. We first write

$$\mu_d = \frac{Z(\beta')}{Z(\beta)} . \quad (7)$$

$Z(\beta')$ is the partition function of the system after applying the stress and the $Z(\beta)$ is the partition function without the stress. $\mu_d$ can also be written as

$$\mu_d = \langle \exp - \sum_{p'} tr U(d) \rangle_0 = \exp(- (F_d - F_0)) , \quad (8)$$

where the subscript 0 refers to the original partition function. We have also expressed $\mu_d$ as the exponential of the free energy difference between the stressed system and the system.
additional thick vortex (having a thickness $d$) is introduced into the original system (which
may already contain thick vortices). Hence the effect of a large number of vortices already
present is subsumed in this free energy difference. It is seen easily that \( \frac{\partial \log \mu_d}{\partial \beta} \) is

\[
\frac{1}{\mu_d} \frac{\partial \mu_d}{\partial \beta} = < \frac{1}{2} \sum_p trU(p) - \frac{1}{2} \sum_{p'} trU(d) >_1 - \frac{1}{2} \sum_p trU(p) >_0 \quad .
\]  

(9)

The subscript 1 indicates that the average is taken with respect to the stressed partition
function, whereas the subscript 0 indicates that the average is taken with respect to the
unstressed partition function. This quantity directly measures the derivative of the free
energy difference because

\[
\frac{\partial \log \mu_d}{\partial \beta} = -\frac{\partial (F_d - F_0)}{\partial \beta} .
\]  

(10)

Before we present the numerical results of this measurement let us see how the quantity
looks in the strong coupling limit. As mentioned before, the quantity $\mu_d$ is also given by

\[
< \exp -\beta \sum_{p'} trU(d) >_0 \quad .
\]  

(11)

Expanding the exponential to second order in $\beta$ we get

\[
1 - \beta \sum_{p'} < trU(d) > + O(\beta^2) \quad .
\]  

(12)

The second term is just a measure of the $dXd$ Wilson loop and goes like the area law in
the strong coupling limit, therefore we get

\[
1 - \beta (N_3 N_4) \exp(-\sigma d^2) + O(\beta^2) \quad .
\]  

(13)

($N_3, N_4$ are the sizes of the lattice in the 3 and 4 directions) If the area of the $dXd$ plaquette
is much larger than the area of the transverse directions, the second term is quite small
and after taking the logarithm we get

\[
(F_d - F_0) = \beta (N_3 N_4) \exp(-\sigma d^2) \quad .
\]  

(14)
\[ \sigma_d = \beta \exp(-\sigma d^2) \]  

We thus find that in the strong coupling region the free energy per unit area of a thick vortex decreases exponentially with the area of the vortex core. Note the relation between \( \sigma_d \) (free energy density of a thick vortex of size \( d \)) and \( \sigma \) (the string tension). This calculation shows that thicker vortices are energetically more favourable than thinner vortices. The above calculation is valid only in the strong coupling region. In addition, it required a delicate handling of the area of the transverse directions as compared with the area of the vortex. The limit \( d \to \infty \) must be taken before the limit \( N_3 N_4 \to \infty \). We also note that the derivative of the free energy of a thick vortex will also fall off exponentially with the area of the vortex core. A direct measurement of the quantity in Eq. 11 is also possible but since this quantity has large fluctuations we have chosen to measure the quantity in Eq. 9. In a simulation we can measure \( \frac{\partial \log u}{\partial \beta} \) in the weak coupling region and we will show that the exponential fall off with the area also persists in the weak coupling region.

In Fig. 1, Fig. 2, and Fig. 3 we present results at three different coupling parameters for \( \frac{\partial \log u}{\partial \beta} \) as a function of the area of the thickness of the vortex. In each case we find that there is an exponential fall-off with respect to the area of the vortex. A logarithm of the curve plotted vs the area of the vortex gives an approximately good straight line fit (see Fig. 4). The simulations were done on a lattice whose spatial extent was 10X10 and the remaining size was 6X6. Vortices of thickness ranging from 1 to 5 were studied, and in order to avoid finite size effects associated with these vortices the lattices were chosen to have a spatial extent of atleast twice the largest vortex size. The other parameters of the lattice were fixed by some limitations in the computer time. It must be mentioned that each point in Figs.1-3 requires a separate simulation because Eq. 9 is to be used for different values of \( d \). 300,000 data points were gathered at each point and the errors were estimated by binning.

We wish to point out that though our result is for the derivative of the free energy of the vortex area as a function of the coupling, the free energy of the vortex will also
immediate implications. It indicates that as we approach the weak coupling limit ($\beta \to \infty$), vortices of larger and larger area are more favourable than vortices of any fixed area. By spreading the core of the vortex over a large area the energy of the vortex loops can be made arbitrarily (exponentially) small. This feature is quite like the thick Peierls contours of ferromagnets with a continuous symmetry. It also means that in the region of the continuum limit we must be able to tackle a many body vortex problem in which vortices can have large overlaps with each other. This feature seems to make the study of such vortex gases quite difficult. However, from the point of view of the continuum limit, very large vortices are almost necessary; if lattice vortices are to survive as continuum excitations they must appear with a length scale which diverges in lattice units as the lattice spacing goes to zero. Only then can they yield a vortex corresponding to some physical thickness. Thick vortices indeed admit this possibility by appearing with arbitrarily large thicknesses.

We now turn to thick monopoles. As mentioned earlier, if the stress is taken to extend only between two points, say $z = z_1$ and $z = z_2$, it corresponds to an external thick monopole-monopole loop running along the 4 direction at $z = z_1$ and $z = z_2$ and separated by a thick vortex sheet between $z = z_1$ and $z = z_2$. We again consider Eq. 9 but this time since the stress is defined only between two points we now write

$$\mu_d(x, y) = \frac{Z(\beta')}{Z(\beta)} .$$

(16)

This quantity can now be written as

$$\mu_d(x, y) = \langle \exp - \sum_{\nu} tr U(d) \rangle_0 = \exp(-(F_d(x, y) - F_0)) .$$

(17)

Eq. 9 is now the quantity

$$\frac{-\partial(F_d(x, y) - F_0)}{\partial \beta} ,$$

(18)

and it measures the derivative of the potential energy with respect to the coupling of an external monopole-monopole pair as a function of their separation.
that if thick monopoles abound in the system there will be a screening mechanism in operation which screens the interactions between an externally introduced monopole-monopole pair. On the other hand, if the density of monopoles is very low then the screening mechanism is no longer operative. Before we study their interaction energy we will first say a few things about thick and thin monopoles. We stress again that the thick monopoles and thick vortices are not suppressed by the term in the action as they are configurations where \( \text{sign}(tr(U(p))) = +1 \) everywhere. It is well known that thin \( Z(2) \) monopoles are suppressed at weak coupling [2, 4, 16]. The thick monopoles need not be (and are not) similarly suppressed. Just for comparison we show the behaviour of thick vs thin monopoles as a function of the coupling in Fig. 5. We see that thick monopoles continue to be dense even after all the thin monopoles have disappeared. However, thick monopoles also start disappearing as we go to weaker and weaker coupling, although much slowly. To see the fall in the density of thick monopoles we have to go to larger and larger values of \( \beta \). Fig. 5 shows that as we go to larger values of \( \beta \) even thick monopoles start falling off but monopoles of even larger thicknesses are still abundant. There is no obvious argument (like there is for thin monopoles) for the suppression of thick monopoles with increasing \( \beta \). The behaviour of thick monopoles seems to be more subtle. Similar considerations apply to thick vortices.

We can also measure the density of thick vortices piercing a loop \( C \) by looking at the instances of the negative sign for the following quantity

\[
tr \ U(C) \prod_{p \in C} \theta(\eta(p))
\]  

(19)

\( \eta(p) \) is the sign of the trace of the plaquette variable \( trU(p) \). A quantity \( \rho_v(C) \) can be defined to take the value 1 whenever the above quantity is negative and 0 otherwise. The \( \theta \) function ensures that all the plaquettes inside the loop have a positive trace. A negative value for the above quantity implies a thick vortex piercing the minimal surface (in this case the plane containing the loop) bounding the loop. Since a direct measurement of the vortex density is complicated by the fact that vortices can bend and twist in every possible
plane. Though this quantity does not give complete information about the structure of the vortices, for example it does not give any information about the size distributions of the vortices, it does indicate the number of vortex lines piercing a plane, the high density of which is atleast a necessary condition for any vortex theory of confinement. The sign of this quantity is measured for different planar (in the 1-2 plane at z=t=1) loops that can be drawn at a point and then the average is taken over all the points in the plane. We have looked at square loops having sizes of up to \( d = 6 \). Fig. 6 shows this density at different values of the coupling. This plot shows that, just like the monopoles, the vortex density also decreases with increasing \( \beta \) but then vortices of larger thicknesses are still significant. The data in Fig. 5 and Fig. 6 were generated on a \( 8^4 \) lattice after collecting 10000 data points.

We now present some measurements of the monopole-monopole potential energy as a function of their separation distance for monopoles of different thicknesses. Fig. 7 shows the potential energy for monopoles of thickness \( d = 1 \) at \( \beta = 1.0 \) and \( \beta = 2.4 \). The approximately linear rise at \( \beta = 2.4 \) shows that their separation energy directly increases with the separation distance. By contrast the curve for \( \beta = 1.0 \) is quite flat. At \( \beta = 2.4 \), the thin monopoles have become very rare and the screening mechanism between an external monopole-monopole pair is absent and their interaction energy is directly proportional to the length of vortex line joining them. On the other hand, at \( \beta = 1.0 \), when the monopole density is quite large, the energy of the external pair does not increase linearly with their separation because of the screening effect of the large number of monopoles in the system. A screening by the monopoles in the system should produce a Yukawa like interaction between the external charges. A similar measurement for thick monopoles shows very different behaviour. Fig. 8 shows the monopole-monopole potential energy for a thick monopole of thickness \( d = 3 \). At \( \beta = 2.4 \) the thick monopole-monopole potential energy curve is quite flat. This can again be understood from the presence of a large number of
$\beta = 4.0$, the thick monopoles appear to show a screening behaviour. The main difference between Fig. 7 and Fig. 8 is the signs of the slopes of the curves in the two plots. The slope of the curve is the negative of the derivative of the free energy of the monopole pair with respect to the coupling. While the first plot (with negative slope) shows a linear rise with the separation distance in the interaction energy of a thin monopole-monopole pair at $\beta = 2.4$, the second plot ($\beta = 4.0$) shows a decrease (with positive slope) in the interaction. This different behaviour for monopoles of finite thickness compared with monopoles of unit thickness again shows that the self-energy of thick monopoles has a more subtle behaviour than that of the thin monopoles. A possible explanation of this difference in behaviour is that thick monopoles tend to repel each other (like similarly charged particles) and lower their potential energy. The data in Fig. 7 and Fig. 8 were taken on a $6^4$ lattice after 50000 iterations.

We wish to put our observations in the context of, what has been called, the hierarchichal $Z(2)$ theory of confinement [15, 16]. A hierarchy of effective $Z(2)$ theories is supposed to be operating at larger and larger values of $\beta$. Each member of this hierarchy is an effective $Z(2)$ theory defined over a different scale. As the weak coupling limit is approached the dynamics can be effectively described by a different effective $Z(2)$ theory at larger and larger scales (we ascend the hierarchy of effective $Z(2)$ theories). For every $\beta$ there is an associated scale $d$ (the square root of the time constant of the exponential in Figs.1-3) at which there is an effective $Z(2)$ like behaviour. As $\beta$ increases, $d$ also increases. This is one way in which the $SU(2)$ LGT can escape the phase transition seen in the $Z(2)$ LGT. It is not easy to write down the effective $Z(2)$ theories for the different members of this hierarchy. In [16] an effective $Z(2)$ theory is derived for the $d = 1$ case in the strong coupling limit. Some properties of the effective $Z(2)$ theory are discussed in [15]. The approach presented in this paper of directly measuring some properties of thick vortices may be useful in elucidating this hierarchy of $Z(2)$ theories. We have also observed that
the weak coupling regime there seems to be a minimum thickness of the monopoles and vortices although there is no maximum thickness.

The aim of this investigation was mainly to study the behaviour of the additional extended objects, thick vortices and thick monopoles, which the continuous and non-abelian nature of the group \( SU(2) \) admits. The behaviour of these excitations is quite different from the usual thin vortices and thin monopoles that one is familiar with. Any vortex theory of confinement must be able to handle these excitations in some, perhaps approximate, way. The method presented in this paper can serve as a useful technique for carrying out further studies of these objects. For instance, we can study the second derivative of the free energy with respect to the coupling and look for peaks to see the different excitations decoupling as we approach the large \( \beta \) limit.

Acknowledgement: This work was supported in part by United States Department of Energy grant DE-FG 05-91 ER 40617.


Figure 1: $\frac{\partial \log \mu}{\partial \beta}$ vs $A$ at $\beta = 2.3$.

Figure 2: $\frac{\partial \log \mu}{\partial \beta}$ vs $A$ at $\beta = 2.4$.

Figure 3: $\frac{\partial \log \mu}{\partial \beta}$ vs $A$ at $\beta = 2.5$.

Figure 4: Fitting the exponential in Fig. 3 to a straight line.

Figure 5: Comparison of thick vs thin monopoles as a function of $\beta$.

Figure 6: $\rho_0(C)$ as a function of $\beta$ for loops of sizes 2-6.

Figure 7: Potential energy of thin $d = 1$ monopoles as a function of distance at $\beta = 1.0$ and $\beta = 2.3$.

Figure 8: Potential energy of thick $d = 3$ monopoles as a function of distance at $\beta = 2.4$ and $\beta = 4.0$. 

17