Oscillation probability of GeV solar neutrinos of all active species

André de Gouvêa

CERN, Theory Division, CH-1211 Geneva 23, Switzerland

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In this paper, I address the oscillation probability of $O(\text{GeV})$ neutrinos of all active flavors produced inside the Sun and detected at the Earth. Flavors other than electron-type neutrinos may be produced, for example, by the annihilation of WIMPs which may be trapped inside the Sun. In the GeV energy regime, matter effects are important both for the “1-3” system and the “1-2” system, and for different neutrino mass hierarchies. A numerical scan of the multidimensional three-flavor parameter space is performed, “inspired” by the current experimental situation. One important result is that, in the three-flavor oscillation case, $P_{\alpha\beta} \neq P_{\beta\alpha}$ for a significant portion of the parameter space, even if there is no CP -violating phase in the neutrino mixing matrix. Furthermore, $P_{\mu\mu}$ has a significantly different behavior from $P_{\tau\tau}$, which may affect expectations for the number of events detected at large neutrino telescopes.

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I. INTRODUCTION

Neutrino experiments have been able to obtain indirect, but recently very strong, evidence for neutrino masses, via neutrino oscillations. The key evidence comes from the angular dependent flux of atmospheric muon-type neutrinos measured at SuperKamiokande [1], combined with a large deviation of the muon-type to electron-type neutrino flux ratio from theoretical predictions. This “atmospheric neutrino puzzle” is best solved by assuming that ν_μ oscillates into ν_τ and that the ν_e does not oscillate [2].

On the other hand, measurements of the solar neutrino flux [3–7] are consistently and significantly smaller than theoretical predictions [8]. Again, this “solar neutrino puzzle” is best resolved by assuming that ν_e oscillates into a linear combination of the other flavor eigenstates [9,10] (for a more conservative analysis of the event rates and the inclusion of the “dark side” of the parameter space, see [11]). For an analysis of the solar neutrino data including the mixing of three active neutrino see, e.g., [12].

Neutrino oscillations were first hypothesized by Pontecorvo in the 1950s [13], while the two-family mixing matrix was originally proposed by Maki, Nakagawa and Sakata [14]. In light of the solar neutrino puzzle, Wolfenstein [15] and Mikheyev and Smirnov (MSW) [16] realized that neutrino-matter interactions could affect in very radical ways the survival probability of electron-type neutrinos which are produced in the solar core and detected at the Earth (MSW effect).

Since then, significant effort has been devoted to understanding the oscillation probabilities of electron-type neutrinos produced in the Sun. For example, in [17] the survival probability of solar-electron-type neutrinos was discussed in the context of three-neutrino mixing including matter effects, and solutions to the solar neutrino puzzle in this context were studied (for example, in [17,18,12]).

In this paper, the understanding of solar neutrino oscillations is extended to the case of other active neutrino species (ν_μ , ν_τ , and antineutrinos) produced in the solar core. Even though only electron-type neutrinos are produced by the nuclear reactions which take place inside the Sun, it is well

known that, in a number of dark matter models, dark matter particles can be trapped gravitationally inside the Sun, and that the annihilation of these should yield a flux of high energy neutrinos ($E_\nu \gtrsim 1 \text{ GeV}$) of all species which may be detectable at the Earth [19]. Indeed, this is one of the goals of very large “neutrino telescopes,” such as AMANDA [20] or BAIKAL [21]. It is important to understand how neutrino oscillations will affect the expected event rates at these experiments.¹

The oscillation probability of all neutrino species has, of course, been studied in different contexts, such as in the case of neutrinos produced in the core of supernovas [23] or in the case of neutrinos propagating in constant electron number densities [24]. The latter case has been receiving a considerable amount of attention from neutrino factory studies [25]. The case at hand (GeV solar neutrinos) differs significantly from these mentioned above, in at least a few of the following: source-detector distance, electron number density average value and position dependence, energy average value and spectrum. Neutrino factory studies, for example, are interested in $O(1000)$ km base lines, $O(10)$ GeV electron-type and muon-type neutrinos produced via muon decay propagating in roughly constant, Earth-like (matter densities around 3 g/cm^3) electron number densities.

The paper is organized as follows. In Sec. II, the well-known case of two-flavor oscillations is briefly reviewed. In Sec. III the case of three-flavor oscillations is discussed in some detail. Special attention is paid to neutrinos produced in the Sun’s core. In Sec. IV the results presented in Sec. III will be analyzed numerically, and the three-neutrino multidimensional parameter space will be explored. Section V contains a summary of the results and the conclusions.

Finally, I would like to comment that the discussions presented here will take advantage of the current experimental situation in order to constrain the (very large) three-flavor oscillation parameter space and of the possibility of produc-

¹Some effects have already been studied, in the two-neutrino case, in [22].

ing neutrinos of all species via dark matter annihilations to constrain the neutrino energies to the range from a few to tens of GeV.

II. TWO-FLAVOR OSCILLATIONS

In this section, the well-studied case of two-flavor oscillations will be briefly reviewed [26].

Neutrino oscillations take place because, similar to what happens in the quark sector, neutrino weak eigenstates are different from neutrino mass eigenstates. The two sets are related by a unitary matrix, which is, in the case of two-flavor mixing, parametrized by one mixing angle ϑ :²

$$\begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} \\ U_{x1} & U_{x2} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \quad (2.1)$$

where ν_1 and ν_2 are neutrino mass eigenstates with masses m_1 and m_2 , respectively, and ν_x is the flavor eigenstate orthogonal to ν_e . All physically distinguishable situations can be obtained if $0 \leq \vartheta \leq \pi/2$ and $m_1^2 \leq m_2^2$ or $0 \leq \vartheta \leq \pi/4$ and no constraint is imposed on the masses squared.

In the case of oscillations in vacuum, it is trivial to compute the probability that a neutrino produced in a flavor state α is detected as a neutrino of flavor β . In the case of non-trivial neutrino-medium interactions, however, the computation of $P_{\alpha\beta}$ can be rather involved. Assuming that the neutrino-medium interactions can be expressed in terms of an effective potential for the neutrino propagation, one has to solve

$$\frac{d}{dt} \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} = -i \left[\begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} + \begin{pmatrix} V_{11}(t) & V_{12}(t) \\ V_{12}(t)^* & V_{22}(t) \end{pmatrix} \right] \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix}, \quad (2.2)$$

with the appropriate boundary conditions (either a ν_e or a ν_x as the initial state, for example).

It is interesting to argue what can be said about $P_{\alpha\beta}$ in very general terms. In the case of only two neutrino families, one can trivially show, assuming that the ‘‘Hamiltonian’’ in Eq. (2.2) is Hermitian, that only one $P_{\alpha\beta}$, is independent. Therefore, it is possible to express all $P_{\alpha\beta}$ in terms of, say, P_{ee} : $P_{ex} = P_{xe} = 1 - P_{ee}$ and $P_{xx} = P_{ee}$. It is important to note that the equality $P_{ex} = P_{xe}$ is *not* a consequence of T invariance, but a consequence of the unitarity of the Hamiltonian evolution and particular only to the two-flavor oscillation case, as will be argued later.

A. Oscillation of neutrinos produced in the Sun’s core

In the presence of electrons, the differential equation satisfied by the two neutrino system is, in the flavor basis,

$$\frac{d}{dx} \begin{pmatrix} \nu_e(x) \\ \nu_x(x) \end{pmatrix} = -i \left[\frac{\Delta m^2}{2E_\nu} \begin{pmatrix} |U_{e2}|^2 & U_{e2}^* U_{x2} \\ U_{e2} U_{x2}^* & |U_{x2}|^2 \end{pmatrix} + \begin{pmatrix} A(x) & 0 \\ 0 & 0 \end{pmatrix} \right] \times \begin{pmatrix} \nu_e(x) \\ \nu_x(x) \end{pmatrix}, \quad (2.3)$$

where terms proportional to the 2×2 identity matrix were neglected, since they play no role in the physics of neutrino oscillations, and

$$A(x) = \sqrt{2} G_F N_e(x) \quad (2.4)$$

is the charged current contribution to the ν_e - e forward scattering amplitude, G_F is Fermi’s constant, and $N_e(x)$ is the position dependent electron number density. In the case of the Sun [8] (see also [27]), $A \equiv A(0) \approx 6 \times 10^{-3} \text{ eV}^2/\text{GeV}$, assuming an average core density of 79 g/cm^3 , and $A(x)$ falls roughly exponentially until close to the Sun’s edge.

In the case of solar neutrinos, it is convenient to express the ν_e produced in the Sun’s core in the basis of the eigenstates of the Hamiltonian evaluated at the production point, $|\nu_e\rangle = \cos \vartheta_M(0) |\nu_L\rangle + \sin \vartheta_M(0) |\nu_H\rangle$, where $|\nu_H\rangle$ ($|\nu_L\rangle$) correspond to the highest (lowest) instantaneous Hamiltonian eigenstate. The matter mixing angle $\vartheta_M \equiv \vartheta_M(0)$ is given by

$$\cos 2\vartheta_M = \frac{\Delta m^2 \cos 2\vartheta - 2E_\nu A}{\sqrt{(\Delta m^2)^2 + A^2 - 4E_\nu A \Delta m^2 \cos 2\vartheta}}. \quad (2.5)$$

Assuming that there is no coherence in the Sun’s core between³ ν_L and ν_H , one arrives at the well-known expression (see, e.g., Refs. [28,29])

$$P_{ee}(x) = P_1 \cos^2 \vartheta + P_2 \sin^2 \vartheta - \cos 2\vartheta_M \sqrt{P_c(1-P_c)} \sin 2\vartheta \times \cos \left(\frac{\Delta m^2 x}{2E_\nu} + \delta \right), \quad (2.6)$$

where δ is a ‘‘matter induced’’ phase of, P_c is the ‘‘level crossing probability,’’ and $P_1 = 1 - P_2 = \frac{1}{2} + \frac{1}{2}(1 - 2P_c) \times \cos 2\vartheta_M$ is interpreted as the probability that the neutrino exits the Sun as a ν_1 .

If the electron number density is in fact exponential [$N_e(x) = N_e(0)e^{-x/r_0}$, where $x=0$ is the center of the Sun], one can solve Eq. (2.3) exactly [30,31]:

$$P_c = \frac{e^{-\gamma \sin^2 \vartheta} - e^{-\gamma}}{1 - e^{-\gamma}} \quad (2.7)$$

[31,32], where

$$\gamma = 2\pi r_0 \frac{\Delta m^2}{2E_\nu} = 1.05 \left(\frac{\Delta m^2}{10^{-6} \text{ eV}^2} \right) \left(\frac{1 \text{ GeV}}{E_\nu} \right), \quad (2.8)$$

²A possible Majorana phase can be ignored, since it plays no role in the physics of neutrino oscillations.

³This is in general the case, because one has to consider that neutrinos are produced at different points in space and time.

for $r_0 = R_\odot/10.54 = 6.60 \times 10^4$ km [27]. In the case of the Sun, the exponential profile approximation has been examined [32], and was shown to be very accurate, especially if one allows r_0 to vary as a function of $\Delta m^2/2E_\nu$.

The exact expression for δ has also been obtained [28], and readers are referred to [33] for details concerning physical implications of the matter phase to solving the solar neutrino puzzle.

B. Case of antineutrinos

Antineutrinos that are produced in the Sun's core obey a differential equation similar to Eq. (2.3), except that the sign of the matter potential changes, i.e., $A(x) \leftrightarrow -A(x)$ and $U_{\alpha i} \leftrightarrow U_{\alpha i}^*$ (this is immaterial since, in the two-flavor mixing case, all $U_{\alpha i}$ can be chosen real).

In the case of two neutrino families, one can easily verify that, $P_{\bar{e}e}(\vartheta) = P_{ee}(\pi/2 - \vartheta)$ [34].⁴ In the case of vacuum oscillations, ϑ is physically equivalent to $\pi/2 - \vartheta$, so $P_{\bar{e}e} = P_{ee}$. In the more general case of nontrivial matter effects, this is clearly not the case, since the presence of matter (or antimatter) explicitly breaks CP invariance.

It is curious to note that, in the case of two-flavor oscillations, there is no T noninvariance, i.e., $P_{\alpha\beta} = P_{\beta\alpha}$, while

there is potentially large CP violation, i.e., $P_{\alpha\beta} \neq P_{\bar{\alpha}\bar{\beta}}$, even if the Hamiltonian for the system is explicitly T noninvariant and CP noninvariant, as is the case of the propagation of neutrinos produced in the Sun.

III. THREE FLAVOR OSCILLATIONS

In this section, the oscillations of three neutrino flavors will be considered. In order to simplify the discussion, I will concentrate on neutrinos with energies ranging from a few to tens of GeV, which is the energy range expected for neutrinos produced by the annihilation of dark matter particles which are possibly trapped inside the Sun. Furthermore, a number of experimentally inspired constraints on the neutrino oscillation parameter space will be imposed, as will become clear later.

The flavor eigenstates, ν_e , ν_μ and ν_τ , and the mass eigenstates ν_i , $i=1,2,3$, with masses m_i are related by a general 3×3 unitary matrix, which is parametrized by three mixing angles (θ , ω , and ξ) and a complex phase ϕ . In shorthand notation $\nu_\alpha = U_{\alpha i} \nu_i$ where $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$. The neutrino mixing matrix will be written, similar to the standard Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [35], as

$$\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c\omega c\xi & s\omega c\xi & s\xi e^{i\phi} \\ -s\omega c\theta - c\omega s\theta s\xi e^{-i\phi} & c\omega c\theta - s\omega s\theta s\xi e^{-i\phi} & s\theta c\xi \\ s\omega s\theta - c\omega c\theta s\xi e^{-i\phi} & -c\omega s\theta - s\omega c\theta s\xi e^{-i\phi} & c\theta c\xi \end{pmatrix}, \quad (3.1)$$

where $c\xi \equiv \cos \xi$ and $s\xi \equiv \sin \xi$ for $\xi = \omega, \theta, \xi$. If the neutrinos are Majorana particles, two extra phases should be added to the neutrino mixing matrix, but since they play no role in the physics of neutrino oscillations, they can be safely ignored. All physically distinguishable situations can be obtained if one allows $0 \leq \phi \leq \pi$, all angles to vary between 0 and $\pi/2$ and no restriction is imposed on the sign of the mass-squared differences, $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$. Note that there are only two independent mass-squared differences, which are chosen here to be Δm_{21}^2 and Δm_{31}^2 .

All experimental evidence from solar, atmospheric, and reactor neutrino experiments [1,3–7,36] can be satisfied,⁵ somewhat conservatively, by assuming [26] $10^{-4} \text{ eV}^2 \leq |\Delta m_{31}^2| \leq |\Delta m_{32}^2| \leq 10^{-2} \text{ eV}^2$, $0.3 \leq \sin^2 \theta \leq 0.7$, $10^{-11} \text{ eV}^2 \leq |\Delta m_{21}^2| \leq 10^{-4} \text{ eV}^2$, $\sin^2 \xi \leq 0.1$, while ω is mostly unconstrained. There is presently no information on ϕ . In deter-

mining these bounds, it was explicitly assumed that only three active neutrinos exist.

A few comments about the constraints imposed above are in order. First, one may complain that ω is more constrained than mentioned above by the solar neutrino data. The situation is far from definitive, however. As pointed out recently in [11], if the uncertainty in the ^8B neutrino flux is inflated or if some of the experimental data are not considered (especially the Homestake data [3]) in the fit, a much larger range of Δm_{21}^2 and ω is allowed.

Second, the limit from the Chooz and Palo Verde reactor experiments [36] does not constrain $\sin^2 \xi$ for $|\Delta m_{31}^2| \leq 10^{-3} \text{ eV}^2$ and, furthermore, their constraints are to $\sin^2 2\xi$, so values of $\sin^2 \xi$ close to 1 should also be allowed. However, constraints from the atmospheric neutrino data require $|\Delta m_{31}^2| > 10^{-3} \text{ eV}^2$ and $\cos^2 \xi$ to be large, such that values of ξ close to $\pi/2$ are ruled out.

In the case of oscillations in vacuum, it is straightforward to compute the oscillation probability $P_{\alpha\beta}$ of detecting a flavor β given that a flavor α was produced. The different vacuum oscillation lengths, L_{osc}^{ij} , are given by

$$L_{\text{osc}}^{ij} = \frac{4\pi E_\nu}{\Delta m_{ij}^2} = 2.47 \times 10^8 \text{ km} \left(\frac{E}{1 \text{ GeV}} \right) \left(\frac{10^{-8} \text{ eV}^2}{\Delta m_{ij}^2} \right). \quad (3.2)$$

⁴If one decides to limit $0 \leq \vartheta \leq \pi/4$, the corresponding relation is $P_{\bar{e}e}(\Delta m^2) = P_{ee}(-\Delta m^2)$.

⁵There is also evidence for neutrino oscillations coming from the Liquid Scintillation Neutrino Detector (LSND) experiment [37]. Such evidence has not yet been confirmed by another experiment, and will not be considered in this paper.

In the energy range of interest, $1 \text{ GeV} \lesssim E_\nu \lesssim 100 \text{ GeV}$ and for $|\Delta m_{31}^2| \simeq |\Delta m_{32}^2| > 10^{-3} \text{ eV}^2$, L_{osc}^{31} and L_{osc}^{32} are much smaller than $1 \text{ a.u.} = 1.496 \times 10^8 \text{ km}$, and their effects “wash out” due to any realistic neutrino energy spectrum, detector energy resolution, or other physical effects. This will be assumed henceforth. In contrast, L_{osc}^{21} maybe as large as (and maybe even much larger than) the Earth-Sun distance.

In the presence of neutrino-medium interactions, the situation is, in general, more complicated (indeed, much more). Similar to the two-neutrino case, it is useful to discuss what is known about the oscillation probabilities. Assuming that propagation of neutrinos from the Sun’s core to the Earth is unitary, it is easy to verify that five independent constraints apply to $P_{\alpha\beta}$ and one may, therefore, express all $P_{\alpha\beta}$ in terms of only four quantities. Here, these are chosen to be P_{ee} , $P_{e\mu}$, $P_{\mu\mu}$, and $P_{\tau\tau}$. The others are given by

$$\begin{aligned} P_{e\tau} &= 1 - P_{ee} - P_{e\mu}, \\ P_{\mu e} &= 1 + P_{\tau\tau} - P_{ee} - P_{\mu\mu} - P_{e\mu}, \\ P_{\mu\tau} &= P_{ee} + P_{e\mu} - P_{\tau\tau}, \\ P_{\tau e} &= P_{\mu\mu} + P_{e\mu} - P_{\tau\tau}, \\ P_{\tau\mu} &= 1 - P_{\mu\mu} - P_{e\mu}. \end{aligned} \quad (3.3)$$

Note that, in general, $P_{\alpha\beta} \neq P_{\beta\alpha}$, in sharp contrast to the two-neutrino case.

A. Oscillation of neutrinos produced in the Sun’s core

The propagation of neutrinos in the Sun’s core can, similar to the two-neutrino case, be described by the differential equation

$$\frac{d}{dx} \nu_\alpha(r) = -i \left[\sum_{i=2}^3 \left(\frac{\Delta m_{i1}^2}{2E_\nu} \right) U_{\alpha i}^* U_{\beta i} + A(x) \delta_{\alpha e} \delta_{\beta e} \right] \nu_\beta(r), \quad (3.4)$$

where $\delta_{\eta\zeta}$ is the Kronecker delta symbol. Terms proportional to the identity $\delta_{\alpha\beta}$ are neglected because they play no role in the physics of neutrino oscillations. The matter induced potential $A(x)$ is given by Eq. (2.4).

As in the two-neutrino case, it is useful to first discuss the initial states ν_α in the Sun’s core and to express them in the basis of instantaneous Hamiltonian eigenstates, which will be referred to as $|\nu_H\rangle$, $|\nu_M\rangle$, and $|\nu_L\rangle$ (H =high, M =medium, and L =low). Therefore

$$|\nu_\alpha\rangle = H_\alpha |\nu_H\rangle + M_\alpha |\nu_M\rangle + L_\alpha |\nu_L\rangle, \quad (3.5)$$

where $\langle \nu_\alpha | \nu_{\alpha'} \rangle = \delta_{\alpha\alpha'}$. The probability of detecting this initial state as a β -type neutrino far away from the Sun (e.g., at the Earth) is given by

$$\begin{aligned} P_{\alpha\beta} &= \left| (U_{\beta 1}^* U_{\beta 2}^* U_{\beta 3}^*) \right. \\ &\quad \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i(\Delta m_{21}^2 x / 2E_\nu)} & 0 \\ 0 & 0 & e^{-i(\Delta m_{31}^2 x / 2E_\nu)} \end{pmatrix} \\ &\quad \left. \times (V_{3 \times 3}) \begin{pmatrix} L_\alpha \\ M_\alpha \\ H_\alpha \end{pmatrix} \right|^2, \end{aligned} \quad (3.6)$$

where $V_{3 \times 3}$ is an arbitrary 3×3 unitary matrix which takes care of propagating the initial state until the edge of the Sun and rotating the state into the mass basis.

In order to proceed, it is useful to take advantage of the constraints on the neutrino parameter space and the energy range of interest. Note that $A \gtrsim |\Delta m_{31}^2| / 2E_\nu \gg |\Delta m_{21}^2| / 2E_\nu$ (remember that the energy range of interest is $1 \text{ GeV} \lesssim E_\nu \lesssim 100 \text{ GeV}$ and that $A \approx 6 \times 10^{-3} \text{ eV}^2 / \text{GeV}$). It has been shown explicitly [17], assuming the neutrino mass-squared hierarchy to be $m_3^2 > m_2^2 > m_1^2$, that, if the mass-squared differences are very hierarchical ($|\Delta m_{31}^2| \gg |\Delta m_{21}^2|$), the three-level system “decouples” into two two-level systems; i.e., one can first deal with matter effects in the “ H - M ” system and then with the matter effects in the “ M - L ” system. One way of understanding why this is the case is to realize that the “resonance point” corresponding to the Δm_{31}^2 is very far away from the resonance point corresponding to Δm_{21}^2 . With this in mind, it is fair to approximate (this is similar to what is done, for example, in [38])

$$V_{3 \times 3} = \begin{pmatrix} A^L & B^L & 0 \\ -B^{L*} & A^{L*} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & A^H & B^H \\ 0 & -B^{H*} & A^{H*} \end{pmatrix}, \quad (3.7)$$

where $|B^H|^2 = 1 - |A^H|^2 \equiv P_c^H$, $|B^L|^2 = 1 - |A^L|^2 \equiv P_c^L$. The superscripts H , L correspond to the “high” and the “low” resonances, respectively.

It is also possible to obtain an approximate expression for the initial states in the Sun’s core. Following the result outlined above, this state should be described by two matter angles, ξ_M and ω_M , corresponding to each of the two-level systems. Both should be given by Eq. (2.5), where, in the case of $\cos 2\xi_M$, ϑ is to be replaced by ξ and Δm^2 by Δm_{31}^2 , while in the case of $\cos 2\omega_M$, ϑ is to be replaced by ω , Δm^2 by Δm_{21}^2 and A is to be replaced by $A \cos \xi$ [38,12]. Furthermore, because $A \cos \xi \gg |\Delta m_{21}^2| / 2E_\nu$, $\cos 2\omega_M$ can be safely replaced by -1 (remember that $\cos^2 \xi \gtrsim 0.9$). Within these approximations, in the Sun’s core,

⁶I will work under this assumption for the time being.

$$\begin{aligned}
|\nu_e\rangle &= \sin \xi_M |\nu_H\rangle + \cos \xi_M |\nu_M\rangle, \\
|\nu_\mu\rangle &= \sin \theta \cos \xi_M |\nu_H\rangle - \sin \theta \sin \xi_M |\nu_M\rangle - \cos \theta |\nu_L\rangle, \\
|\nu_\tau\rangle &= \cos \theta \cos \xi_M |\nu_H\rangle - \cos \theta \sin \xi_M |\nu_M\rangle + \sin \theta |\nu_L\rangle.
\end{aligned} \tag{3.8}$$

The accuracy of this approximation has been tested numerically in the range of parameters of interest, and the difference between the “exact” result and the approximate result presented in Eq. (3.8) is negligible.

Keeping all this in mind, it is straightforward to compute all oscillation probabilities, starting from Eq. (3.6). From here on, $\phi=0$ (no T -violating phase in the mixing matrix, such that all $U_{\alpha i}$ are real) will be assumed, in order to simplify expressions and render the results cleaner. At the end of the day one obtains

$$\begin{aligned}
P_{\alpha\beta} &= a_\alpha^2 (U_{\beta 1})^2 + b_\alpha^2 (U_{\beta 2})^2 + c_\alpha^2 (U_{\beta 3})^2 \\
&\quad + 2a_\alpha b_\alpha (U_{\beta 1} U_{\beta 2}) \cos \left(\frac{\Delta m_{21}^2 x}{2E_\nu} + \delta^L \right)
\end{aligned} \tag{3.9a}$$

or

$$\begin{aligned}
P_{\alpha\beta} &= (a_\alpha U_{\beta 1} + b_\alpha U_{\beta 2})^2 + c_\alpha^2 (U_{\beta 3})^2 \\
&\quad - 4a_\alpha b_\alpha (U_{\beta 1} U_{\beta 2}) \sin^2 \left(\frac{\Delta m_{21}^2 x}{4E_\nu} + \delta^L \right),
\end{aligned} \tag{3.9b}$$

where δ^L is the matter phase, induced in the low resonance, and

$$\begin{aligned}
a_e &= \sqrt{P_2^H P_c^L}, \\
b_e &= \sqrt{P_2^H (1 - P_c^L)}, \\
c_e &= \sqrt{P_3^H}, \\
a_\mu &= -\sqrt{(1 - P_c^L)} \cos \theta - \sqrt{P_3^H P_c^L} \sin \theta, \\
b_\mu &= \sqrt{P_c^L} \cos \theta - \sqrt{P_3^H (1 - P_c^L)} \sin \theta, \\
c_\mu &= \sqrt{P_2^H \sin^2 \theta}, \\
a_\tau &= \sqrt{(1 - P_c^L)} \sin \theta - \sqrt{P_3^H P_c^L} \cos \theta, \\
b_\tau &= -\sqrt{P_c^L} \sin \theta - \sqrt{P_3^H (1 - P_c^L)} \cos \theta, \\
c_\tau &= \sqrt{P_2^H \cos^2 \theta},
\end{aligned} \tag{3.10}$$

and $P_2^H = 1 - P_3^H = (|A^H|^2 \cos^2 \xi_M + |B^H|^2 \sin^2 \xi_M)$, which can also be written as $P_2^H = \frac{1}{2} + \frac{1}{2}(1 - 2P_c^H) \cos 2\xi_M$. This is to be compared with the expression for P_1 obtained in the two-flavor case. Note that $a_\alpha^2 + b_\alpha^2 + c_\alpha^2 = 1$.

The effects of the matter phase are (under most conditions) negligible when the neutrinos are detected far away from the Sun’s surface (at the Earth, for example). The matter phase is in general smaller than the “vacuum phase” by

a factor proportional to the radius of the Sun divided by the distance traveled by the neutrino. In practice, one should worry about δ^L when dealing with monochromatic neutrino sources and “short” term effects, such as seasonal variations (for a recent discussion, see [39]). This is certainly not the case here, as will become very clear in the next section, and δ^L will be set to zero henceforth.

Many comments are in order. First, in the extreme nonadiabatic limit which can be obtained for very large energies, $P_c^H \rightarrow \cos^2 \xi$, $P_c^L \rightarrow \cos^2 \omega$ and $\cos 2\xi_M \rightarrow -1$. It is trivial to check that in this limit $a_\alpha \rightarrow U_{\alpha 1}$, $b_\alpha \rightarrow U_{\alpha 2}$, $c_\alpha \rightarrow U_{\alpha 3}$, and the vacuum oscillation result is reproduced, up to the matter induced phase δ^L .

Second, P_{ee} can be written as

$$P_{ee} = P_2^H \cos^2 \xi (P_{ee}^{2\nu}) + P_3^H \sin^2 \xi, \tag{3.11}$$

where $P_{ee}^{2\nu}$ is the two-neutrino result obtained in the previous section [see Eq. (2.6)] in the limit $\cos 2\vartheta_M \rightarrow -1$. It is easy to check that Eq. (2.6) would be exactly reproduced (with ϑ_M replaced by ω_M , of course) if the $\cos 2\omega_M = -1$ approximation were dropped.

For solar neutrino energies ($100 \text{ keV} \lesssim E_\nu \lesssim 10 \text{ MeV}$), $\xi_M \rightarrow \xi$, $P_c^H \rightarrow 0$ and therefore $P_2^H (P_3^H) \rightarrow \cos^2 \xi (\sin^2 \xi)$, reproducing correctly the result of the survival probability of electron-type solar neutrinos in a three-flavor oscillation scenario (see [26] and references therein). In this scenario there is no “ H - L ” resonance inside the Sun, because $|\Delta m_{31}^2|/2E_\nu \gg A$ for solar neutrino energies.

On the other hand, in the case $P_c^H \rightarrow 0$ and $\cos 2\xi_M \rightarrow -1$, $P_3^H \rightarrow 1$ and electron-type neutrinos exit the Sun as a pure ν_3 mass eigenstate, and do not undergo vacuum oscillations even if Δm_{21}^2 is very small. In contrast, ν_μ and ν_τ always undergo vacuum oscillations if Δm_{21}^2 is small enough. The reason for this is simple. The generic feature of matter effects is to “push” ν_e into the heavy mass eigenstate, while ν_μ and ν_τ are “pushed” into the light mass eigenstates. This situation is changed by nonadiabatic effects, as argued above.

Finally, it is important to note that all equations obtained are also valid in the case of inverted hierarchies ($m_3^2 < m_{1,2}^2$ or $m_2^2 < m_1^2$). This has been discussed in detail in the two-neutrino oscillation case [40], and is also applicable here. It is worthwhile to point out that, in the approximation $\Delta m_{31}^2 \simeq \Delta m_{32}^2$, the transformation $\Delta m_{21}^2 \rightarrow -\Delta m_{21}^2$ can be reproduced by transforming $\omega \rightarrow \pi/2 - \omega$, $\theta \rightarrow \pi - \theta$ and redefining the sign of ν_τ . Therefore, one is in principle allowed to fix the sign of Δm_{21}^2 as long as θ is allowed to vary between 0 and π .

In the case of inverted hierarchies (especially when $\Delta m_{31}^2 < 0$) one expects to see no “level crossing” (indeed, matter effects tend to increase the distance between the “energy” levels in this case), but matter effects are still present, because the initial state in the Sun’s core can be nontrivial

(i.e., $\vartheta_M \neq \vartheta$). Note that ν_e is still “pushed” towards ν_H , even in the case of inverted hierarchies, and the expressions for the matter mixing angles, Eq. (2.5), and the initial states inside the Sun, Eq. (3.8), are still valid. The consequence of no “level crossing” is that the adiabatic limit does not connect, for example, $\nu_H \rightarrow \nu_3$ but $\nu_H \rightarrow \nu_2$ (or ν_1 , depending on the sign of Δm_{21}^2). This information is in fact contained in the equations above. The crucial feature is that, for example, when $\Delta m_{31}^2 < 0$, $P_c^H \rightarrow 1$ in the “adiabatic limit” and the matrix $V_{3 \times 3}$ correctly “connects” $\nu_H \rightarrow \nu_2$ (or ν_1). Another curious feature is that, in the limit $|\Delta m_{31}^2|/2E_\nu \gg A$, $\cos 2\xi_M \rightarrow -\cos 2\xi$, $P_c^H \rightarrow 1$ and Eq. (3.11) correctly reproduces the survival probability of electron-type solar neutrinos in the three-flavor oscillation case. Note that on this case the sign of Δm_{31}^2 does not play any role, as expected. On the other hand, it is still true that $P_c^{(H,L)} \rightarrow \cos^2(\xi, \omega)$ in the extreme nonadiabatic limit, and vacuum oscillation results are reproduced, as expected. Again, in this limit, one is not sensitive to the sign of Δm_{31}^2 , as expected.

B. Case of antineutrinos

As in the two-neutrino case, the difference between neutrinos and antineutrinos is that the equivalent of Eq. (3.4) for antineutrinos can be obtained by changing $A(x) \rightarrow -A(x)$ and $U_{\alpha i} \leftrightarrow U_{\alpha i}^*$. Unlike the two-flavor case, however, there is no set of variable transformations that allows one to exactly relate the differential equation for the neutrino and antineutrino systems. One should, however, note that if the signs of both Δm^2 are changed and $U_{\alpha i} \leftrightarrow U_{\alpha i}^*$, the neutrino equation turns into the antineutrino equation, up to an overall sign. This means, for example, that the instantaneous eigenvalues of the antineutrino Hamiltonian can be read from the eigenvalues of the neutrino Hamiltonian with $\Delta m_{ij}^2 \leftrightarrow -\Delta m_{ij}^2$, $U_{\alpha i} \leftrightarrow U_{\alpha i}^*$ plus an overall sign.

When it comes to computing $P_{\bar{\alpha}\bar{\beta}}$ this global sign difference is not relevant, and therefore $P_{\bar{\alpha}\bar{\beta}}(\Delta m_{ij}^2, U_{\alpha i}) = P_{\alpha\beta}(-\Delta m_{ij}^2, U_{\alpha i}^*)$.

IV. RESULTS AND DISCUSSIONS

This section contains the compilation and discussion of a number of results concerning the oscillation of GeV neutrinos of all species produced in the Sun’s core. The goal here is to explore the multidimensional parameter space spanned by Δm_{21}^2 , Δm_{31}^2 , $\sin^2 \omega$, $\sin^2 \theta$, and $\sin^2 \xi$ (and E_ν).

It will be assumed throughout that the electron number density profile of the Sun is exponential, so that Eq. (2.7) can be used. As mentioned before, the numerical accuracy of this approximation is quite good and certainly good enough for the purposes of this paper. Therefore, both P_c^H and P_c^L which appear in Eq. (3.9) will be given by Eqs. (2.7), (2.8), with $\vartheta \rightarrow \xi$, $\Delta m^2 \rightarrow \Delta m_{31}^2$ in the former and $\vartheta \rightarrow \omega$, $\Delta m^2 \rightarrow \Delta m_{21}^2$ in the latter.

When computing $P_{\alpha\beta}$, an averaging over “seasons” is performed, which “washes out” the effect of very small oscillation wavelengths. Furthermore, integration over neutrino energy distributions is performed. Finally, all $P_{\alpha\beta}$ to be computed should be understood as the value of $P_{\alpha\beta}$ in the Earth’s surface; i.e., Earth matter effects are not included. This is done in order to make the Sun matter effects in the evaluation of $P_{\alpha\beta}$ more clear. It should be stressed that Earth matter effects may play a significant role for particular regions of the parameter space, but the discussion of such effects will be left for another opportunity.

Because the parameter space to be explored is multidimensional, it is necessary to make two-dimensional projections of it, such that “illustrative” points are required. The following points in the parameter space are chosen, all inspired by the current experimental situation:

-
- (i) atmospheric (ATM): $\Delta m_{31}^2 = 3 \times 10^{-3} \text{ eV}^2$, $\sin^2 \theta = 0.5$, and $\sin^2 \xi = 0.01$,
  - (ii) large mixing angle (LMA): $\Delta m_{21}^2 = 2 \times 10^{-5} \text{ eV}^2$, $\sin^2 \omega = 0.2$,
  - (iii) small mixing angle (SMA): $\Delta m_{21}^2 = 6 \times 10^{-6} \text{ eV}^2$, $\sin^2 \omega = 0.001$,
  - (iv) low mass, low probability (LOW): $\Delta m_{21}^2 = 1 \times 10^{-7} \text{ eV}^2$, $\sin^2 \omega = 0.4$,
  - (v) vacuum (VAC): $\Delta m_{21}^2 = 1 \times 10^{-10} \text{ eV}^2$, $\sin^2 \omega = 0.55$.
-

ATM corresponds to the best fit point of the solution to the atmospheric neutrino puzzle [2], and a value of $\sin^2 \xi = 0.01$ which is consistent with all the experimental bounds. Note that some “subset” of ATM will always be assumed [for example, $\sin^2 \theta$ is fixed while exploring the $(\Delta m_{31}^2 \times \sin^2 \xi)$ plane]. For each analysis, it will be clear what are the “variables” and what quantities are held fixed at their “preferred point” values. All other points refer to sample points in the regions which best solve the solar neutrino puzzle [9–11],

and the notation should be obvious. Initially a flat neutrino energy distribution with $E_\nu^{\min} = 1 \text{ GeV}$ and $E_\nu^{\max} = 5 \text{ GeV}$ is considered (for concreteness), and the case of higher average energies is briefly discussed later.

A. Case of vacuum oscillations

If neutrinos were produced and propagated exclusively in vacuum, the oscillation probabilities are very simple to com-

FIG. 1. Constant $P_{\alpha\beta}$ contours in the $(\Delta m_{21}^2 \times \sin^2 \omega)$ plane, at ATM (see text), in the case of pure vacuum oscillations.

pute. This would be the case of neutrinos produced in the Sun's core if very low energy neutrinos ($E_\nu \lesssim 100$ keV) were being considered. Nonetheless, it is still useful to digress some on the “would be” vacuum oscillation probabilities of GeV neutrinos in order to understand better the matter effects.

In the case of pure vacuum oscillations, it is trivial to check that $P_{\alpha\beta} = P_{\beta\alpha}$ (remember that the neutrino mixing matrix phase ϕ has been set to zero), and therefore all $P_{\alpha\beta}$ can be parametrized by three quantities, namely $P_{\alpha\alpha}$, $\alpha = e, \mu, \tau$. It is easy to show that

$$P_{\alpha\beta} = P_{\beta\alpha} \Leftrightarrow P_{e\mu} = \frac{1}{2}(1 + P_{\tau\tau} - P_{\mu\mu} - P_{ee}). \quad (4.1)$$

Explicitly,

$$P_{\alpha\alpha} = U_{\alpha 1}^4 + U_{\alpha 2}^4 + U_{\alpha 3}^4 + 2U_{\alpha 1}^2 U_{\alpha 2}^2 \cos\left(\frac{\Delta m_{21}^2 x}{2E_\nu}\right) \quad (4.2a)$$

or

$$P_{\alpha\alpha} = (1 - U_{\alpha 3}^2)^2 + U_{\alpha 3}^4 - 4U_{\alpha 1}^2 U_{\alpha 2}^2 \sin^2\left(\frac{\Delta m_{21}^2 x}{4E_\nu}\right). \quad (4.2b)$$

Note that there is no dependency on Δm_{31}^2 . Two particularly simple limits are reached when L_{osc}^{21} is either very small or very large compared with the Earth-Sun distance. In both limits $P_{\alpha\alpha}$ is independent of Δm_{21}^2 and, in the latter case, $P_{\alpha\alpha}$ depends only on $U_{\alpha 3}^2$. Figure 1 depicts constant $P_{\alpha\beta}$ contours in the $(\Delta m_{21}^2 \times \sin^2 \omega)$ plane, at ATM. Remember that, here, $P_{e\mu}$ is not an independent quantity but is a linear combination of all $P_{\alpha\alpha}$. Note that P_{ee} is symmetric for

FIG. 2. Contours of constant P_3^H (see text) in the $(\Delta m_{31}^2/E_\nu \times \sin^2 \xi)$ plane.

$\omega \rightarrow \pi/2 - \omega$ and that $P_{\mu\mu} \leftrightarrow P_{\tau\tau}$ when $\omega \rightarrow \pi/2 - \omega$. The latter property is a consequence of $\theta = \pi/4$.

Finally, one should note that oscillatory effects are present for $10^{-6} \text{ eV}^2 \gtrsim \Delta m_{21}^2 \gtrsim 10^{-9} \text{ eV}^2$ and are maximal for $\Delta m_{21}^2 \approx 2 \times 10^{-8} \text{ eV}^2$. For this value of Δm_{21}^2 , $\cos(\Delta m_{21}^2 x / 2E_\nu) \approx -1$, and the largest suppression to all $P_{\alpha\alpha}$ is obtained when $U_{\alpha 1}^2 U_{\alpha 2}^2$ is maximum. For example, P_{ee} is smallest when $\omega = \pi/4$, since $U_{e1}^2 U_{e2}^2 \propto \sin^2 2\omega$. There are no “localized” maxima for $P_{\alpha\alpha}$ because $U_{\alpha 1}^2 U_{\alpha 2}^2$ is positive definite.

B. “Normal” neutrino hierarchy

When matter effects are “turned on,” the situation can be dramatically different. This is especially true in the case of normal neutrino mass hierarchies ($m_1^2 < m_2^2 < m_3^2$), which will be discussed first.

1. Preliminaries

The first effect one will observe is that, even though $L_{\text{osc}}^{31} \ll 1$ a.u., $P_{\alpha\beta}$ depend rather nontrivially on Δm_{31}^2 . This dependence comes from the terms P_3^H and $P_2^H = 1 - P_3^H$ in Eq. (3.9). Remember that P_3^H is interpreted as the probability that a ν_e produced in the Sun's core exits the Sun as a ν_3 mass eigenstate. When matter effects are negligible (such as in the limit of small neutrino energies) $P_3^H \rightarrow \sin^2 \xi$, its “vacuum limit.” Figure 2 depicts constant P_3^H contours in the $(\Delta m_{31}^2/E_\nu \times \sin^2 \xi)$ plane.

Note that, for $\Delta m_{31}^2/E_\nu \sim 10^{-2} \text{ eV}^2/\text{GeV}$, $P_3^H \rightarrow 1$, even for small values of $\sin^2 \xi$. In this region, ν_e 's produced in the Sun's core exit the Sun as pure ν_3 's. Therefore, $P_{ee} \approx U_{e3}^2$. Because of unitarity in the propagation, ν_μ 's and ν_τ 's exit the Sun as linear combinations of the light mass eigenstates, and may not only undergo vacuum oscillations but are also susceptible to further matter effects (dictated by the “ M - L ” system, as described in Sec. III). For future reference, at ATM, $P_3^H \approx 0.87$ when averaged over the energy range mentioned in the beginning of this section.

FIG. 3. Contours of constant $1 - P_c^L$ (see text) in the $(\Delta m_{21}^2/E_\nu \times \sin^2 \omega)$ plane.

As $\Delta m_{31}^2/E_\nu$ decreases (as is the case for higher energy neutrinos) the nonadiabaticity of the “ H - M ” system starts to become relevant, and $P_3^H \rightarrow \sin^2 \xi$, as argued in Sec. III B. A hint of this behavior can already be seen in Fig. 2, for small values of $\Delta m_{31}^2/E_\nu$.

The information due to the “ M - L ” matter effect is encoded in P_c^L , present in Eq. (3.9). Figure 3 depicts contours of constant $1 - P_c^L$ in the $(\Delta m_{21}^2/E_\nu \times \sin^2 \omega)$ plane. One should note that $1 - P_c^L$ reaches its extreme nonadiabatic limit, $\sin^2 \omega$, when $\Delta m_{21}^2/E_\nu \lesssim 10^{-7}$ eV²/GeV. For $\Delta m_{21}^2/E_\nu \gtrsim 10^{-7}$ eV²/GeV, matter effects increase the value of $1 - P_c^L$.

One can use the intuition from the two-flavor solution to the solar neutrino puzzle to better appreciate the results presented here. In the case of the solutions to the solar neutrino puzzle, the energies of interest range from 100 keV to 10 MeV, and large matter effects happen around $\Delta m^2 \sim 10^{-5}$ eV². Furthermore, at $\Delta m^2 \sim 10^{-10}$ eV² one encounters the “just-so” solution, which is characterized by very long wavelength vacuum oscillations. Rescaling to $O(\text{GeV})$ energies, the equivalent of the “just-so” solution happens for $\Delta m_{21}^2 \sim (10^{-8} - 10^{-7})$ eV², while large matter effects would be present at $\Delta m^2 \sim (10^{-3} - 10^{-2})$ eV². Indeed, one observes large matter effects for $\Delta m_{31}^2 \sim (10^{-3} - 10^{-2})$ eV². $\Delta m_{21}^2 \sim (10^{-5} - 10^{-6})$ eV² corresponds to the region between the LOW and VAC solutions, where matter effects distort $P_{\alpha\beta}$ from its pure vacuum value, but no dramatic suppression or enhancement takes place. Incidentally, this behavior has physical consequences in the solution to the solar neutrino problem, as was first pointed out in [41].

2. $(\Delta m_{12}^2 \times \sin^2 \omega)$ plane

Having understood the behavior of P_3^H and P_c^L , one is ready to explore the parameter space. Figure 4 depicts constant $P_{\alpha\beta}$ contours in the $(\Delta m_{21}^2 \times \sin^2 \omega)$ plane, at ATM. The shapes and forms can be readily understood, and will be discussed in the remainder of this subsection.

FIG. 4. Constant $P_{\alpha\beta}$ contours in the $(\Delta m_{21}^2 \times \sin^2 \omega)$ plane, at ATM (see text).

First note that the shapes of the constant P_{ee} and $P_{e\mu}$ regions resemble those of the pure vacuum oscillations depicted in Fig. 1, with two important differences. First, the constant values of the contours are quite different. For example, P_{ee} varies from a few percent to less than 15%, while in the case of pure vacuum oscillations, P_{ee} varies from 30% to 100%. This can be roughly understood numerically by noting that $P_{(ee)} \approx P_2^H P_{(ee)}^{\text{vac}}$ (remember that $P_2^H = 1 - P_3^H \approx 0.13$ when averaged over the energy range of interest). Second, at high Δm_{21}^2 , the regions are distorted. This is due to nontrivial matter effects in the “ M - L ” system. Note that the contours follow the constant $1 - P_c^L$ curves depicted in Fig. 3.

The $P_{\mu\mu}$ and $P_{\tau\tau}$ contours are a lot less familiar, and require some more discussion. Many features are rather prominent. For example, the plane is roughly divided into a $\sin^2 \omega > 0.5$ and $\sin^2 \omega < 0.5$ structure, and large (small) values of $P_{\mu\mu}$ ($P_{\tau\tau}$) are constrained to the $\sin^2 \omega > 0.5$ half and vice versa. Also, there is a rough $P_{\mu\mu}(\omega) \leftrightarrow P_{\tau\tau}(\pi/2 - \omega)$ symmetry in the picture, which was present in the pure vacuum case (see Fig. 1). This symmetry is absent for large values of Δm_{21}^2 , similar to what happens in the case of P_{ee} , and is due, as mentioned in the previous paragraph, to the fact that P_c^L is significantly different from $\cos^2 \omega$ in this region.

All other features are also fairly simple to understand and are all due to fact that $P_3^H \gg \sin^2 \xi$. To guide the discussion it is convenient to rewrite expressions in the limit $P_3^H = 1$, $\sin^2 \theta = 0.5$:

$$c_\mu^2 = c_\tau^2 = 0, \quad (4.3)$$

$$a_\mu^2 = b_\tau^2 = 0.5[1 + 2\sqrt{P_c^L(1 - P_c^L)}], \quad (4.4)$$

$$b_\mu^2 = a_\tau^2 = 0.5(1 - 2\sqrt{P_c^L(1 - P_c^L)}), \quad (4.5)$$

$$a_\mu b_\mu = -a_\tau b_\tau = 0.5(1 - 2P_c^L), \quad (4.6)$$

and

$$P_{(\mu\mu, \tau\tau)} = \frac{1}{2}(1 - U_{(\mu, \tau)3}^2) \pm \sqrt{P_c^L(1 - P_c^L)}(U_{(\mu, \tau)1}^2 - U_{(\mu, \tau)2}^2) \\ \pm (1 - 2P_c^L)U_{(\mu, \tau)1}U_{(\mu, \tau)2}\cos\left(\frac{\Delta m_{21}^2 x}{2E_\nu}\right). \quad (4.7)$$

For $\Delta m_{12}^2 \gtrsim 10^{-7} \text{ eV}^2$, the oscillatory term averages out to zero, while for $\Delta m_{12}^2 \lesssim 10^{-9} \text{ eV}^2$ $\cos(\Delta m_{21}^2 x / 2E_\nu) = 1$.

In order to understand the behavior at $\Delta m^2 \lesssim 10^{-7} \text{ eV}^2$, one should take advantage of the fact that $1 - P_c^L \rightarrow \sin^2 \omega$. In this case, it proves more advantageous to use the second form of Eq. (3.9) in order to express all $P_{\alpha\beta}$:

$$P_{ee} = P_2^H \cos^2 \xi + P_3^H \sin^2 \xi - (\text{Osc})_{ee},$$

$$P_{e\mu} = (P_2^H \sin^2 \xi + P_3^H \cos^2 \xi) \sin^2 \theta - (\text{Osc})_{e\mu},$$

$$P_{\mu\mu} = (\cos^2 \theta + \sqrt{P_3^H} \sin \xi \sin^2 \theta)^2 + P_2^H \cos^2 \xi \sin^4 \theta - (\text{Osc})_{\mu\mu},$$

$$P_{\tau\tau} = (\sin^2 \theta + \sqrt{P_3^H} \sin \xi \cos^2 \theta)^2 + P_2^H \cos^2 \xi \cos^4 \theta - (\text{Osc})_{\tau\tau}, \quad (4.8)$$

where

$$(\text{Osc})_{\alpha\beta} = 4a_\alpha b_\alpha U_{\beta 1} U_{\beta 2} \sin^2\left(\frac{\Delta m_{12}^2 x}{4E_\nu}\right) \quad (4.9)$$

are the oscillatory terms.

When $L_{\text{osc}}^{21} \gg 1 \text{ a.u.}$, the oscillatory terms are zero, and $P_{\alpha\beta}$ are particularly simple. It is, therefore, convenient to start the discussion in this “region” (very small Δm_{21}^2). Note that here many simplifications take place: $P_{\alpha\beta}$ is independent of ω and Δm_{21}^2 , and $P_{\mu\mu} = P_{\tau\tau}$ if $\sin^2 \theta = \cos^2 \theta$. It is trivial to compute the value of $P_{\alpha\beta}$ given that we are at ATM and that $P_3^H \approx 0.87$.

The next curious feature is that there is a “band” around $\sin^2 \omega = 1/2$ where $P_{\mu\mu} \approx P_{\mu\mu}(L_{21}^{\text{osc}} \rightarrow \infty)$. The same is true of $P_{\tau\tau}$. This is due to the fact that, around $\sin^2 \omega \approx 1/2$, $a_\mu b_\mu$ and $a_\tau b_\tau$ vanish when P_3^H is large. This can be seen, in the limit $P_3^H = 1$, from Eq. (4.6) by noting that indeed both $a_\mu b_\mu$ and $a_\tau b_\tau$ vanish at $P_c^L = 1/2$ and remembering that for values of $\Delta m_{21}^2 \lesssim 10^{-7} \text{ eV}^2$ $P_c^L \approx \cos^2 \omega$. This explains why the band is around $\omega \approx \pi/4$. Slight distortions are due to the fact that $P_3^H \neq 1$, and are easily computed from the exact expressions. At larger values of Δm_{21}^2 , the band “curves” following the constant P_c^L contours present in Fig. 3.

Again in the limit $P_3^H = 1$, $P_c^L = \cos^2 \omega$, the coefficient of the $\cos(\Delta m_{21}^2 x / 2E_\nu)$ term, Eq. (4.7), is

$$\pm \frac{1}{2}(1 - 2\cos^2 \omega) \left(-\frac{1}{2} \sin 2\omega \mp 0.1 \cos 2\omega \right), \quad (4.10)$$

if $\sin^2 \xi$ terms are neglected. The $+$, $-$ signs are for $P_{\mu\mu}$ while the $-$, $+$ signs for $P_{\tau\tau}$. It is trivial to verify numeri-

cally (if a little tedious) that the $P_{\mu\mu}$ term has a maximum at $\sin^2 \omega \approx 0.1$ and a minimum at $\sin^2 \omega \approx 0.8$. For $P_{\tau\tau}$ the maximum (minimum) is at $\sin^2 \omega \approx 0.9(0.2)$. It is important to comment that the minima are negative numbers. On the other hand, from Fig. 1 (as mentioned before) it is easy to see that $\cos(\Delta m_{21}^2 x / 2E_\nu) \approx -1$ for $\Delta m_{21}^2 \approx 2 \times 10^{-8} \text{ eV}^2$ (this is where all $P_{\alpha\alpha}$ are maximally suppressed in Fig. 1). Combining both pieces of information, it is simple to understand the maxima/minima of $P_{\mu\mu}$ and $P_{\tau\tau}$ at $\Delta m_{21}^2 \approx 2 \times 10^{-8} \text{ eV}^2$: Minima occur when the coefficient is maximum (e.g. at $\sin^2 \omega \approx 0.1$ for $P_{\mu\mu}$) while maxima occur when the coefficient is minimum (e.g., at $\sin^2 \omega \approx 0.8$ for $P_{\mu\mu}$).

A pictorial description of what has happened is, therefore, the following: The matter effects “compress” the constant $P_{\mu\mu}$ ($P_{\tau\tau}$) contours from the pure vacuum oscillation case (presented in Fig. 1) to the $\sin^2 \omega < 1/2$ ($> 1/2$) half of the plane, and a new region “appears” on the other half. This other region is characterized by negative values to the coefficients of the oscillatory terms, which are not attainable in the case of pure vacuum oscillations [see Eq. (4.2a)].

At last, the contours in the region where the oscillatory effects average out, $P_{\mu\mu}$ and $P_{\tau\tau}$ are also best understood from Eq. (4.7) in the limit that $\cos(\Delta m_{21}^2 x / 2E_\nu) \rightarrow 0$. It is simple to see, for example, that $P_{\mu\mu} < P_{\tau\tau}$ if $\cos 2\omega > 0$ ($\sin^2 \omega > 1/2$), while the situation is reversed if $\cos 2\omega < 0$. This is indeed what one observes in Fig. 4.

3. Dependence on $\sin^2 \theta$ and T -violating matter effects

Figures 5 and 6 depict $P_{\alpha\beta}$ as a function of $\sin^2 \theta$ at LMA and SMA, and at LOW and VAC, respectively. In these figures, all $P_{\alpha\beta}$ are plotted, in order to illustrate that $P_{\alpha\beta} \neq P_{\beta\alpha}$ at LMA, SMA and LOW. This means that, as advertised in Sec. III, even though the CP -violating phase in the neutrino mixing matrix was set to zero, matter effects are producing observable T -violating effects.

At VAC, even though $P_3^H \neq \sin^2 \xi$ (strong matter effect in the “ M - H ” system), $P_{\alpha\beta} = P_{\beta\alpha}$. This is simple to understand, and is related to the fact that, at VAC, $P_c^L = \cos^2 \omega$ and “1-2” oscillations do not have “time” to happen. As argued before, at LOW and VAC, Eqs. (4.8) apply, and it is easy to verify directly that, if the oscillatory terms vanish, $2P_{e\mu} = 1 + P_{\tau\tau} - P_{\mu\mu} - P_{ee}$. This condition implies $P_{\alpha\beta} = P_{\beta\alpha}$, as shown before. This is not the case in the presence of non-vanishing oscillatory terms or when $P_c \neq \cos^2 \omega$. Both statements are trivial to verify directly. For example,

$$4a_e b_e U_{\mu 1} U_{\mu 2} = P_2^H \sin 2\omega [\sin 2\omega (\sin^2 \xi \sin^2 \theta - \cos^2 \theta) \\ - \sin \xi \sin 2\theta \cos 2\omega] \quad (4.11)$$

while

$$4a_\mu b_\mu U_{e 1} U_{e 2} = \cos^2 \xi \sin 2\omega [\sin 2\omega (P_3^H \sin^2 \theta - \cos^2 \theta) \\ - \sqrt{P_3^H} \sin 2\theta \cos 2\omega], \quad (4.12)$$

such that $\text{Osc}_{e\mu} \neq \text{Osc}_{\mu e}$. Therefore, at LOW, even though $P_c^L = \cos^2 \omega$, $P_3^H \neq \sin^2 \xi$ and nontrivial oscillatory terms render $P_{\alpha\beta} \neq P_{\beta\alpha}$.

FIG. 5. $P_{\alpha\beta}$ as a function of $\sin^2\theta$, at LMA (top) and SMA (bottom) (see text).

From Eqs. (4.8) one can also roughly understand the dependency of $P_{\alpha\beta}$ on $\sin^2\theta$. Obviously P_{ee} does not depend on θ [by the very form of the neutrino mixing matrix, Eq. (3.1)], while $P_{e\mu}(P_{e\tau})$ depends almost exclusively on $\sin^2\theta$ ($\cos^2\theta$). This is guaranteed by the fact that $P_3^H \gg P_2^H$ even at LMA and LOW, when one expects the interference terms to play a significant role.

C. “Inverted” neutrino hierarchy

Here I turn to the case of an “inverted” neutrino hierarchy, namely $\Delta m_{31}^2 < 0$. Currently, there is no experimental hint as to what the sign of Δm_{31}^2 should be, so there is no reason to believe that the “normal” hierarchy is to be preferred over the “inverted” hierarchy. Indeed, even from a theoretical and model building point of view, there are no strong reasons for or against a particular neutrino mass hierarchy [42].

FIG. 6. $P_{\alpha\beta}$ as a function of $\sin^2\theta$, at LOW (top) and VAC (bottom) (see text).

The discussion will be restricted to $\Delta m_{21}^2 > 0$ for two reasons. First, the $\Delta m_{21}^2 < 0$ can be approximately read off from the $\Delta m_{21}^2 > 0$ case by changing $\omega \rightarrow \pi/2 - \omega$, as mentioned before. Second and most important, there is some experimental hints as to what is the sign of Δm_{21}^2 [12,11]. For example, the SMA solution only exists for one sign of Δm_{21}^2 , while the LMA and LOW solutions prefer one particular sign. Therefore, the notation introduced in the beginning of this section (ATM, SMA, LMA, LOW, VAC) still applies, and one should simply remember that here $\Delta m_{31}^2 < 0$.

As advertised, the largest effect of $\Delta m_{31}^2 < 0$ is the typical values of P_c^H . From Eq. (2.7), keeping in mind that here γ is negative,

$$P_c^H = \frac{1 - e^{-|\gamma|\cos^2\xi}}{1 - e^{-|\gamma|}}, \quad (4.13)$$

FIG. 7. $P_{\alpha\beta}$ as a function of $\sin^2\xi$, at LMA, SMA, LOW, and VAC (see text), for $\Delta m_{31}^2 < 0$.

where γ is given by Eq. (2.8) with $\Delta m^2 \rightarrow \Delta m_{31}^2$. Since $|\gamma| \gg 1$ [see Eq. (2.8)], $P_c^H = 1$ for all values of Δm_{31}^2 and $\sin^2\xi$ of interest. Indeed, this is true for any value of $\sin^2\xi$ as long as $\Delta m_{31}^2 \gg 10^{-6} \text{ eV}^2$. This is to be contrasted to the normal hierarchy case, when there is always some value of $\sin^2\xi$ (which is a function of Δm_{31}^2) below which P_c deviates significantly from its adiabatic limit.

All of the Δm_{31}^2 dependence of $P_{\alpha\beta}$ is therefore encoded in ξ_M . However, in the case $\Delta m_{31}^2 < 0$ it is trivial to show that $-1 < \cos 2\xi_M < -\cos 2\xi$, where the upper bound is reached in the limit $|\Delta m_{31}^2/2E_\nu| \gg A$. (This has been mentioned before. The minus sign takes care of the “unorthodox” $P_c^H \rightarrow 1$ adiabatic limit.) Since one is interested in $\sin^2\xi < 0.1$ ($-\cos 2\xi < -0.8$), the range for ξ_M is rather limited, and therefore any Δm_{31}^2 effects are bound to be very small. Larger Δm_{31}^2 effects are expected for larger $\sin^2\xi$.

In light of this, Fig. 7 depicts $P_{\alpha\alpha}$ and $P_{e\mu}$ as a function of $\sin^2\xi$ at the various points (LMA, SMA, LOW, VAC), for $\Delta m_{31}^2 = -3 \times 10^{-3} \text{ eV}^2$ and $\sin^2\theta = 0.5$.

It is interesting to compare the results which are obtained here with the pure vacuum case. In the limit $P_2^H = 1$,

$$P_{ee} = \cos^2\xi \left[P_c^L \cos^2\omega + (1 - P_c^L) \sin^2\omega + \sqrt{P_c^L(1 - P_c^L)} \sin 2\omega \cos\left(\frac{\Delta m_{21}^2 L}{2E_\nu}\right) \right], \quad (4.14)$$

while the pure vacuum result in the same region of the parameter space is

$$P_{ee}^{\text{vac}} = \cos^4\xi \left[\cos^4\omega + \sin^4\omega + 2\sin^2\omega \cos^2\omega \cos\left(\frac{\Delta m_{21}^2 L}{2E_\nu}\right) \right] + \sin^4\xi. \quad (4.15)$$

In the limit $P_c^L = \cos^2\omega$, the difference

$$P_{ee} - P_{ee}^{\text{vac}} = (P_{ee}^{2\nu, \text{vac}}) \cos^2\xi \sin^2\xi - \sin^4\xi, \quad (4.16)$$

where $P_{ee}^{2\nu, \text{vac}}$ is the electron neutrino survival probability in the two-flavor case with $\Delta m^2 = \Delta m_{21}^2$ and vacuum mixing angle ω . This difference vanishes at $\sin^2\xi = 0$, and $\sin^2\xi = P_{ee}^{2\nu, \text{vac}}/(1 + P_{ee}^{2\nu, \text{vac}})$, (which is between 0 and 0.5). Furthermore, it is a convex function of $\sin^2\xi$, which means that P_{ee} is larger than the pure vacuum case for values of $\sin^2\xi < P_{ee}^{2\nu, \text{vac}}/(1 + P_{ee}^{2\nu, \text{vac}})$. Away from the limit $P_c^L = \cos^2\omega$, keeping in mind that the oscillatory terms average out, P_{ee} is still larger than the pure vacuum case if $\cos^2\omega > \sin^2\omega$ since $P_c^L \leq \cos^2\omega$, as one can easily verify.

Also, in the limit $P_2^H = 1$, $\sin^2\theta = 1/2$,

$$P_{\mu\mu} = \frac{1}{2} \left[(1 - P_c^L) U_{\mu 1}^2 + P_c^L U_{\mu 2}^2 + U_{\mu 3}^2 + 2\sqrt{P_c^L(1 - P_c^L)} U_{\mu 1} U_{\mu 2} \cos\left(\frac{\Delta m_{21}^2 L}{2E_\nu}\right) \right]. \quad (4.17)$$

The same expression applies for $P_{\tau\tau}$ with $U_{\mu i} \rightarrow U_{\tau i}$. This is a consequence of $\sin^2\theta = \cos^2\theta$. Furthermore, in the limit $\sin^2\xi \rightarrow 0$ (and for $\sin^2\theta = \cos^2\theta$), $U_{\mu i} = U_{\tau i}$, which explains why $P_{\mu\mu} = P_{\tau\tau}$ for $\sin^2\xi \leq 10^{-2}$. At VAC this equality remains for all values of $\sin^2\xi$. The reason for this is that, at VAC, the expression simplifies tremendously and $P_{\mu\mu} = P_{\tau\tau} = \frac{1}{4}(1 + \cos^2\xi)$. In the same region of the parameter space, the pure vacuum oscillation case yields $P_{\mu\mu}^{\text{vac}} = P_{\tau\tau}^{\text{vac}} = \frac{1}{2} \cos^4\xi - \cos^2\xi + 1$. Note that, in this region of the parameter space $P_{\mu\mu}^{\text{vac}} \geq P_{\mu\mu}$, the inequality being saturated at $\cos^2\xi = 1$.

The same result also applies (approximately) at SMA, since the oscillatory terms are proportional to $\sqrt{P_c^L(1 - P_c^L)}$ and $1 - P_c^L$ is very small at SMA (see Fig. 3). The equality $P_{\mu\mu} = P_{\tau\tau}$ is broken at larger values of $\sin^2\xi$ because $P_c^L \neq \cos^2\omega$ at SMA.

It remains to discuss how $P_{\mu\mu}$ and $P_{\tau\tau}$ diverge from the pure vacuum case at LMA and LOW. In the limit $P_c^L = \cos^2\omega$, and averaging out the oscillatory terms,

$$P_{\mu\mu} - P_{\mu\mu}^{\text{vac}} = \frac{\sin\xi}{2} \{ \sin\xi [U_{\mu 3}^2 - (\cos^2\omega U_{\mu 1}^2 + \sin^2\omega U_{\mu 2}^2)] - \sin 2\omega (U_{\mu 1}^2 - U_{\mu 2}^2) \}. \quad (4.18)$$

This difference goes to zero as $\sin^2\xi \rightarrow 0$. This is to be expected, since in this limit the difference of P_2^H and $\cos^2\xi$ disappears. For small values of $\sin^2\xi$, the last term in Eq. (4.18) dominates, and, as discussed before, $U_{\mu 1}^2 - U_{\mu 2}^2 = -0.5 \cos 2\omega + O(\sin\xi)$. Therefore, $P_{\mu\mu} - P_{\mu\mu}^{\text{vac}} > 0$ (< 0) for $\cos 2\omega > 0$ (< 0). The expression for $P_{\tau\tau}$ can be obtained from Eq. (4.18) by replacing $U_{\mu i} \rightarrow U_{\tau i}$ and changing the sign of the last term. Therefore, since $U_{\tau 1}^2 - U_{\tau 2}^2 = -0.5 \cos 2\omega + O(\sin\xi)$, $P_{\tau\tau} - P_{\tau\tau}^{\text{vac}} > 0$ (< 0) for $\cos 2\omega < 0$ (> 0). When the oscillatory terms do not average out, it is easy to verify explicitly that the behavior of the oscillatory

FIG. 8. Constant $P_{\alpha\beta}$ contours in the $(\Delta m^2_{21} \times \sin^2 \omega)$ plane, at ATM (see text), for $\Delta m^2_{31} < 0$.

terms follows the behavior of the average terms, discussed above, and the inequalities obtained above still apply.

The situation, however, changes, when $P_c^L \neq \cos^2 \omega$, i.e., when matter effects due to the “ M - L ” system are relevant. In this region, a behavior similar to the one observed in the “normal” hierarchy case is expected, since $\Delta m^2_{21} > 0$. Figure 8 depicts constant $P_{\alpha\beta}$ contours in the $(\Delta m^2_{21} \times \sin^2 \omega)$ plane. One should be able to see upon close inspection that the region $P_{ee} < 30\%$ is smaller in Fig. 8 than the same region in the pure vacuum oscillation case, Fig. 1. Also, the constant $P_{\mu\mu}$ ($P_{\tau\tau}$) contours are shifted to larger (smaller) values of $\sin^2 \omega$. The other prominent (and expected, as mentioned above) feature is the distortion of the contours at large values of Δm^2_{21} . This behavior is similar to the one observed in Fig. 4.

I conclude this subsection with a comment on antineutrinos. As discussed previously, $P_{\bar{\alpha}\bar{\beta}}(\Delta m^2_{21}, \Delta m^2_{31}) = P_{\alpha\beta}(-\Delta m^2_{21}, -\Delta m^2_{31})$, such that the “normal” hierarchies yield “inverted” hierarchy results for antineutrinos and vice versa. One cannot, however, apply Fig. 4 and Fig. 8 for the antineutrinos because both Δm^2_{ij} have to change sign, not just Δm^2_{31} . Qualitatively, however, it is possible to understand the constant $P_{\bar{\alpha}\bar{\beta}}$ contours by examining figures Fig. 4 and Fig. 8 reflected in a mirror positioned at $\sin^2 \omega = 0.5$, meaning that $P_{\bar{\alpha}\bar{\beta}}(\sin^2 \omega, \Delta m^2_{31}) \approx P_{\alpha\beta}(\cos^2 \omega, -\Delta m^2_{31})$. The equality is not complete because one is also required to exchange $\theta \rightarrow \pi - \theta$, as mentioned earlier.

D. Higher neutrino energies

As the average neutrino energy increases, the values of $P_{\alpha\beta}$ start to resemble more the pure vacuum case. This is easy to see from Figs. 2 and 3. Any deviation of $1 - P_c^L$

from $\sin^2 \omega$ goes away even at LMA for $E_\nu \approx 50$ GeV, while “ H - M ” effects remain important up to $E_\nu \approx 1$ TeV, even though quantitatively the effect decreases noticeably. This can be illustrated by the value of P_3^H at ATM, for example, which drops from 0.87 for energies which range from 1 to 5 GeV (see the previous subsections) to 0.058, for energies which range from 100 to 110 GeV.

Furthermore, all L_{ij}^{osc} increase as the energy increases, for fixed values of Δm^2_{ij} . Therefore, LOW becomes indistinguishable from VAC at $E_\nu \approx 100$ GeV. For $O(\text{TeV})$ neutrinos the sensitivity to Δm^2_{21} remains only for its highest allowed values, while one should start worrying about nontrivial oscillatory effects due to L_{31}^{osc} .

The case of higher energy neutrinos contains a more serious complication: neutrino absorption inside the Sun. As the neutrino energy increases, one has to start worrying about the fact that absorptive neutrino interactions can take place. According to [43], for neutrinos produced in the Sun’s core, absorption becomes important for $E_\nu \gtrsim 200$ GeV. In this case, ν_e and ν_μ interact with nuclear matter and produce electrons and muons, respectively. The former are capture and “lost” inside the Sun, while the latter stop before decaying into low energy neutrinos. The case of ν_τ -Sun interactions is more interesting, because the τ leptons produced via charged current interactions decay before “stopping,” yielding ν_τ ’s with slightly reduced energies. Therefore, it is possible to get a flux of very high energy initial state τ neutrinos but not muon or electron-type neutrinos. Such effects have been studied for high energy galactic neutrinos traversing the Earth [44].

The effect of neutrino oscillations inside the Sun in the presence of non-negligible neutrino absorption is certainly of great interest but is beyond the scope of this paper.

V. SUMMARY AND CONCLUSIONS

The oscillation probability of $O(\text{GeV})$ neutrinos of all flavors produced in the Sun’s core has been computed, including matter effects, which are, in general, nontrivial. All the results presented here can be obtained from Eq. (3.9), which is one of the main results of this paper.

In particular, it was shown that, unlike the two-flavor oscillation case, in the three-flavor case the probability of a neutrino produced in the flavor eigenstate α to be detected as a flavor eigenstate β ($P_{\alpha\beta}$) is (in general) different from $P_{\beta\alpha}$, even if the CP -violating phase of the neutrino mixing matrix vanishes. This is, of course, expected since Sun-neutrino interactions explicitly break T invariance. Indeed, it is the case of two-flavor oscillations which is special, in the sense that the number of independent oscillation probabilities is too small because of unitarity.

The results of a particular scan of the parameter space are presented in Sec. IV. In this case, special attention was paid to the regions of the parameter space which are preferred by the current experimental situation. The effect of neutrino-matter interactions is illustrated by the obvious differences between Fig. 1 (pure vacuum), Fig. 4 (“normal” hierarchy), and Fig. 8 (“inverted” hierarchy). Note that the oscillation

probabilities depend rather dramatically on the “sign of Δm_{31}^2 .”

It turns out that, in the case of a “normal” neutrino mass hierarchy, it is possible to suppress P_{ee} tremendously with respect to its pure vacuum oscillation values, by a mechanism that is similar to the well-known MSW effect in the case of two-flavor oscillations: the parameters are such that electron-type neutrinos produced in the Sun’s core exit the Sun (almost) as pure mass eigenstates, and the ν_e component of this eigenstate is small. Both $P_{\mu\mu}$ and $P_{\tau\tau}$ can be significantly suppressed, and the constant $P_{\mu\mu}$ and $P_{\tau\tau}$ contours as a function of the “solar” angle and the smaller mass-squared differences are nontrivial. One important feature is that when $P_{\mu\mu}$ is significantly suppressed, $P_{\tau\tau}$ is not, and vice versa. One consequence of this is that, for some regions of the parameter space, it is possible to have an enhancement of ν_τ ’s detected in the Earth with respect to the number of ν_μ ’s (or vice versa). This may have important implications for solar weekly interacting massive particle (WIMP) annihilation searches at neutrino telescopes, and will be studied in another opportunity. In order to study such implications, it is imperative to know the expected production rate, as a function of energy and for each neutrino species, in the core of the Sun. Such information will depend very strongly on the dark matter candidate and “model.”

In the case of an “inverted” mass hierarchy, the situation is very similar to the pure vacuum case, and no particular suppression of any $P_{\alpha\alpha}$ is possible. Indeed, for a large region of the parameter space P_{ee} is in fact enhanced, a feature which is also observed in the two-flavor case [40].

The case of higher energy neutrinos was very briefly discussed, and the crucial point is to note that, for neutrino energies above a few hundred GeV, the absorption of neutrinos by the Sun becomes important. The study of absorption effects is beyond the scope of this paper.

Finally, it is important to reemphasize that the values of $P_{\alpha\beta}$ computed here are to be understood as if they were evaluated at the Earth’s surface. No Earth-matter effects have been included. It is possible that Earth-matter effects are important, especially the ones related to Δm_{31}^2 , in the advent that $U_{e3}^2 \equiv \sin^2 \xi$ turns out to be “large.”

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