ACCELERATION IN THE LEP HADRON COLLIDER

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1. Evaluation of R.F. Parameters

In this section we consider the RF requirements for efficient injection, accumulation, acceleration and storage of protons in a LEP hadron collider \(^1\),\(^2\).

The main parameters which need examination in the appraisal of an RF system for such a collider are as follows:

- **Longitudinal Injection Efficiency.** For high efficiency the bunch (from the SPS) must be injected into the centre of an RF bucket having phase plane trajectories which are identical to those of the bunch. In addition the bucket area must be significantly larger than the bunch area so that the lifetime at injection is acceptable.

- **Acceleration rate.** The beams should be accelerated from 400 Gev to around 8000 Gev in a reasonably short time. The rate of acceleration determines to a large extent the peak RF voltage required.

- **Longitudinal Lifetime of the Stored Beam.** The particles of the stored beam must remain inside the RF buckets if they are to collide with the bunches of the counter-rotating beam. Hence, the RF bucket must be appreciably larger than the bunch, and the spectral density of the RF noise at the synchrotron frequency must be kept to an absolute minimum. It is therefore prudent to avoid synchrotron frequencies which are harmonics of the mains frequency (50 Hz).
Considerations of cavity size, shunt impedance and economy of RF power generation favour the choice of a high accelerating frequency; considerations of beam dynamics often point in the opposite direction. For LEP $e^+e^-$ a frequency of 352 MHz has been chosen essentially as the lowest frequency at which multicell cavities of acceptable size, compact klystron power generators and waveguides for power distribution can be used. Since the same criterion also applies to the hardware of the RF system considered here and as it will be very desirable to profit from existing designs and, possibly, from existing equipment of the LEP $e^+e^-$ RF system the same choice of frequency - 352 MHz - has been tentatively assumed throughout this paper.

Table 1 shows a summary of RF related parameters, for 352 MHz, and the criteria used for their derivation. All parameters comfortably meet the requirements stated above. In a single p+ ring the quoted peak voltage per revolution would require approximately one quarter of the $e^+e^-$ RF equipment of LEP phase 1. Details of these calculations are given in Appendix I.

<table>
<thead>
<tr>
<th>Beam Energy (Gev)</th>
<th>R.F. parameters</th>
<th>$Q_s$</th>
<th>Criterion used for evaluation of the RF parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>$V_{RF}$ (MV)</td>
<td></td>
<td>(.0037 matching bucket to bunch)</td>
</tr>
<tr>
<td>400 + 8000</td>
<td>3.6 + 72.5</td>
<td>174.0</td>
<td>(.0042 acceleration time = 5 minutes (constant $Q_s$))</td>
</tr>
<tr>
<td>8000</td>
<td>72.5</td>
<td>180.0</td>
<td>(.0042 maximum installed voltage so as to minimise effect of RF noise)</td>
</tr>
</tbody>
</table>
2. The accelerating structure

The conclusion of the previous section is that approximately 350 MHz is a reasonable choice of frequency and about 80 MV per revolution a desirable voltage at that frequency. At a much higher frequency the buckets would be too short to accept the bunches from the SPS, a substantially lower frequency would lead to uncomfortably large cavities and make it very difficult to use klystrons and waveguides for the generation and distribution of RF power (which is in the Megawatt range in any case). In addition 352 MHz is the frequency chosen for electrons in LEP so that much of the design - possibly of the hardware too - can be taken over from the LEP phase 1 RF system.

A circumferential voltage of 80 MV may seem relatively modest compared with the requirement for the electron ring (400 MV in phase 1) but this relative modesty would be completely lost if we were obliged to deviate substantially from the extreme values of shunt impedance and from the very economic way of power generation developed for e⁻e⁺ storage rings \(^3,^4,^5\).

For a single ring \(p\bar{p}\) collider a common RF system using \(\pi\)-mode cavities would obviously be used. The building blocks of the LEP Phase 1 RF system are one megawatt klystrons, waveguide power dividers and five-cell \(\pi\)-mode cavities which have a shunt impedance of 26 \(\Omega\) per active meter.

Each cavity (without storage cavity) yields 2.63 MV at 125 kW input. An attractive single ring RF system for hadrons, constructed from these elements, would have a total of 32 such cavities arranged in two pairs of stations – one station on either side of two diametrically opposite collision points. Each station would contain a single one-megawatt klystron, powering 8 cavities via three stages of power dividers. The chain of 8 cavities requires about 21 m actual length along the beam. In total this system would give about 84 MV peak voltage per turn at 4 MW RF power.

For \(pp\) twin rings the same system, common to both beams, can be employed with obvious economic advantage, provided the cavities can be placed into an extension of the lattice insertion where the beams are kept essentially collinear. This will be discussed under 2.6 below. It is the preferred solution, by far, from the point of view of RF system design.
If this solution is found impossible or uneconomical, a serious difficulty arises from the fact that the distance between the two counter rotating beams is about 0.2 m in the regular arcs and that it seems very undesirable to change to a substantially different separation in the RF sections. The inner diameter of an optimum 352 MHz cell is 600 mm. Thus, placing two such cavities side by side is excluded and even making one beam pass along the outer wall while the other passes through the centre would require an unacceptable increase of the frequency to well above 500 MHz. The following facts must be considered in trying to find a solution.

2.1 Flat cavities

In a cavity of high shunt impedance the electric field is predominantly longitudinal and the wave propagation predominantly radial. In this configuration only little space can be gained by making the cavity oval since, for large aspect ratios, it is the minimum transverse dimension which determines the frequency with an asymptotic lower limit of \( \lambda/2 \). This is best illustrated by the \( E_0 \) mode in a rectangular box of width \( a \) and height \( b \). The resonant frequency is given by

\[
f = \frac{c}{2} \sqrt{a^{-2} + b^{-2}}
\]

2.2 Re-entrant cavities, septum cavities

Even when transverse dimensions are of no concern it is customary to introduce short re-entrant elements ("nose cones") with concomitant transverse electric field components in order to optimize the shunt impedance (including transit time). At optimum shunt impedance these nose cones remain rudimentary and cause only a small reduction in cavity diameter - from 0.65 m for an \( E_{01} \) pill-box to the 0.60 m of our actual LEP cavity.
In order to obtain a radical reduction of cavity width the wave propagation has to be made predominantly longitudinal i.e. the electric field becomes predominantly transverse and the structure takes the shape of a transmission line resonator (coaxial or otherwise). This leads to a drastic reduction of shunt impedance. To illustrate this we may take coaxial lines with 10 cm and 6 cm diameter of the outer and inner conductors respectively. This is an extreme choice of dimensions for side-by-side cavities having axis spacing of 20 cm and sufficient beam aperture. The Q-factor \(^6\) of a half-wavelength resonator \(*)\) of copper at 352 MHz is about 5200, the shunt impedance (defined as peak voltage at the center squared over power loss) is 403 kΩ. By comparison the Q-factor of the LEP cavity is 40000 and the shunt impedance of one cell is 11 MΩ.

The degradation of shunt impedance by roughly a factor 30 is completely unacceptable and the general conclusion seems inescapable that separate side by side cavities for pp beams at 20 cm distance are excluded.

The situation becomes more favourable if the cavities are longitudinally staggered so that beam 2, say, can scrape along the outer wall of a cavity centered on beam 1. Figure 1 shows a sketch of such a "septum cavity". The cavity shown has been made to resonate at 353 MHz and the Q-factor (for copper) and shunt impedance have been computed with SUPERFISH\(^7\) as 24900 and 12.4 MΩ/m (5.3 MΩ per cell) respectively **). The deterioration by a factor 2.1 from the optimum value may be acceptable and the following tentative choice of parameters may be proposed.

\(*\) A real cavity would be made of two approximately quarter-wave stubs facing each other across a gap, the shunt impedance of that will be even less than what is calculated here.

\(**\) The values given are 85% of the theoretical values – as in the LEP cavities – to allow for the coupling slots and spurious losses.
A total of 128 five-cell septum cavities (64 cavities per beam each cell like Fig. 1) powered by a total of 8 klystrons of 1MW each (four per beam) gives 82 MV per revolution and per beam. Such a system would contain the same number of cavities as the LEP phase 1 RF system and occupy the same total length in the tunnel but use only half the total RF power.

Thus, "septum cavities" at 350 MHz and with 20 cm beam spacing do form a possible solution. These cavities will not be easy to build, however, and may be expensive. The re-entrant electrodes need intense cooling from built-in water channels (the short nose cones of the LEP cavities are cooled by conduction to the discs) and a satisfactory method for making the septum has yet to be found. In any case it appears that 350 MHz and 20 cm beam separation are lower limits for this solution.

2.3 Optimized cavities with side holes

Instead of using septum cavities we may simply thread beam 2 through side holes of beam 1 cavities which are normal and fully optimized in all other respects. Neglecting the spurious voltage contribution of the side channel we may propose a total of 64 five-cell cavities of the LEP geometry, powered by a total of 8 klystrons of 1 MW each. This corresponds to exactly one half of the LEP phase 1 layout, and delivers 84 MV per turn to each proton beam. The side holes are at about two thirds of the cavity radius. This may look awkward, but fully optimized cavities can be used and the side holes would not seem to present great constructional problems.

Beam 2 is exposed to an appreciable voltage in the side channels of the beam 1 cavities but this can be made to contribute fully to acceleration by proper location and phasing of the cavities (c.f. below); it does bring about a coupling of the beams. The main and possibly prohibitive drawback is that beam 2 can see nearly the full magnetic field of about 2.5 mT (peak value) in the beam 1 cavity. Due to the π-mode the deflections add up cell per cell. A particle passing at phase φ with respect to the zero crossing of the accelerating voltage suffers cos φ times the peak deflection.
2.4 Common $E_{01}$ cavities for separated beams

At 20 cm separation the two beams could pass through separate holes (say 5 cm in diameter, $\pm$ 10 cm off axis) in a common cavity. Even for a simple $E_{01}$ pillbox each beam would see over 85% of the maximum field and separate sets of small nose cones could be applied to improve shunt impedance. A further improvement may be obtained by giving the cavity the shape of a (horizontal) ellipse with the nose-cones at or near the foci provided mechanical problems and cost permit this. The absence of computer codes for non axially symmetric structures would be a difficulty to overcome. It appears probable that the shunt impedance per unit length will not be far below the figure for the LEP cavities. Consequently this solution is superior to the solutions discussed under 2.3 from the point of view of cost, length of structure and power. Unfortunately the RF magnetic field will be strong (for a pill box the magnetic field at 0.31 times the radius is over half of the maximum field). Whether this is tolerable remains to be studied.

The proper phasing for simultaneous acceleration of counter-rotating beams with equal charge is obtained by placing the $\pi$-mode cavities at odd multiples of $\lambda/4$ ($\lambda$: wavelength in vacuum) from the collision point. If cavities are placed symmetrically on either side of a collision point they have to be driven in antiphase.

2.5 Common $E_{11}$ cavities for both beams

Figure 2 shows the $E_{11}$ mode ("push-pull", as used in RF separators) in a circular cavity. This mode would accelerate both beams at maximum electric field and zero magnetic field provided the beam separation could be made to coincide with the field maxima. The simple example of the flat pillbox can again be used to investigate the feasibility of this. At 352 MHz the diameter of an $E_{11}$ pillbox cavity is 1.04 m and the maxima of electric field (and zeros of magnetic field) are spaced by 0.5 m. To reduce this distance to 0.2 m by re-entrant conductors at acceptable shunt impedance seems hopeless by the same argument as given under 2.2 above. Indeed the $E_{11}$ cavity is identical to a pair of side by side single cavities excited in antiphase, with the separation wall removed.

We have to conclude therefore, that $E_{11}$ cavities do not form an acceptable solution.
2.6 Optimized cavities common to collinear beams

Even pp twin rings will contain collinear lattice insertions (or insertions with very small beam separation\textsuperscript{8}) in the neighbourhood of the collision points. At first sight only modest extensions of these collinear insertions (21 m on either side of two insertions) are required to accommodate the total of 32 normal LEP cavities, as proposed above for a p\bar{p} ring.

However the very high values of the beta function found immediately adjacent to a high luminosity insertion make these locations unsuitable for cavities. This is obvious in an e^+e^- ring where a combination of large values of beta and the high transverse impedance of an RF structure will lead to instability by transverse turbulence\textsuperscript{9}). In a hadron collider with many bunches the bunch charge is low and the bunches are long. Nevertheless an instability threshold of around 45 \mu A per bunch is found in Appendix II (for a total of 32 LEP cavities at \beta = 1600 m, and 0.4 TeV injection). This corresponds to around 25 mA average bunch current (540 bunches) and therefore constitutes a serious luminosity limitation. Substantial extensions of the collinear lattice will, therefore, be required to reduce the beta function to tolerable values at the locations of the RF cavities.

Such extensions are likely to bring the length of the collinear sections to well over a hundred meters from the collision point and will make unwanted bunch crossings unavoidable. It is not clear whether a limited number of unwanted bunch collisions is really harmful\textsuperscript{10}). If so, small beam separations (obtained with static magnetic fields) may be required inside the "collinear" section.
3. Conclusions

i) In a single p̅p ring one quarter of the LEP phase 1 RF system - 32 five-cell cavities (without storage cavities) and four 1 MW klystrons - would seem to form a very attractive acceleration system. It yields over 80 MV circumferential voltage and 5 minutes acceleration time to 8 TeV.

ii) In a pp twin-ring collider with approximately 20 cm spacing of the counter-rotating beams completely separate RF systems using "septum cavities" as shown in Fig.1 form the safest and most versatile solution, but also the one with the most expensive RF equipment. A total of 128 five-cell cavities (the same number as in LEP phase 1) and eight 1 MW klystrons (half the LEP number) would provide the same voltage as the p̅p system quoted above. A septum cavity is likely to be more expensive than a normal LEP cavity. The two sets of cavities are staggered along the tunnel and, hence, occupy a total active length of 272 m (as in LEP phase 1). A choice of frequency much below 350 MHz or a beam separation much below 20 cm would make this solution impossible.

iii) Side by side cavities of acceptable shunt impedance at acceptable frequency seem to be excluded in any case. The same seems to be true of E₁₁-mode ("push-pull") cavities common to separated beams.

iv) The attractive single ring system given under i) above may be retained for pp rings by placing the RF system into the collinear lattice insertions around the collision points. The cavities themselves will occupy only a modest amount of space there (e.g. 4 x 20 m) but considerable extension of the collinear lattice beyond the collision optics would seem to be required to arrive at reasonably low values of the beta functions. If necessary, spurious bunch-bunch collisions in this part of the lattice might be avoided by a small amount of beam separation. This solution would lead to the lowest RF cost, but the cost of the special lattice insertions has to be added (if indeed such insertion can be designed).
v) As a compromise twin-bore $E_0$ ("push-push") cavities common to separated beams might be worth further study. Nominally this solution requires the same hardware as the single-ring under i) but the twin-bore cavities will be mechanically awkward, correspondingly expensive and yield somewhat smaller shunt impedance. In spite of this, symmetric twin-bore $E_0$ cavities appear more favourable than separate sets of normal cavities with side holes for the opposite beam. In either case the off-center beams are subject to first-order deflection by the RF magnetic field, externally driven, and beam induced. This may turn out intolerable and, hence, exclude this solution.
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Fig. 1 Septum cavity for 353 MHz
Fig 2  a) $E_{11}$ mode
   b) $E_{01}$ mode
   in a pillbox cavity
Appendix 1
Details of Calculation of RF Parameters

SPS Parameters at 400 GeV

The longitudinal parameters in the SPS at 270 GeV are\(^{11}\)
\[ V = 4.0 \text{ MV, } h = 4620, \quad \gamma_t = 23.22, \quad f = 43376 \text{ (revolution frequency)} \]
\[ \Delta E \Delta t = 0.832 \text{ eV} \cdot \text{s} \quad \text{(longitudinal emittance, full width, full length)} \]

At 400 GeV

\[ Q_s = \left( \frac{e h V}{2 \pi E Y_t^2} \right)^{1/2} = 3.69 \times 10^{-3} \quad \text{(A1)} \]

The bucket area is much larger than the bunch area (at 270 GeV) hence the aspect of the RF bucket is given by

\[ \Delta t = \frac{1}{2 \pi f Y_t^2 Q_s} \frac{\Delta E}{E} = 1.84 \times 10^{-6} \frac{\Delta E}{E} \quad \text{(A2)} \]

Solving A1 and A2 gives

\[ \frac{\Delta E}{E} = 1.062 \times 10^{-3} \quad \text{(A3)} \]

\[ \Delta t = 1.9586 \text{ ns} \approx 0.587 \text{ m} \]

LEP at 400 GeV

The wavelength of the 352 MHz system is 85 cm, which is only about 50% greater than the calculated full bunch length. Hence the full equation (without approximation for small amplitudes) must be used for the aspect ratio of the bucket trajectories. This is (for \( \phi_s = 0 \)) given by

\[ \frac{\Delta E}{E} = \left( \frac{2 \sqrt{2} \gamma_t}{h} \right)^2 \left( 1 - \cos \pi f_{RF} \Delta t \right)^{1/2} \quad \text{(A4)} \]
Substituting in (A4) the bunch length and width (A3) and assuming \( \gamma_t = 50.6 \) gives

\[
Q_s = \frac{0.0037}{h}
\]

and

\[
\frac{v}{Q_s} = \frac{Q_s^2 2\pi E_t^2}{2.8MV} = \frac{2Q_s \gamma_t^2}{h}
\]

and

\[
\frac{(\Delta E)}{E_{\text{bucket}}} = \pm \left( \frac{2Q_s \gamma_t^2}{h} \right) = \pm \frac{0.000601}{(20\% \text{ larger than the bunch spread})}
\]

Note: The matching condition could of course be met with a larger voltage in LEP and hence a more comfortable bucket to bunch area ratio if the longitudinal emittance in the SPS was less. In fact this may turn out to be the case since the emittance quoted here is for very high intensity per bunch. For the hadron collider the number of bunches will be higher and hence the intensity per bunch may be less. If this is not the case the bunch length in the SPS could be increased by using a higher RF voltage or by jumping to the unstable fixed point just before ejection.

**Acceleration from 400 to 8000 GeV**

Acceleration from injection energy to design energy can be performed at constant synchrotron frequency or at maximum voltage (which means the synchrotron frequency decreases with the square root of energy). The latter solution would result in a relatively large value of \( Q_s \) just before acceleration and hence a wide range to be traversed in synchrotron frequency at injection energy since the matching condition dictates the above mentioned value for \( Q_s \). For this reason acceleration with a constant \( Q_s \) is slightly preferred. In this case the RF voltage increases linearly with energy and the acceleration time is given by (for constant \( Q_s \))

\[
t_{\text{acc}} = \frac{h}{\frac{2\pi f Q_s}{2\pi f Q_s \gamma_t^2 \tan \phi_s}} \ln \left( \frac{E_d}{E_{\text{inj}}} \right)
\]
From this equation an acceleration time of 5 minutes requires (for tan $\phi_S = 0.1$)

$$Q_S = 0.0042 \quad (f_S = 47.2 \text{ Hz})$$

This value is fortunately close to the value needed for injection matching.

At 8000 GeV this value of $Q_S$ requires a peak RF voltage of

$$\hat{V} = 72.5 \text{ MV}$$

The half width of the bucket is ($\phi_S = 0$)

$$(\Delta E)_{E_{\text{bucket}}} = \pm \left(\frac{2Q_S \gamma^2_i}{h}\right) = \pm 6.9 \times 10^{-4}$$

For constant longitudinal emittance during acceleration the bunch parameters at 8 TeV are

$$\Delta t = 0.37 \text{ ns} \equiv 0.111 \mu\text{m}$$

$$\frac{\Delta E}{E} = 2.81 \times 10^{-4}$$

Hence $$(\frac{\Delta E}{E})/(\Delta E)_{E_{\text{bucket}}} = 4.90 \quad (\text{c.f. 1.47 in SPS at 270 GeV where the lifetime is } \sim 40 \text{ hours})$$

For stable beams the peak voltage could of course be reduced so as to move the synchrotron frequency further from the mains frequency. The avoidance of this frequency should also be a criterion for the choice of the $\gamma_t$ of the lattice.
Appendix II
Transverse Turbulence Threshold For pp LEP

A. II 1. Estimates of the Transverse Impedance \( R_{st} \)

- Vacuum Chamber

The low frequency longitudinal impedance of the LEP \( e^+e^- \) vacuum chamber (i.e. everything except the RF cavities) has been estimated \(^{12}\) to be

\[
|Z_n| = 0.08 \Omega
\]

It appears reasonable to assume that the impedance will scale inversely with the effective chamber radius. This gives a value of 0.18\( \Omega \) for the longitudinal impedance of the hadron collider. The transverse impedance is approximately related to the longitudinal impedance by

\[
R_{st} = \frac{2R}{b^2} |Z_n|
\]

where \( R \) is the average radius (4242.9m) and \( b \) is the effective chamber radius (assumed = 2.5 cm) giving

\[ R_{st} \text{ (vac. chamber) } = 2.4 \text{ M\Omega/m} \]

For the broadband resonator model \(^{13}\) the assumed resonant frequency is proportional to the cut-off frequency of the lowest mode in the structure. For LEP this has been evaluated at 1.3 GHz; for pp LEP the vacuum chamber is smaller and hence a reasonable value is

\[ f_{res} = 2.2 \text{ GHz} \]
RF Cavities

The transverse impedance of 128 LEP cavities has been calculated\(^{12}\) as

\[ R_{st} = 1.7 \, \text{M}\Omega/m \]

Thus for approximately one quarter of this system the impedance is

\[ R_{st} \text{ (32 cavities)} = 0.45 \, \text{M}\Omega/m \]

Again for the broadband resonator model the resonant frequency is approximately

\[ f_{res} = 2.2 \, \text{GHz} \]

A.II.2 Threshold For Transverse Turbulence

The condition for longitudinal matching (Appendix 1) dictates a synchrotron tune of \(3.7 \times 10^{-3}\) and bunch parameters given by

\[ \frac{\Delta E}{E} = 3.7 \left( \frac{\sigma_E}{E} \right) = 1.062 \times 10^{-3} \]

\[ (\Delta t) = 3.7 \, \sigma_t = 1.959 \, \text{ns} \quad \text{i.e.} \quad \sigma_t = 5.3 \times 10^{-10} \, \text{s} \]

Consequently \(\omega_r \sigma_t = 2\pi f_{res} \sigma_t = 7.3\) and (from fig. 1 LEP note 363)

\[ F = 4.5 \]

where the threshold current is given by

\[ I_{th} \text{ (per bunch)} = \frac{7.5 \, F \, Q_s \, E/e}{\beta_v \, h_r \, R_{st}} \]

where \(h_r = f_{res} / f = 1.23 \times 10^6\)
For the case where the accelerating cavities are to be common to both beams, the cavities must be installed close to the interaction point and almost certainly close to the maximum value of $\beta_v$. In contrast the vacuum chamber transverse impedance will be associated with the average value of $\beta_v (= R/Q)$. Thus the threshold current per bunch is

$$I_{th} = \frac{7.5 F Q_s E/e}{h_r [\beta_v R_{st}(cav) + \beta_v R_{st}(vac.ch)]}$$

Thus for the cavities $\beta_v R_{st} = 1600 \times 0.45 \times 10^6 = 720 \, \text{M} \Omega$

and for the vac. chamber $\beta_v R_{st} = 75.0 \times 1.08 \times 10^6 = 180 \, \text{M} \Omega$

(the $\beta_v$ of 1600 is given in ref. 8)

and $I_{th}$ (per bunch) = $45.1 \mu\text{A}$