DAMPING APERTURE OF LEP

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Abstract

An improved calculation of the variation of damping partition numbers with momentum deviation in a separated function storage ring is presented and applied to LEP. The effects of non-linear fields and closed orbit distortions are included, exactly to second order in momentum deviation.

The damping aperture of LEP is smaller than that of existing machines and ways of improving it are discussed. The emittance control wigglers help to reduce the sensitivity of the damping to RF frequency and momentum variations.
1. Introduction

Variation of the damping partition numbers will be an important means of controlling beam characteristics in LEP.\(^1\) In practice this is done by making a small change \( \Delta f \) in the RF frequency \( f \). This changes the equilibrium beam energy from \( E_0 \) to \( E_0 (1 + \delta) \)

\[
\frac{\Delta f}{f} = \alpha \delta
\]

(1)

where \( \alpha \) denotes the momentum compaction.

The particles then no longer oscillate about the design orbit but about some off-momentum orbit along which the bending fields are different because of the quadrupole (and higher multipole) gradients present in the lattice.

Furthermore, the energy loss \( U(\delta) \) along this orbit and its dependence on deviations from the new equilibrium energy will be different. The longitudinal damping partition number for such an orbit is

\[
J_s(\delta) = \frac{d}{d\delta} \log U(\delta)
\]

(2)

2. Calculation of \( J_s \) for off-momentum orbits

For a horizontal displacement \( x \) from the design orbit at azimuth \( s \), the vertical magnetic field, including dipoles, quadrupoles and sextupoles, is

\[
B(s,x) = \frac{E_0}{ec} \left( 1 + K(s)x + 1/2 K'(s) x^2 \right) + O(x^3)
\]

(3)

In a separated function lattice

\[
K(s)/\varphi(s) = K'(s)/\varphi(s) = K(s)K'(s) = 0
\]

(4)

for each \( s \).

The displacement of an off-momentum particle is

\[
x(\delta) = x_c + \eta_o \delta + \eta_1 \delta^2 + \eta_2 \delta^3 + O(\delta^4)
\]

(5)

where \( x_c \) is the local closed orbit distortion, \( \eta_o \) is the usual dispersion and \( \eta_1, \eta_2 \) are higher order corrections to it. We suppress the argument \( s \) here and from now on.

The energy lost by synchrotron radiation in one turn is

\[
U(\delta) = C_1 \int ds \ B( x(\delta) )^2 ( 1 + \delta )^2
\]

(6)

where the integral is along the design orbit. \( C_1 \) is independent of \( \delta \).
Combining (3), (4), (5) and (6) one finds

\[
\log U(\mathcal{F}) = C_2 + \log \left| A_0 + A_1 \mathcal{F} + A_2 \mathcal{F}^2 + A_3 \mathcal{F}^3 + O(\mathcal{F}^4) \right|
\]  

(7)

where \( C_2 \) is independent of \( \mathcal{F} \) and the \( A_i \) are the following 

*synchrotron radiation integrals* \(^*\)

\[
A_0 = \int \! ds \left\{ \frac{1}{\mathcal{F}^2} + \frac{\eta_0}{\rho^3} + \frac{\eta_1}{\rho^3} + \frac{\eta_2}{\rho^3} + \frac{2\eta_0}{\rho^3} + \frac{2\eta_1}{\rho^3} + \frac{2\eta_2}{\rho^3} \right\}
\]

(8)

\[
A_1 = \int \! ds \left\{ \frac{2}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + 2K^2\mathcal{F} + \frac{1}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + 2K^2\mathcal{F} + \frac{1}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + 2K^2\mathcal{F} + \frac{1}{\rho^3} \right\}
\]

(9)

\[
A_2 = \int \! ds \left\{ \frac{2}{\rho^3} + K^2\eta_0 + \frac{4K^2\eta_0}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} + \frac{4K^2\eta_0}{\rho^3} \right\}
\]

(10)

\[
A_3 = \int \! ds \left\{ \frac{2}{\rho^3} + 2K^2\eta_0 + \frac{4K^2\eta_0}{\rho^3} + 2K^2\eta_0 \right\}
\]

(11)

From (2) it follows that \(^**\)

\[
J_s(\mathcal{F}) = \frac{A_1}{A_0} + \left[ \frac{A_2}{A_0} - \left( \frac{A_1}{A_0} \right)^2 \right] \mathcal{F} + \left[ \frac{A_3}{A_0} - \frac{2A_1A_2}{A_0^2} \right] \mathcal{F}^2 + O(\mathcal{F}^3).
\]

(12)

Making some numerical estimates for LEP : \(4,5\)

\[
\eta_0 \ll 2m, \quad \rho_0 \approx 3000 \text{ m},
\]

\[
K \ll 0.02 \text{ m}^{-2}, \quad K' \ll 0.2 \text{ m}^{-3},
\]

\[
\eta_1 \ll 100 \text{ m}, \quad \eta_2 \ll 100 \text{ m}
\]

quickly shows that we can neglect several terms in the integrands of the \( A_i \); in particular those with factors \( \rho^{-3} \).

\(^*\) The REDUCE program \(7\) was useful here and later.

\(^**\) To clarify the connection with the formula \( J_s = 2 + D \) appearing, e.g. in reference 2, note that \( D = I_4/I_2 \) is equal to the second term in \( A_1 \) divided by the first term of \( A_0 \) in a separated function lattice. This is very small for large machines but, if \( I_4 \) is evaluated by integration along an off-momentum orbit other terms contribute and its value can be much more significant. Indeed this is a common way of calculating \( dJ_s/d\mathcal{F} \). In other words, a machine is only separated function on a particular orbit.
If we assume that the closed orbit distortion has no systematic component and is uncorrelated with the nature of the magnets we may neglect terms containing odd powers of $x_c$ and factor out average values of the even powers from the integrals. For formal convenience we even assume gaussian statistics for the process $x_c$

$$<x_c^4> = <x_c^2>^2$$

Then we have, with $I_2$ the usual synchrotron radiation integral,

$$A_0 = I_2 + <x_c^2> \int ds \kappa^2 + 1/4 <x_c^2>^2 \int ds \kappa'^2$$

$$A_1 = 2A_0 + \int ds \frac{\eta_o}{\varphi^3}$$

$$A_2 = A_0 + \int ds \kappa^2 \eta_o^2 + 3/2 <x_c^2> \int ds \kappa'^2 \eta_o^2$$

$$A_3 = \int ds \kappa^2 \eta_o (\eta_0 + \eta_1) + 3 <x_c^2> \int ds \kappa'^2 \eta_o (\eta_0 + \eta_1)$$

In $A_0$ and $A_1$ we have not neglected the terms with factors $1/\varphi^3$.

From (12) we find a very good approximation for the damping partition number in LEPI.

$$J_s(\vec{B}) \approx J_s(0) + J_s'(0) \vec{B} + 1/2 J_s''(0) \vec{B}^2$$

where

$$J_s(0) = 2 + \int ds \frac{\eta_o}{\varphi^3}$$

$$J_s'(0) = -3 + \frac{1}{A_0} \int ds \kappa^2 \eta_o^2 + 3/2 <x_c^2> \frac{1}{A_0} \int ds \kappa'^2 \eta_o^2$$

$$J_s''(0) = -8 + \frac{1}{A_0} \int ds \kappa^2 \eta_o (\eta_1 - \eta_0) + 6 <x_c^2> \frac{1}{A_0} \int ds \kappa'^2 \eta_o (\eta_1 - \eta_0)$$

3. Sensitivity to closed orbit distortion

If we now assume that the closed orbit has been corrected $5)$

$$<x_c^2> \approx (1 \text{mm})^2$$

we may expand these formulae to second order in $<x_c^2>$. With the abbreviations

$$Q_0 = \frac{1}{I_2} \int ds \kappa^2, \quad S_0 = \frac{1}{I_2} \int ds \kappa'^2,$$

$$Q_1 = \frac{1}{I_2} \int ds \kappa^2 \eta_o^2, \quad S_1 = \frac{1}{I_2} \int ds \kappa'^2 \eta_o^2$$

$$Q_2 = \frac{1}{I_2} \int ds \kappa^2 \eta_o \eta_1, \quad S_2 = \frac{1}{I_2} \int ds \kappa'^2 \eta_o \eta_1$$

...
we find

\[ J_s'(0) = Q_1 - 3 + \langle x_c^2 \rangle \left( \frac{3}{2} S_1 - Q_0 Q_1 \right) + \langle x_c^2 \rangle^2 \left( Q_0^2 Q_1 - \frac{1}{4} S_0 Q_1 - \frac{3}{2} Q_0 S_1 \right) + O(\langle x_c^2 \rangle^3) \]  

(24)

(here the first term corresponds to the standard formula\(^1,8\) as computed by the program BEAMPARAM)

\[ J_s''(0) = -8 + 4 (Q_2 - Q_1) + 2 \langle x_c^2 \rangle \left[ 3(S_1 - S_0)(Q_0^2 + 5)(Q_0^2 Q_1) \right] - 2 \langle x_c^2 \rangle^2 \left[ 3Q_0(S_1 - S_0) + 2(Q_2 - Q_1)(Q_0^2 - \frac{1}{4} S_0) \right] + O(\langle x_c^2 \rangle^3). \]  

(25)

This shows that the first effect of closed orbit distortion is actually to reduce the sensitivity of the damping partition number to momentum. Physically this is easy to understand: off-centre closed orbits lead to greater energy loss in quadrupoles and sextupoles (characterized by \(Q_0\) and \(S_0\)) and this would increase the value of \(I_2\) if it were evaluated on the distorted closed orbit (the \(I_2\) appearing in our formulae is always evaluated on the design orbit).

We can make very crude estimates of the \(Q_i\) and \(S_i\) for the 60° lattice e.g.

\[ Q_0 \approx \frac{L_Q R^2 \rho^2}{L_B} \approx 110 \text{ m}^{-2} \]

where for each cell in the arcs we take

\[ K = 0.016 \text{ m}^{-2}, \quad \rho = 3103 \text{ m}, \]
\[ L_Q = 1.6 \text{ m}, \quad L_B = 35 \text{ m}, \]
\[ L_s = 0.58 \text{ m}, \quad K' = 0.1 \text{ m}^{-3} \]
\[ \eta_0 = 3.3 \text{ m}^2 \text{ in sextupoles}, \]
\[ \eta_1 = 25 \eta_0 \text{ in all magnets}. \]  

(26)

The results are

\[ J_s'(0) = 720 - (7 \times 10^4) \langle x_c^2 \rangle + (8 \times 10^6) \langle x_c^2 \rangle^2 \]  

(27)

\[ J_s''(0) = 10^4 - 10^6 \langle x_c^2 \rangle - 10^8 \langle x_c^2 \rangle^2 \]  

(28)

with \(x_c\) measured in metres.

Above a certain threshold in closed orbit error the sensitivity of \(J_s\) to momentum errors will increase. It goes without saying that the closed orbit correction should be as good as possible.
With the expected quality of the LEP correction scheme (22), the perturbation of $J_s'(0)$ will be negligible. Furthermore, the quadratic variation of $J_s$ can be neglected over the usable range of momentum deviation.

4. Range of variation of momentum and RF frequency

To retain damping horizontally and longitudinally one must have

$$0 < J_s < 3$$

so the allowable range of variation of the fractional momentum deviation (neglecting the quadratic variation) is

$$-0.28\% \approx -\frac{2}{J_s'(0)} < \frac{1}{J_s'(0)} \approx 0.14\%$$

for the 60° lattice without wigglers. This corresponds to a variation in RF frequency of a few hundred Hz. It will therefore be important to be able to control the RF frequency within a few tens of Hz.

It may be observed that the natural momentum spread in the beam already exceeds these limits. This is not in itself a cause for concern. Only the central momentum matters because the synchrotron frequency is much larger than the damping rate (at least until $J_s'(0)$ becomes very large).

A further important benefit of the use of wigglers to control emittance is an increase of the damping aperture. With a coupled horizontal emittance $\varepsilon_{\text{exc}} = 98.4$ nm, which fills the dynamic aperture of the 60° lattice with nominal $\beta$-function values at the interaction points, $J_s'$ can be brought down to 500 at injection energy. If a detuned lattice were used for injection optics the increase in damping aperture could be even greater. In Fig. 1 we give curves of $J_s'$ versus energy for two values of the emittance.

5. Measurements of CESR's damping aperture.

With the help of R. Siemann, one of us was able to measure the damping aperture of CESR with well-defined lattice conditions.

The experiment is rather simple to do: vary the RF frequency until the beam lifetime starts to decrease rapidly. The CESR RF frequency is actually determined as 0.7 times the frequency of a master oscillator which controls the frequency of the Cornell synchrotron. After taking this into account we found an allowable variation of

$$\Delta f = \begin{cases} 
18.03 \text{ kHz for the positron beam} \\
21.12 \text{ kHz for the electron beam}
\end{cases}$$
With the hypothesis that the electron beam is more stable due to accretion of positive ions, the figure for the positrons is more likely to correspond to the theoretical damping aperture. This gives a value

\[ J_{s}'(0) = 3\alpha \left( \frac{f}{\Delta f} \right) = 980 \]

The conventional calculation, incorporated in the program BEAMPARAM gives 602.6.

Estimates of the corrections to this calculation from (12) are again very small.

The discrepancy between measured and calculated values of \( J_{s}' \) suggests that the beams were probably lost because of some other effect although no dependence on beam intensity was observed in this very brief experiment. However, the beams were colliding and we feel that the conclusion that the damping aperture was less than predicted needs more support.

Another measurement is reported in Reference 8.

6. Comparison with other machines

In PETRA \( J_{s}' \) is about 200 and in PEP it is about 400. CESR has the smallest damping aperture of these three machines but the damping aperture of LEP will be somewhat smaller still, with \( J_{s}' \approx 720 \). With the 90° LEP lattice \( J_{s}' \approx 390 \).

7. Conclusions

With 60° phase advance per cell the damping aperture of LEP will be rather small but not unduly sensitive to the residual closed orbit error. The RF frequency will have to be controlled within a few tens of Hz.

The damping aperture can be increased if necessary by using dipole-sextupole wigglers\(^6\)) or by increasing the length of the quadrupoles while keeping their integrated focussing strength constant. In the latter case the damping aperture will increase in proportion to the quadrupole length.

If a dipole-sextupole wiggler were used, substitution of reasonable field and gradient values in formula (8) of Reference 6 shows that, at 50 GeV, a few tens of metres of wiggler would be needed to reduce \( J_{s}' \) by 100 units. The necessary length increases in proportion to energy in first approximation. The accompanying synchrotron radiation loss would be very large.
Acknowledgement

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References

4. Extended parameter list for LEP Version 12 Phase 1, compiled by M. Placidi, LEP Note 394 (1982)
5. G. Guignard, Closed orbit correction in LEP Version 11 with a phase advance of 60° per cell, LEP Note 403 (1982)
LEP 60 degree lattices

\[ \frac{\partial J}{\partial \delta} \]

\( \phi = 98.4 \)
\( \phi = 240.0 \)

Fig. 1 Derivative of longitudinal damping partition number with \( \phi \) held constant.