Type Ia Supernova Explosion Models

Wolfgang Hillebrandt and Jens C. Niemeyer
Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85740 Garching, Germany

KEYWORDS: stellar evolution, supernovae, hydrodynamics

ABSTRACT

Because calibrated light curves of Type Ia supernovae have become a major tool to determine the local expansion rate of the Universe and also its geometrical structure, considerable attention has been given to models of these events over the past couple of years. There are good reasons to believe that perhaps most Type Ia supernovae are the explosions of white dwarfs that have approached the Chandrasekhar mass, \( M_{\text{chan}} \approx 1.39\, M_{\odot} \), and are disrupted by thermonuclear fusion of carbon and oxygen. However, the mechanism whereby such accreting carbon-oxygen white dwarfs explode continues to be uncertain. Recent progress in modeling Type Ia supernovae as well as several of the still open questions are addressed in this review. Although the main emphasis will be on studies of the explosion mechanism itself and on the related physical processes, including the physics of turbulent nuclear combustion in degenerate stars, we also discuss observational constraints.

1 INTRODUCTION

Changes in the appearance of the night sky, visible with the naked eye, have always called for explanations (and speculations). But, although “new stars”, i.e. novae and supernovae, are observed by humans for thousands of years, the modern era of supernova research began only about one century ago on August 31, 1885, when Hartwig discovered a “nova” near the center of the Andromeda galaxy, which became invisible about 18 months later. In 1919 Lundmark estimated the distance of M31 to be about \( 7 \times 10^5 \) ly, and by that time it became obvious that Hartwig’s nova had been several 1000 times brighter than a normal nova (Lundmark 1920). It was also Lundmark (1921) who first suggested an association between the supernova observed by Chinese astronomers in 1054 and the Crab nebula.

A similar event as S Andromeda was observed in 1895 in NGC 5253 (“nova” Z Centauri), and this time the “new star” appeared to be 5 times brighter than the entire galaxy, but it was not before 1934 that a clear distinction between classical novae and supernovae was made (Baade & Zwicky 1934). Systematic searches, performed predominantly by Zwicky, lead to the discovery of 54 supernovae in the years up to 1956 and, due to improved observational techniques, 82 further supernovae were discovered in the years from 1958 to 1963, all of course in external galaxies (e.g., Zwicky 1965).

Until 1937 spectrograms of supernovae were very rare, and what was known
seemed to be not too different from common novae. This changed with the very bright $(m_V \simeq 8.4)$ supernova SN1937c in IC 4182 which had spectral features very different from any object that had been observed before (Popper 1937). All of the other supernovae discovered in the following years showed very little dispersion in their maximum luminosity and their post-maximum spectra looked very similar at a given time. Based on this finding Wilson (1939) and Zwicky (1938a) suggested to use supernovae as distance indicators.

In 1940 it became clear, however, that there exist at least two distinctly different classes of supernovae. SN1940c in NGC 4725 had a spectrum very different from all other previously observed supernovae for which good data were available at that time, leading Minkowski (1940) to introduce the names “Type I” for those with spectra like SN1937c and “Type II” for SN 1940c-like events, representing supernovae without and with Balmer-lines of hydrogen near maximum light.

Whether or not the spectral differences also reflect a different explosion mechanism was not known. In contrast, the scenario originally suggested by Zwicky (1938b), that a supernova occurs as the transition from an ordinary star to a neutron star and gains its energy from the gravitational binding of the newly born compact object, was for many years the only explanation. Hoyle & Fowler (1960) were the first to discover that thermonuclear burning in an electron-degenerate stellar core might trigger an explosion and (possibly) the disruption of the star. Together with the idea that the light curves could be powered by the decay-energy of freshly produced radioactive $^{56}$Ni (Truran, Arnett, & Cameron 1967; Colgate & McKee 1969) this scenario is now the generally accepted one for a sub-class of all Type I supernovae called Type Ia today. It is a bit amusing to note that all supernovae (besides the Crab nebula) on which Zwicky had based his core-collapse hypothesis were in fact of Type Ia and most likely belonged to the other group, whereas the first core-collapse supernova, SN1940c, was observed only about one year after he published his paper.

To be more precise, supernovae which do not show hydrogen lines in their spectra but a strong silicon P Cygni – feature near maximum light are named Type Ia (Wheeler & Harkness 1990). They are believed to be the result of thermonuclear disruptions of white dwarfs, either consisting of carbon and oxygen with a mass close to the Chandrasekhar-mass, or of a low-mass C+O core mantled by a layer of helium, the so-called sub-Chandrasekhar-mass models (see the recent reviews by Woosley (1997b); Woosley & Weaver (1994a, 1994b) and Nomoto et al. (1994b, 1997)). The main arguments in favor of this interpretation include: (i) the apparent lack of neutron stars in some of the historical galactic supernovae (e.g. SN1006, SN1572, SN1604); (ii) the rather homogeneous appearance of this sub-class; (iii) the excellent fits to the light curves, which can be obtained from the simple assumption that a few tenths of a solar mass of $^{56}$Ni is produced during the explosion; (iv) the good agreement with the observed spectra of typical Type Ia supernovae. Several of these observational aspects are discussed in some detail in Section 2, together with their cosmological implications. Questions concerning the nature of the progenitor stars are addressed in Section 3, and models of light curves and spectra are reviewed in Section 5.

But having good arguments in favor of a particular explosion scenario does not mean that this scenario is indeed the right one. Besides that one would like to understand the physics of the explosion, the fact that the increasing amount of data also indicates that there is a certain diversity among the Type Ia supernovae seems to contradict a single class of progenitor stars or a single explosion
mechanism. Moreover, the desire for using them as distance indicator makes it necessary to search for possible systematic deviations from uniformity. Here, again, theory can make important contributions. In Section 4, therefore, we discuss the physics of thermonuclear combustion, its implementation into numerical models of exploding white dwarfs, and the results of recent computer simulations. A summary and conclusions follow in Section 6.

2 OBSERVATIONS

The efforts to systematically obtain observational data of SNe Ia near and far have gained tremendous momentum in recent years. This is primarily a result of the unequalled potential of SNe Ia to act as “standardizable” candles (Branch & Tammann 1992; Riess, Press, & Kirshner 1996; Hamuy et al. 1995; Tripp 1998) for the measurement of the cosmological expansion rate (Hamuy et al. 1996b; Branch 1998) and its variation with lookback time (Perlmutter et al. 1999; Schmidt et al. 1998; Riess et al. 1998). For theorists, this development presents both a challenge to help understand the correlations among the observables and an opportunity to use the wealth of new data to constrain the zoo of existing explosion models. There exist a number of excellent reviews about SNe Ia observations in general (Filippenko 1997b), their spectral properties (Filippenko 1997a), photometry in the IR and optical bands (Meikle et al. 1996; Meikle et al. 1997), and their use for measuring the Hubble constant (Branch 1998). Recent books that cover a variety of observational and theoretical aspects of type Ia supernovae are Ruiz-Lapuente, Canal, & Isern (1997) and Niemeyer & Truran (2000). Below, we highlight those aspects of SN Ia observations that most directly influence theoretical model building at the current time.

2.1 General Properties

The classification of SNe Ia is based on spectroscopic features: the absence of hydrogen absorption lines, distinguishing them from Type II supernovae, and the presence of strong silicon lines in the early and maximum spectrum, classifying them as Types Ia (Wheeler & Harkness 1990).

The spectral properties, absolute magnitudes, and light curve shapes of the majority of SN Ia are remarkably homogeneous, exhibiting only subtle spectroscopic and photometric differences (Branch & Tammann 1992; Hamuy et al. 1996c; Branch 1998). It was believed until recently that approximately 85% of all observed events belong to this class of normal (“Branch-normal”, Branch, Fisher, & Nugent 1993) SNe Ia, represented for example by SNe 1972E, 1981B, 1989B, and 1994D. However, the peculiarity rate can be as high as 30 % as suggested by Li et al. (2000).

The optical spectra of normal SN Ia’s contain neutral and singly ionized lines of Si, Ca, Mg, S, and O at maximum light, indicating that the outer layers of the ejecta are mainly composed of intermediate mass elements (Filippenko 1997b). Permitted Fe II lines dominate the spectra roughly two weeks after maximum when the photosphere begins to penetrate Fe-rich ejecta (Harkness 1991). In the nebular phase of the light curve tail, beginning approximately one month after peak brightness, forbidden Fe II, Fe III, and Co III emission lines become the dominant spectral features (Axelrod 1980). Some Ca II remains observable in absorption even at late times (Filippenko 1997a). The decrease of Co lines
(Axelrod 1980) and the relative intensity of Co III and Fe III (Kuchner et al. 1994) give evidence that the light curve tail is powered by radioactive decay of $^{56}\text{Co}$ (Truran et al. 1967; Colgate & McKee 1969).

The early spectra can be explained by resonant scattering of a thermal continuum with P Cygni-profiles whose absorption component is blue-shifted according to ejecta velocities of up to a few times $10^4$ km/s, rapidly decreasing with time in the early phase (Filippenko 1997a). Different lines have different expansion velocities (Patat et al. 1996), suggesting a layered structure of the explosion products.

Photometrically, SN Ia rise to maximum light in the period of approximately 20 days (Riess et al. 1999b) reaching

$$M_B \approx M_V \approx -19.30 \pm 0.03 + 5 \log(H_0/60)$$  \hspace{1cm} (1)

with a dispersion of $\sigma_M \leq 0.3$ (Hamuy et al. 1996b). It is followed by a first rapid decline of about three magnitudes in a matter of one month. Later, the light curve tail falls off in an exponential manner at a rate of approximately one magnitude per month. In the I-band, normal SNe Ia rise to a second maximum approximately two days after the first maximum (Meikle et al. 1997).

It is especially interesting that the two most abundant elements in the universe, hydrogen and helium, so far have not been unambiguously detected in SN Ia spectra (Filippenko 1997a), but see Meikle et al. (1996) for a possible identification of He) and there are no indications for radio emission of SNe Ia. Cumming et al. (1996) failed to find any signatures of H in the early-time spectrum of SN 1994D and used this fact to constrain the mass accretion rate of the progenitor wind (Lundqvist & Cumming 1997). The later spectrum of SN 1994D also did not exhibit narrow H$\alpha$ features (Filippenko 1997b). Another direct constraint for the progenitor system accretion rate comes from the non-detection of radio emission from SN 1986G (Eck et al. 1995), used by Boffi & Branch (1995) to rule out symbiotic systems as a possible progenitor of this event.

### 2.2 Diversity and Correlations

Early suggestions (Pskovskii 1977; Branch 1981) that the existing inhomogeneities among SN Ia observables are strongly intercorrelated are now established beyond doubt (Hamuy et al. 1996a; Filippenko 1997a). Branch (1998) offers a recent summary of correlations between spectroscopic line strengths, ejecta velocities, colors, peak absolute magnitudes, and light curve shapes. Roughly speaking, SNe Ia appear to be arrangeable in a one-parameter sequence according to explosion strength, wherein the weaker explosions are less luminous, redder, and have a faster declining light curve and slower ejecta velocities than the more energetic events (Branch 1998). The relation between the width of the light curve around maximum and the peak brightness is the most prominent of all correlations (Pskovskii 1977; Phillips 1993). Parameterized either by the decline rate $\Delta m_{15}$ (Phillips 1993; Hamuy et al. 1996a), a “stretch parameter” (Perlmutter et al. 1997), or a multi-parameter nonlinear fit in multiple colors (Riess et al. 1996), it was used to renormalize the peak magnitudes of a variety of observed events, substantially reducing the dispersion of absolute brightnesses (Riess et al. 1996; Tripp 1998). This correction procedure is a central ingredient of all current cosmological surveys that use SNe Ia as distance indicators (Perlmutter et al. 1999; Schmidt et al. 1998).
SN 1991bg and SN 1992K are well-studied examples for red, fast, and subluminous supernovae (Filippenko et al. 1992a; Leibundgut et al. 1993; Hamuy et al. 1994; Turatto et al. 1996). Their V, I, and R-band light curve declined unusually quickly, skipping the second maximum in I, and their spectrum showed a high abundance of intermediate mass elements (including Ti II) with low expansion velocities but only little iron (Filippenko et al. 1992a). Models for the nebular spectra and light curve of SN 1991bg consistently imply that the total mass of $^{56}$Ni in the ejecta was very low ($\sim 0.07 \, M_\odot$) (Mazzali et al. 1997a). On the other side of the luminosity function, SN 1991T is typically mentioned as the most striking representative of bright, energetic events with broad light curves (Phillips et al. 1992; Jeffery et al. 1992; Filippenko et al. 1992b; Ruiz-Lapuente et al. 1992; Spyromilio et al. 1992). Rather than the expected Si II and Ca II, its early spectrum displayed high-excitation lines of Fe III but returned to normal a few months after maximum (Filippenko et al. 1992b).

Peculiar events like SN 1991T and SN 1991bg were suggested to belong to different subgroups of SNe Ia than the normal majority, created by different explosion mechanisms (Mazzali et al. 1997a; Filippenko et al. 1992b; Fisher et al. 1999). The overall SN Ia luminosity function seems to be very steep on the bright end (Vaughan et al. 1995), indicating that “normal” events are essentially the brightest while the full class may contain a large number of undetected subluminous SNe Ia (Livio 2000). New results (Li et al. 2000) indicate, however, that the luminosity function may be shallower than anticipated.

There is also mounting evidence that SN Ia observables are correlated with the host stellar population (Branch 1998). SNe Ia in red or early-type galaxies show, on average, slower ejecta velocities, faster light curves, and are dimmer by $\approx 0.2$ to $0.3$ mag than those in blue or late-type galaxies (Hamuy et al. 1995, 1996a; Branch, Romanishin, & Baron 1996). The SN Ia rate per unit luminosity is nearly a factor of two higher in late-type galaxies than in early-type ones (Cappellaro et al. 1997). In addition, the outer regions of spirals appear to give rise to similarly dim SNe Ia as ellipticals whereas the inner regions harbor a wider variety of explosion strengths (Wang, Höflich, & Wheeler 1997). When corrected for the difference in light curve shape, the variation of absolute magnitudes with galaxy type vanishes along with the dispersion of the former. This fact is crucial for cosmological SN Ia surveys, making the variations with stellar population consistent with the assumption of a single explosion strength parameter (Perlmutter et al. 1999; Riess et al. 1998).

2.3 Nearby and Distant SNe Ia

Following a long and successful tradition of using relatively nearby ($z \leq 0.1$, comprised mostly of the sample discovered by the Calán/Tololo survey (Hamuy et al. 1996a)) SNe Ia for determining the Hubble constant (Branch 1998), the field of SN Ia cosmology has recently seen a lot of activity expanding the range of observed events out to larger redshift, $z \approx 1$. Systematic searches involving a series of wide-field images taken at epochs separated by 3-4 weeks, in addition to pre-scheduled follow-up observations to obtain detailed spectroscopy and photometry of selected events, have allowed two independent groups of observers – the Supernova Cosmology Project (SCP) (Perlmutter et al. 1999) and the High-$z$ Supernova Search Team (Schmidt et al. 1998) – to collect data of more than 50 high-redshift SNe. Extending the Hubble diagram out to $z \approx 1$ one can, given
a sufficient number of data points over a wide range of $z$, determine the density parameters for matter and cosmological constant, $\Omega_M$ and $\Omega_\Lambda$, independently (Goobar & Perlmutter 1995) or, in other words, constrain the equation of state of the universe (Garnavich et al. 1998). Both groups come to a spectacular conclusion (Riess et al. 1998; Perlmutter et al. 1999): The distant SNe are too dim by $\approx 0.25$ mag to be consistent with a purely matter dominated, flat or open FRW universe. Interpreted as being a consequence of a larger than expected distance, this discrepancy can be resolved only if $\Omega_\Lambda$ is non-zero, implying the existence of an energy component with negative pressure. In fact, the SN Ia data is consistent with a spatially flat universe made up of two parts vacuum energy and one part matter.

Both groups discuss in detail the precautions that were taken to avoid systematic contaminations of the detection of cosmological acceleration, including SN Ia evolution, extinction, and demagnification by gravitational lensing. All of these effects would, in all but the most contrived scenarios, give rise to an increasing deviation from the $\Omega_\Lambda = 0$-case for higher redshift, while the effect of a non-zero cosmological constant should become less significant as $z$ grows. Thus, the degeneracy between a systematic overestimation of the intrinsic SN Ia luminosity and cosmological acceleration can be broken when sufficiently many events at $z \geq 0.85$ are observed (Filippenko & Riess 2000). Meanwhile, the only way to support the cosmological interpretation is by “...adding to the list of ways in which they are similar while failing to discern any way in which they are different” (Riess et al. 1999a). This program has been successful until recently: The list of similarities between nearby and distant SNe Ia includes spectra near maximum brightness (Riess et al. 1998) and the distributions of brightness differences, light curve correction factors, and $B-V$ color excesses of both samples (Perlmutter et al. 1999). Moreover, while the nearby sample covers a range of stellar populations similar to the one expected out to $z \approx 1$, a separation of the low-$z$ data into sub-samples arising from different progenitor populations shows no systematic shift of the distance estimates (Filippenko & Riess 2000). However, a recent comparison of the rise times of more than 20 nearby SNe (Riess et al. 1999b) with those determined for the SCP high-redshift events gives preliminary evidence for a difference of roughly 2.5 days. This result was disputed by Aldering, Knop, & Nugent (2000) who conclude that the rise times of local and distant supernovae are statistically consistent.

2.4 Summary: Observational Requirements for Explosion Models

To summarize the main observational constraints, any viable scenario for the SN Ia explosion mechanism has to satisfy the following (necessary but probably not sufficient) requirements:

1. Agreement of the ejecta composition and velocity with observed spectra and light curves. In general, the explosion must be sufficiently powerful (i.e., produce enough $^{56}$Ni) and produce a substantial amount of high-velocity intermediate mass elements in the outer layers. Furthermore, the isotopic abundances of “normal” SNe Ia must not deviate significantly from those found in the solar system.

2. Robustness of the explosion mechanism. In order to account for the homogeneity of normal SNe Ia, the standard model should not give rise to widely
different outcomes depending on the fine-tuning of model parameters or initial conditions.

3. Intrinsic variability. While the basic model should be robust with respect to small fluctuations, it must contain at least one parameter that can plausibly account for the observed sequence of explosion strengths.

4. Correlation with progenitor system. The explosion strength parameter must be causally connected with the state of the progenitor white dwarf in order to explain the observed variations as a function of the host stellar population.

3 LIGHT CURVE AND SPECTRA MODELING

Next we have to discuss the problem of coupling the interior physics of an exploding white dwarf to what is finally observed, namely light curves and spectra, by means of radiative transfer calculations. For many astrophysical applications this problem is not solved, and SN Ia are no exceptions. In fact, radiation transport is even more complex in Type Ia’s than for most other cases.

A rough sketch of the processes involved can illustrate some of the difficulties (see, e.g., Mazzali & Lucy (1993); Eastman & Pinto (1993)). Unlike most other objects we know in astrophysics SN Ia do not contain any hydrogen. Therefore the opacities are always dominated either by electron scattering (in the optical) or by a huge number of atomic lines (in the UV). In the beginning, the supernova is an opaque expanding sphere of matter into which energy is injected from radioactive decay. This could happen in a very inhomogeneous manner, as will discussed later. As the matter expands diffusion times eventually get shorter than the expansion time and the supernova becomes visual. However, because the star is rapidly expanding the Doppler-shift of atomic lines causes important effects. For example, a photon emitted somewhere in the supernova may find the surrounding matter more or less transparent until it finds a line Doppler-shifted such that it is trapped in that line and scatters many times. As a consequence, the spectrum might look thermal although the photon “temperature” has nothing in common with the matter temperature.

It is also obvious that radiation transport in SN Ia is very non-local and that the methods used commonly in models of stellar atmospheres need refinements. As a consequence, there is no agreement yet among the groups modeling light-curves and spectra as to what the best approach is. Therefore it can happen that even if the same model for the interior physics of the supernova is inserted into one of the existing codes for modeling light-curves and spectra the predictions for what should be “observed” could be different, again a very unpleasant situation. Things get even worse because all such models treat the exploding star as being spherically symmetric, an assumption that is at least questionable, given the complex combustion physics discussed below.

In the following subsections we outline some of the commonly used numerical techniques and also discuss their predictions for SN Ia spectra and light curves. For more details on the techniques used by the various groups, we refer readers to the articles by Eastman (1997); Blinnikov (1997); Pinto (1997); Baron, Hauschildt, & Mezzacappa (1997); Mazzali et al. (1997b); Höflich et al. (1997), and Ruiz-Lapuente (1997).
3.1 Radiative transfer in Type Ia supernovae

In principle, the equations which have to be solved are well-known, either in form of the Boltzmann transport equation for photons or as a transport equation for the monochromatic intensities. However, to solve this time dependent, frequency dependent radiation transport problem, including the need of treating the atoms in non-LTE, is very expensive, even in spherically symmetric situations. Therefore, approximations of various kinds are usually made which give rise to controversial discussions.

Conceptually, it is best to formulate and solve the transport equation in the co-moving (Lagrangian) frame (cf. Mihalas & Weibel (1984)). This makes the transport equation appear simpler, but causes problems in calculating the “co-moving” opacity, in particular if the effect of spectral lines on the opacity of an expanding shell of matter is important, as in the case of SN Ia (Karp et al. 1977).

There are different ways to construct approximate solutions of the transport equation. One can integrate over frequency and replace the opacity terms by appropriate means, leaving a single (averaged) transport equation. Unfortunately, in order to compute the flux-mean opacity one has to know the solution of the transport equation. Frequently the flux-mean is replaced, for example, by the Rosseland mean, allowing for solutions, but at the expense of consistency (see, e.g., Eastman (1997)).

Another way out is to replace the transport equation by its moment expansion introducing, however, the problem of closure. In its simplest form, the diffusion approximation, the radiation field is assumed to be isotropic, the time rate of change of the flux is ignored, and the flux is expressed in terms of the gradient of the mean intensity of the radiation field. Replacing the mean intensity by the Planck function and closing the moment expansion by relating the radiation energy density and pressure via an Eddington factor (equal to 1/3 for isotropic radiation) finally leads to a set of equations that can be solved (Mihalas & Weibel 1984).

Again, this simple approach has several short-comings that are obvious. First of all, the transition from an optically thick to thin medium at the photosphere requires a special treatment mainly because the radiation field is no longer isotropic. One can compensate for this effect by either putting in a flux-limiter or a variable Eddington factor to describe the transition from diffusion to free streaming, but both approaches are not fully satisfactory since it is difficult to calibrate the newly invented parameters (e.g., Kunasz (1984); Fu (1987); Blinnikov & Nadyoshin (1991); Mair et al. (1992); Stone, Mihalas, & Norman (1992); Yin & Miller (1995)).

Alternatively, one can bin frequency space into groups and solve the set of fully time dependent coupled monochromatic transport equations for each bin. In this approach the problem remains to compute average opacities for each frequency bin. Moreover, because of computer limitations, in all practical applications the number of bins cannot be large which introduces considerable errors, given the strong frequency dependence of the line-opacities (Blinnikov & Nadyoshin 1991; Eastman 1997) (see also Fig. 1).

Finally, in order to get synthetic spectra one might apply Monte-Carlo techniques, as was done by Mazzali et al. (1997b) and Lucy (1999). Here the assumption is that the supernova envelope is in homologous spherical expansion.
and that the luminosity and the photospheric radius are given. The formation of spectral lines is then computed by considering the propagation of a wave packet emitted from the photosphere subject to electron scattering and interaction with lines. Line formation is assumed to occur by coherent scattering, and the line profiles and escape probabilities are calculated in the Sobolev approximation. While this approach appears to be a powerful tool to get synthetic spectra it lacks consistency since the properties of the photosphere have to be calculated by other means.

But having a numerical scheme at hand to solve the transport equation is not sufficient. It is even more important to have accurate opacities. The basic problem, namely that at short wavelengths the opacity is dominated by a huge number of weak lines, was mentioned before. In practice this means that because the list included in anyone’s code is certainly incomplete and the available information may not always be very accurate it is difficult to estimate possible errors. Moreover, there is no general agreement among the different groups calculating SN Ia lightcurves and spectra on how to correct the opacities for Doppler-shifts of the lines, caused by the expansion of the supernova. The so-called “expansion opacity” (see Fig. 1) that should be used in approaches based on the diffusion equation as well as on moment expansions of the transport equation is still discussed in a controversial manner (Pauldrach et al. 1996; Blinnikov 1997; Baron et al. 1997; Eastman 1997; Höflich et al. 1997; Mazzali et al. 1997b; Pinto 1997).

Other open questions include the relative importance of absorption and scattering of photons in lines, and whether or not one can calculate the occupation numbers of atomic levels in equilibrium (LTE) or has to do it by means of the Saha-equation (NLTE) (Pauldrach et al. 1996; Nugent et al. 1997; Höflich, Wheeler, & Thielemann 1998b).

3.2 Results of numerical studies

Despite of the problems discussed in the previous Subsection radiation hydrodynamic models have been used widely as a diagnostic tool for SN Ia. These studies include computations of $\gamma$-ray (Burrows & The 1990; Müller, Höflich, & Khokhlov 1991; Burrows, Shankar, & van Riper 1991; Shigeyama et al. 1993; Ruiz-Lapuente et al. 1993b; Timmes & Woosley 1997; Höflich, Wheeler, & Khokhlov 1999a; Watanabe et al. 1999), UV and optical (Branch & Venkatakrishna 1986; Ruiz-Lapuente et al. 1992; Nugent et al. 1995; Pauldrach et al. 1996; Höflich et al. 1997; Nugent et al. 1997; Hatano, Branch, & Baron 1998; Höflich et al. 1998b; Lentz et al. 2000; Fisher et al. 1999; Lucy 1999; Lentz et al. 1999), and of infrared lightcurves and spectra (Spyromilio, Pinto, & Eastman 1994; Höflich 1995; Wheeler et al. 1998). All studies are based on the assumption, that the explosion remains on average spherically symmetric, an assumption which is questionable, as will be discussed in Sect. 5. Although spherical symmetry might be a good approximation for temperatures, densities, and velocities, the spatial distribution of the products of explosive nuclear burning is expected to be very non-spherical, and it is the distribution of the heavier elements, both in real and in velocity space, which determines to a large extent lightcurves and spectra.

With the possible exception of SN 1991T, where a $2 - 3\sigma$ detection of the $^{56}\text{Co}$ decay-lines at 847keV and 1238keV has been reported (Morris et al. 1997) (see, however, Leising et al. (1995)), only upper limits on $\gamma$-ray line-emission from SN Ia are known. On the basis of the models this is not surprising since the
Figure 1: Mass opacity in a hot (23 000K) plasma of cobalt at a density of \(10^{-12}\) g/cm\(^3\). The upper plot shows the line opacities (calculated by Iglesias, Rogers & Wilson (1987) with OPAL), the lower one the bound-free (long dashed), electron scattering (short dashed), and the “line expansion opacity” (solid curve). (Courtesy of R. Eastman (1999), personal communication)
flux limits of detectors such as COMPTEL on GRO \((10^{-5} \text{ photons per cm}^2 \text{ and second})\) allows detections out to distances of about 15 Mpc in the most favorable cases, i.e. delayed-detonation models producing lots of \(^{56}\text{Ni}\) in the outer parts of the supernova (Timmes & Woosley 1997). In fact, the tentative detection of decay-lines from SN 1991T at a distance of about 13 Mpc can be explained by certain delayed-detonation models and was even predicted by some of them (Müller et al. (1991); see also Sect. 5).

Synthetic (optical and UV) spectra of hydrodynamic models of SN Ia have been computed by several groups (Höflich & Khokhlov 1996; Nugent et al. 1997) and have been compared with the observations. The bottom line of these investigations is that Chandrasekhar-mass deflagration models are in good agreement with observations of "Branch-normals" such as SN 1992A and SN 1994D (Höflich & Khokhlov 1996; Nugent et al. 1997), and delayed-detonation are equally good. The reason is that in both classes of models the burning front starts by propagating out slowly, giving the star some time to expand. The front then speeds up to higher velocities, e.g. to a fair fraction of the sound velocity for deflagration models and to supersonic velocity for detonations, which is necessary to match the observed high velocities of the ejecta. But as far as the amount of radioactive Ni is concerned, the predictions of both classes of models are not too different (Nugent et al. 1997). It also appears that sub-Chandrasekhar models cannot explain the observed UV-flux and the colors of normal SNe Ia (Khokhlov, Müller, & Höflich 1993). Moreover, although sub-Chandrasekhar models eject considerable amounts of He, according to the synthetic spectra He-lines should not be seen, eliminating them as a tool to distinguish between the models (Nugent et al. 1997).

In the infrared SN Ia do show non-monotonic behavior (Elias et al. 1985) and, as for the bolometric lightcurves, a correlation between peak-brightness and lightcurve-shape seems to exist (Contardo & Leibundgut 1998; Contardo 1999). Therefore calculations of IR lightcurves and spectra are of importance and they might prove to be a good diagnostic tool. Broad-band IR lightcurves have been computed by Höflich, Khokhlov, & Wheeler (1995) with the result that the second IR-peak can be explained as an opacity effect. Although the fits were not perfect the general behavior, again, was consistent with both, the deflagration and the delayed-detonation models. Detailed early IR-spectra have been calculated only recently (Wheeler et al. 1998) and the models provide a good physical understanding of the spectra. Again, a comparison between several of the delayed-detonation models and SN 1994D gave good agreement, but one might suspect that certain deflagrations would do equally well. However, in principle, synthetic IR spectra are sensitive to the boundary between explosive C and O and between complete and incomplete Si burning (Wheeler et al. 1998) and should provide some information on the progenitors and the explosion mechanism.

In conclusion, models of SN Ia lightcurves and spectra can fit the observations well but, so far, their predictive power is limited. The fact that multi-dimensional effects are ignored and that the opacities as well as the radiative-transfer codes have obvious shortcomings make it difficult to derive strong constraints on the explosion mechanism. It appears, however, that while it seems to be difficult to distinguish between pure deflagrations and delayed-detonations on the basis of synthetic lightcurves and spectra, sub-Chandrasekhar models cannot fit normal SN Ia equally well.
4 PROGENITOR SYSTEMS

In contrast to supernovae from collapsing massive stars for which in two cases the progenitor star was identified and some of its properties could be inferred directly from observations (SN1987A in the LMC (Blanco 1987; Gilmozzi 1987; Gilmozzi et al. 1987; Hillebrandt et al. 1987) and SN1993J in M81 (Benson et al. 1993; Schmidt et al. 1993; Nomoto et al. 1993; Podsiadlowski 1993)), there is not a single case known where we have this kind of information for the progenitor of a SN Ia. This is not too surprising, given the fact that their progenitors are most likely faint compact dwarf stars and not red or blue supergiants. Therefore we have to rely on indirect means if we want to determine their nature.

The standard procedure is then to eliminate all potential candidates if some of their properties disagree with either observations or physical principles, and to hope that one is left with a single and unique solution. Unfortunately, for the progenitors of Type Ia supernovae this cannot be done unambiguously, the problem being the lack of strong candidates that pass all possible tests beyond doubt.

In this Section we will first repeat the major constraints which have to be imposed on the progenitor systems and then discuss the presently favored candidates, Chandrasekhar-mass C+O white dwarfs and low-mass C+O white dwarf cores embedded in a shell of helium, in some detail. It will be shown, however, that even if we could single out a particular progenitor system this would narrow the parameter space for the initial conditions at the onset of the explosion, but might not determine them sufficiently well, in particular if we are aiming at a quantitative understanding. Some of the discussion given below follows recent reviews of Renzini (1996) and Livio (2000).

4.1 Observational constraints on Type Ia progenitors

As was already discussed in Sect. 2, SNe Ia are (spectroscopically) defined by the absence of emission lines of hydrogen and the presence of a (blue-shifted) Si II absorption line with a rest-wavelength of 6355Å near maximum light. The first finding requires that the atmosphere of the exploding star contains no or at most 0.1 M⊙ of hydrogen, and the second one indicates that some nuclear processing takes place and that products of nuclear burning are ejected in the explosion. Mean velocities of the ejecta, as inferred from spectral fits, are around 5,000 km/s and peak velocities exceeding 20,000 km/s are observed, which is consistent with fusing about 1 M⊙ of carbon and oxygen into Fe-group elements or intermediate-mass elements such as Si or Ca. The presence of some UV-flux, the width of the peak of the early light curve, and the fact that radioactive-decay models (56Ni → 56Co → 56Fe) can fit the emission very well, all point towards compact progenitor stars with radii of less than about 10,000 km.

After about two weeks the typical SN Ia spectrum changes from being dominated by lines of intermediate-mass nuclei to being dominated by Fe II. Since also a Co III feature is identified at later stages this adds evidence to the interpretation that they are indeed thermonuclear explosions of rather compact stars, leaving the cores of stars with main sequence masses near 6 to 8 M⊙ or white dwarfs as potential candidates. Moreover, the energetics of the explosion and the spectra seem to exclude He white dwarfs (Nomoto & Sugimoto 1977; Woosley, Taam, & Weaver 1986), mainly because such white dwarfs would undergo very
Next one notes that most SNe Ia, of order 85%, have very similar peak luminosities, light curves, and spectra. The dispersion in peak blue and visual brightness is only of order 0.2 - 0.3 magnitudes calling for a very homogeneous class of progenitors. It is mainly this observational fact that seems to single out Chandrasekhar-mass white dwarfs as their progenitors. Since the ratio of energy to mass determines the velocity profile of the exploding star the homogeneity would be explained in a very natural way. However, as has been discussed in Sect. (2), there exist also significant differences among the various SNe Ia which may indicate that this simple interpretation is not fully correct. The difference in peak-brightness, ranging from sub-luminous events like SN 1991bg in NGC 4374 ($B_{\text{max}} = -16.54$ (Turatto et al. 1996), as compared to the mean of the “Branch-normals” of $B_{\text{max}} \simeq -19$ (Hamuy et al. 1996c)) to bright ones like SN 1991T, which was about 0.5 magnitudes brighter in $B$ than a typical Type Ia in the Virgo cluster (Mazzali, Danziger, & Turatto 1995), is commonly attributed to different $^{56}\text{Ni}$-masses produced in the explosion. They range from about 0.07 $M_\odot$ for SN 1991bg (see, e.g., Mazzali et al. (1997a)) to at least 0.92 $M_\odot$ for SN 1991T (Khokhlov et al. 1993; see however Fisher et al. (1999)), with typically 0.6 $M_\odot$ for normal SNe Ia (Höflich & Khokhlov 1996; Nugent et al. 1997). It is hard to see how this rather large range can be accommodated in a single class of models.

The stellar populations in which SNe Ia show up include spiral arms as well as elliptical galaxies, with some weak indication that they might be more efficiently produced in young populations (Bartunov, Tsvetkov, & Filimonova 1994). Again, if we insist on a single class of progenitors, the very fact that they do occur in ellipticals would rule out massive stars as potential candidates. On the other hand, the observations may tell us that there is not a unique class of progenitors. In particular, the fact that the bright and slowly declining ones (like SN 1991T) are absent in elliptical and S0 galaxies may point towards different progenitor classes (Hamuy et al. 1996c).

All in all, the observational findings summarized so far are consistent with the assumption that Type Ia supernovae are the result of thermonuclear disruptions of white dwarfs, C+O white dwarfs being the favored model. The diversity among them must then be attributed to the history and nature of the white dwarf prior to the explosion and/or to the physics of thermonuclear burning during the event. It cannot be excluded, however, that at least some SNe Ia have a different origin, such as accretion-induced collapse of massive O-Ne-Mg (or O-Ne) white dwarfs for SN 1991bg-like objects (Nomoto et al. 1994a, 1995, 1996; Fryer et al. 1999). Also it is not clear whether or not there is a clear-cut distinction between Type Ib/c supernovae, defined by the absence of the Si II feature, and the (faint) SNe Ia. The former are believed to reflect the core-collapse of a massive star, its hydrogen-rich envelope being peeled off due to mass-loss in a binary system. For example, SN1987K started out as a SN II with H lines in its spectrum, but changed into a SN Ib/c-like spectrum after 6 months (Filippenko 1988), supporting this interpretation. It should be noted that SN 1991bg-like objects are not often observed, but that this may well be a selection effect. Suntzeff (1996), for example, argues that up to 40% of all Type Ia’s could perhaps belong to that sub-group.
4.2 Pre-supernova evolution of binary stars

Despite of all these uncertainties it is the current understanding and believe that the progenitors of SNe Ia are C+O white dwarfs in binary systems evolving to the stage of explosion by mass-overflow from the companion (single-degenerate scenario) or by the merger of two white dwarfs (double-degenerate scenario). Binary evolution of some sort is necessary because C+O white dwarfs a typically born with a mass around 0.6 $M_\odot$ (Homeier et al. 1998) but need to be near the Chandrasekhar mass or to accumulate a shell of helium in order to explode. In this Subsection we will summarize the arguments in favor and against both scenarios.

Double-degnerates as potential Type Ia progenitors had many ups and downs in the past, beginning with the classic papers of Iben & Tutukov (1984) and Webbink (1984). The arguments in favor are that such binaries should exist as a consequence of stellar evolution, they would explain very naturally the absence of hydrogen, and they could, in principle, be an easy way to approach a critical mass. In fact, several candidate-systems of binary white dwarfs have recently been identified but most of the short-period ones (at present 8 systems are known with orbital periods of less than half a day), which could merge in a Hubble-time due to the emission of gravitational radiation, have a mass less than $M_{\text{chan}}$ (Saffer, Livio, & Yungelson 1998); see also Livio (2000) for a recent review). There is only one system known (KPD 0422+5421; Koen, Orosz, & Wade (1998)) with a mass which, within the errors, could exceed $M_{\text{chan}}$, a surprisingly small number. None-the-less it is argued that from population synthesis one could arrive at about the right frequency of sufficiently massive mergers (Livio 2000).

Besides the lack of convincing direct observational evidence for sufficiently many appropriate binary systems, the homogeneity of “typical” SNe Ia may be an argument against this class of progenitors. It is not easy to see how the merging of two white dwarfs of (likely) different mass, composition, and angular momentum with different impact parameters, etc., will always lead to the same burning conditions and, therefore, the production of a nearly equal amount of $^{56}\text{Ni}$. Moreover, some investigations of white dwarf mergers seem to indicate that an off-center ignition will convert carbon and oxygen into oxygen, neon, and magnesium, leading to gravitational collapse rather than a thermonuclear disruption (Woosley & Weaver 1986a; Saio & Nomoto 1985, 1998; Mochkovitch & Livio 1990). Finally, based on their galactic chemical evolution model, Kobayashi et al. (1998) claim that double-degenerate mergers lead to inconsistencies with the observed O/Fe as a function of metallicity, but this statement is certainly model-dependent. In any case, mergers might, if they are not responsible for the bulk of the SNe Ia, still account for some peculiar ones, such as the super-luminous SN 1991T -like explosions.

Single-degenerate models are in general favored today. They consist of a low-mass white dwarf accreting matter from the companion-star until either it reaches $M_{\text{chan}}$ or a layer of helium has formed on-top of its C+O core that can ignite and possibly drive a burning front into the carbon and oxygen fuel. This track to thermonuclear explosions of white dwarfs was first discussed by Whelan & Iben (1973); Nomoto (1982b); Iben & Tutukov (1984) and Paczynski (1985). The major problem of these models has always been that nearly all possible accretion rates can be ruled out by rather strong arguments (Munari & Renzini 1992; Cassisi, Castellani, & Tornambe 1996; Tutukov & Yungelson 1996; Livio
Figure 2: The likely outcome of hydrogen accretion onto white dwarfs of different masses is shown. (From Nomoto 1982a.)

et al. 1996; King & Van Teeseling 1998; Kato & Hachisu 1999; Cassisi, Iben, & Tornambe 1998). In short, it is believed that white dwarfs accreting hydrogen at a low rate undergo nova eruptions and lose more mass in the outburst than they have accreted prior to it (e.g. Beer (1974); Gehrz, Truran, & Williams (1993)). At moderate accretion rates, a degenerate layer of helium is thought to form which might flash and could give rise to sub-Chandrasekhar explosions (which have other problems, as will be discussed later). Next, still higher accretion rates can lead to quiet hydrostatic burning of H and He, but these systems should be so bright that they could easily be detected, but it is not clear beyond doubt that they coincide with any of the known symbiotic or cataclysmic binaries. Very high accretion rates, finally would form an extended H-rich red giant envelope around the white dwarf the debris of which are not seen in the explosions (Nomoto, Nariai, & Sugimoto 1979) (see also Fig. 2). Therefore, it is very uncertain if white dwarfs accreting hydrogen from a companion star can ever reach the $M_{\text{chan}}$ (Cassisi et al. 1998).

Some of these arguments may be questioned, however. Firstly, a class of binary systems has recently been discovered, the so-called “Supersoft X-ray Sources”, which are best interpreted as white dwarfs accreting hydrogen-rich matter at such a high rate that H burns steadily (Truemper et al. 1991; Greiner, Hasinger, & Kahabka 1991; Van Den Heuvel et al. 1992; Southwell et al. 1996; Kahabka & Van Den Heuvel 1997). It appears that if these white dwarfs could retain the
accreted gas they might be good candidates for SN Ia progenitors. In principle, they could accrete a few tenths of a solar mass with a typical accretion rate of a few $10^{-7} \text{M}_\odot/\text{yr}$ over the estimated lifetime of such systems of several $10^9$ years. Since most of them are heavily extinct their total number might be sufficiently high (Di Stefano & Rappaport 1994); see also Livio (1996) and Yungelson et al. (1996), although this statement is certainly model-dependent. However, some of the supersoft X-ray sources are known to be variable in X-rays (but not in the optical wave-bands) on time-scales of weeks (Pakull et al. 1993), too short to be related with the H-burning shell, possibly indicating substantial changes in the accretion rates. It may therefore not be justified to assume that the accretion rates we see now are sustained over several $10^9$ years. But their very existence provides a first and strong case for the single-degenerate scenario.

Secondly, also the minimum accretion rate at which hydrogen burns quietly without a nova outburst is rather uncertain. All models that compute this rate ignore important pieces of physics and, therefore, their predictions could be off by orders of magnitude. For example, classical nova outbursts require that the accreted hydrogen-rich envelope of the white dwarf is also heavily enriched in C and O from the white dwarf’s core (see, e.g., Starrfield et al. (1972); Sparks, Starrfield, & Truran (1976); Starrfield, Truran, & Sparks (1978); Truran (1982)). One possible explanation has been that convective mixing and dredge-up might happen during the thermonuclear runaway, but recent numerical simulations indicate that this mechanism is insufficient (Kercek, Hillebrandt, & Truran 1999). In contrast to spherically symmetric models their 3-D simulations lead to a phase of quiet H-burning for accretion rates as low as $5 \times 10^{-9} \text{M}_\odot/\text{yr}$ for a white dwarf of 1 M$_\odot$ rather than a nova outburst with mass-loss from the core. Other shortcomings include the assumption of spherical accretion with zero entropy, the neglect of magnetic fields, etc. So the dividing line between steady hydrogen burning and nova eruptions might leave some room for SN Ia progenitors.

Finally, it has been argued that the interaction of a wind from the white dwarf with the accretion flow from lobe-filling low mass red giant may open a wider path to Type Ia supernovae. In a series of papers Hachisu, Kato, & Nomoto (1996); Hachisu et al. (1999b); Hachisu, Kato, & Nomoto (1999a) discuss the effect that when the mass accretion rate exceeds a certain critical value the envelope solution on the white dwarf is no longer static but corresponds to a strong wind. The strong wind stabilizes the mass transfer and limits the accretion rate and the white dwarf can burn hydrogen steadily. However, their model assumes spherical accretion onto and a spherical wind from the white dwarf which seem to be contradicting assumptions. But the idea should certainly be followed up.

4.3 Evolution to ignition

In what follows we will assume most of the time that SN Ia progenitors are Chandrasekhar-mass C+O white dwarfs because, as was discussed in the previous sections, this class of models seems to fit best the “typical” or “average” Type Ia. In this subsection we also will not discuss models in which two degenerate white dwarfs merge and form a critical mass for the ignition of carbon, mainly because the merging process will, in reality, be very complex and it is difficult to construct realistic explosion models (although with increasing computational resources it may be possible in the future).

But even if we consider only Chandrasekhar-mass white dwarfs as progenitor
Type Ia Supernova Explosion Models

17

candidates the information that is needed in order to model the explosion cannot be obtained easily. In particular, the thermal structure and the chemical composition are very uncertain. The C/O-ratio, for example, has to be know throughout the white dwarf, but this ratio depends on the main sequence mass of its progenitor and the metallicity of the gas from which it formed (Umeda et al. 1999; Wellstein & Langer 1999). It was found that, depending on the main sequence mass, the central C/O can vary from 0.4 to 0.6, considerably less than assumed in most supernova models.

Next, the thermal structure of a white dwarf on its way to an explosion depends on the (convective) URCA-process (Paczynski 1973; Iben 1978, 1982; Barkat & Wheeler 1990; Mochkovitch 1996). The URCA-pairs $A = 21, 23, \text{and } 25$ (such as, i.e., $^{21}\text{Ne}/^{21}\text{F}$, ...) can lead to either heating or cooling, and possibly even to a temperature inversion near the center of the white dwarf. The abundances of the URCA-pairs depends again on the initial metallicity which could, thus, affect the thermal structure of the white dwarf. Unfortunately, the convection in the degenerate star is likely to be non-local, time-dependent, 3-dimensional, and very sub-sonic, but needs to be modeled over very long (secular) time-scales. It is not likely that in the near future we will be able to model these processes in a realistic manner, even on super-computers.

Due to these difficulties, numerical studies of the explosion rely on ad-hoc assumptions fixing the initial conditions, which are usually chosen to be as simple as possible. Realistic simulations have to be multi-dimensional, as will be explained in the next Section, and therefore numerical studies can only investigate a small fraction of the available parameter space. The failure or success of a particular model to explain certain observational results may, therefore, not be conclusive.

5 EXPLOSION MODELING

Numerical models are needed to provide the density, temperature, composition, and velocity fields of the supernova ejecta that result from the thermonuclear explosion of a white dwarf, accepted by most researchers as the “standard model” for SNe Ia (Sec. (2), (4)). This information can then be used to compute the resulting light curve and spectra with the help of radiation transport codes (Sec. (3)) or to compare the relative distribution of isotopes with the observed solar abundances.

To a very good approximation, the exploding white dwarf material can be described as a fully ionized plasma with varying degrees of electron degeneracy, satisfying the fluid approximation. The governing equations are the hydrodynamical equations for mass, species, momentum, and energy transport including gravitational acceleration, viscosity, heat and mass diffusion (Landau & Lifshitz 1963), and nuclear energy generation (Arnett 1996). They must be supplemented by an equation of state for an ideal gas of nuclei, an arbitrarily relativistic and degenerate electron gas, radiation, and electron-positron pair production and annihilation (Cox & Giulì 1968). The gravitational potential is calculated with the help of the Poisson equation. In numerical simulations that fully resolve the relevant length scales for dissipation, diffusion, and nuclear burning it is possible to obtain the energy generation rate from a nuclear reaction network (Timmes 1999, for a recent overview, see) and the diffusion coefficients from an evaluation of the kinetic transport mechanisms (Nandkumar & Pethick 1984). If, on the other
hand, these scales are unresolved – as is usually the case in simulations on scales of the stellar radius – subgrid-scale models are required to compute (or parameterize) the effective large-scale transport coefficients and burning rates, which are more or less unrelated to the respective microphysical quantities (Khokhlov 1995; Niemeyer & Hillebrandt 1995b).

Initial conditions can be obtained from hydrostatic spherically symmetric models of the accreting white dwarf or – for Chandrasekhar mass progenitors – from the Chandrasekhar equation for a fully degenerate, zero temperature white dwarf (Kippenhahn & Weigert 1989). Given the initial conditions and symmetries specifying the boundary conditions, the dynamics of the explosion can in principle be determined by numerically integrating the equations of motion. Müller (1998) gives a detailed account of some current numerical techniques used for modeling supernovae.

Until the mid-nineties, most work on SN Ia explosions was done studying one-dimensional (1D), spherically symmetric models. This approach inherently lacks some important aspects of multidimensional thermonuclear burning relevant for $M_{\text{chan}}$-explosion models, e.g. off-center flame ignition, flame instabilities, and turbulence, which have to be mimicked by means of a spherical flame front with an undetermined turbulent flame speed e.g., Nomoto, Sugimoto, & Neo (1976); Nomoto, Thielemann, & Yokoi (1984); Woosley & Weaver (1986a); Woosley (1990). In spite of these caveats, 1D models still represent the only reasonable approach to combine the hydrodynamics with detailed nucleosynthesis calculations and to carry out parameter studies of explosion scenarios. In fact, most of the phenomenology of SN Ia explosions and virtually all of the model predictions for spectra and light curves are based on spherically symmetric models. Several recent articles (Woosley 1990; Nomoto et al. 1996; Höflich & Khokhlov 1996; Iwamoto et al. 1999) describe the methodology and trends observed in these studies, as well as their implications regarding the cosmological supernova surveys (Höflich et al. 1998b; Ruiz-Lapuente & Canal 1998; Umeda et al. 1999; Sorokina, Blinnikov, & Bartunov 1999).

Following the pioneering work of Müller & Arnett (1982, 1986), some groups have explored the dynamics of two-dimensional (2D) (Livne 1993; Arnett & Livne 1994a, 1994b; Niemeyer & Hillebrandt 1995b; Niemeyer, Hillebrandt, & Woosley 1996; Arnett 1997; Reinecke, Hillebrandt, & Niemeyer 1999a) and three-dimensional (3D) (Khokhlov 1994, 1995; Bravo & Garcia-Senz 1997; Hillebrandt, Woosley, & Arnett 1997; Niemeyer & Hillebrandt 1997) explosion models, triggering the development of numerical algorithms for representing thin propagating surfaces in large-scale simulations (Khokhlov 1993a; Niemeyer & Hillebrandt 1995b; Bravo & Garcia-Senz 1995; Arnett 1997; Garcia-Senz, Bravo, & Serichol 1998; Reinecke et al. 1999b). It has also become possible to perform 2D and 3D direct numerical simulations (DNS), i.e. fully resolving the relevant burning and diffusion scales, of microscopic flame instabilities and flame-turbulence interactions (Niemeyer & Hillebrandt 1995a; Khokhlov 1995; Niemeyer & Hillebrandt 1997; Niemeyer, Bushe, & Ruetsch 1999).

5.1 Chandrasekhar Mass Explosion Models

Given the overall homogeneity of SNe Ia (Sec. (2.1)), the good agreement of parameterized 1D $M_{\text{chan}}$-models with observed spectra and light curves, and their reasonable nucleosynthetic yields, the bulk of normal SNe Ia is generally assumed to consist of exploding $M_{\text{chan}}$ C+O white dwarfs (Hoyle & Fowler 1960;
Type Ia Supernova Explosion Models

In spite of three decades of work on the hydrodynamics of this explosion mechanism (beginning with Arnett (1969)), no clear consensus has been reached whether the star explodes as a result of a subsonic nuclear deflagration that becomes strongly turbulent (Ivanova, Imshen- nik, & Chechetkin 1974; Buchler & Mazurek 1975; Nomoto et al. 1976, 1984; Woosley, Axelrod, & Weaver 1984), or whether this turbulent flame phase is followed by a delayed detonation during the expansion (Khokhlov 1991a, 1991b; Woosley & Weaver 1994a) or after one or many pulses (Khokhlov 1991b; Arnett & Livne 1994a, 1994b). Only the prompt detonation mechanism is agreed to be inconsistent with SN Ia spectra as it fails to produce sufficient amounts of intermediate mass elements (Arnett 1969; Arnett, Truran, & Woosley 1971).

This apparently slow progress is essentially a consequence of the overwhelming complexity of turbulent flame physics and deflagration-detonation transitions (DDTs) (Williams 1985; Zeldovich et al. 1985) that makes first-principle predictions based on $M_{\text{chan}}$-explosion models nearly impossible. The existence of an initial subsonic flame phase is, it seems, an unavoidable ingredient of all $M_{\text{chan}}$-models (and only those) where it is required to pre-expand the stellar material prior to its nuclear consumption in order to avoid the almost exclusive production of iron-peaked nuclei (Nomoto et al. 1976, 1984; Woosley & Weaver 1986a).

Guided by parameterized 1D models that yield estimates for the values for the turbulent flame speed $S_t$ and the DDT transition density $\rho_{\text{DDT}}$ (e.g., Höflich & Khokhlov 1996), a lot of work has been done recently on the physics of buoyancy-driven, turbulent thermonuclear flames in exploding $M_{\text{chan}}$-white dwarfs. The close analogy with thin chemical premixed flames has been exploited to develop a conceptual framework that covers all scales from the white dwarf radius to the microscopic flame thickness and dissipation scales (Khokhlov 1995; Niemeyer & Woosley 1997). In the following discussion of nuclear combustion (Sec. (5.1.1)), flame ignition (Sec. (5.1.2)), and the various scenarios for $M_{\text{chan}}$ explosions characterized by the sequence of combustion modes (Sec. (5.1.3) – Sec. (5.1.6)) we will emphasize the current understanding of physical processes rather than empirical fits of light curves and spectra.

5.1.1 FLAMES, TURBULENCE, AND DETONATIONS

Owing to the strong temperature dependence of the nuclear reaction rates, $\dot{S} \sim T^{12}$ at $T \approx 10^{10}$ K (Hansen & Kawaler 1994, p. 247), nuclear burning during the explosion is confined to microscopically thin layers that propagate either conductively as subsonic deflagrations (“flames”) or by shock compression as supersonic detonations (Courant & Friedrichs 1948; Landau & Lifshitz 1963, chap. XIV). Both modes are hydrodynamically unstable to spatial perturbations as can be shown by linear perturbation analysis. In the nonlinear regime, the burning fronts are either stabilized by forming a cellular structure or become fully turbulent – either way, the total burning rate increases as a result of flame surface growth (Lewis & von Elbe 1961; Williams 1985; Zeldovich et al. 1985). Neither flames nor detonations can be resolved in explosion simulations on stellar scales and therefore have to be represented by numerical models.

When the fuel exceeds a critical temperature $T_c$ where burning proceeds nearly instantaneously compared with the fluid motions (see Timmes & Woosley (1992) for a suitable definition of $T_c$), a thin reaction zone forms at the interface between burned and unburned material. It propagates into the surrounding fuel.
by one of two mechanisms allowed by the Rankine-Hugoniot jump conditions: a deflagration ("flame") or a detonation (cf. fig. 2.5 in Williams (1985)).

If the overpressure created by the heat of the burning products is sufficiently high, a hydrodynamical shock wave forms that ignites the fuel by compressional heating. A self-sustaining combustion front that propagates by shock-heating is called a detonation. Detonations generally move supersonically and therefore do not allow the unburned medium to expand before it is burned. Their speed depends mainly on the total amount of energy released per unit mass, $\epsilon$, and is therefore more robustly computable than deflagration velocities. A good estimate for the velocity of planar strong detonations is the Chapman-Jouget velocity (Lewis & von Elbe 1961; Zeldovich et al. 1985, and references therein). The nucleosynthesis, speed, structure, and stability of planar detonations in degenerate C+O material was analyzed by Imshennik & Khokhlov (1984); Khokhlov (1988, 1989, 1993b), and recently by Kriminski, Bychkov, & Liberman (1998) and Imshennik et al. (1999) who claim that C+O detonations are one-dimensionally unstable and therefore cannot occur in exploding white dwarfs above a critical density of $\sim 2 \times 10^7$ g cm$^{-3}$ (Kriminski et al. 1998) (cf. Sec. (5.1.3)).

If, on the other hand, the initial overpressure is too weak, the temperature gradient at the fuel-ashes interface steepens until an equilibrium between heat diffusion (carried out predominantly by electron-ion collisions) and energy generation is reached. The resulting combustion front consists of a diffusion zone that heats up the fuel to $T_c$, followed by a thin reaction layer where the fuel is consumed and energy is generated. It is called a deflagration or simply a flame and moves subsonically with respect to the unburned material (Landau & Lifshitz 1963). Flames, unlike detonations, may therefore be strongly affected by turbulent velocity fluctuations of the fuel. Only if the unburned material is at rest, a unique laminar flame speed $S_l$ can be found which depends on the detailed interaction of burning and diffusion within the flame region (e.g., Zeldovich et al. 1985). Following Landau & Lifshitz (1963), it can be estimated by assuming that in order for burning and diffusion to be in equilibrium, the respective time scales, $\tau_b = \epsilon/\dot{w}$ and $\tau_d = \delta^2/\kappa$, where $\delta$ is the flame thickness and $\kappa$ is the thermal diffusivity, must be similar: $\tau_b \sim \tau_d$. Defining $S_l = \delta/\tau_b$, one finds $S_l \sim (\kappa \dot{w}/\epsilon)^{1/2}$, where $\dot{w}$ should be evaluated at $T \approx T_c$ (Timmes & Woosley 1992). This is only a crude estimate due to the strong $T$-dependence of $\dot{w}$. Numerical solutions of the full equations of hydrodynamics including nuclear energy generation and heat diffusion are needed to obtain more accurate values for $S_l$ as a function of $\rho$ and fuel composition. Laminar thermonuclear carbon and oxygen flames at high to intermediate densities were investigated by Buchler, Colgate, & Mazurek (1980); Ivanova, Imshennik, & Chechetkin (1982); Woosley & Weaver (1986b), and, using a variety of different techniques and nuclear networks, by Timmes & Woosley (1992). For the purpose of SN Ia explosion modeling, one needs to know the laminar flame speed $S_l \approx 10^7 \ldots 10^4$ cm s$^{-1}$ for $\rho \approx 10^9 \ldots 10^7$ g cm$^{-3}$, the flame thickness $\delta = 10^{-4} \ldots 1$ cm (defined here as the width of the thermal pre-heating layer ahead of the much thinner reaction front), and the density contrast between burned and unburned material $\mu = \Delta \rho/\rho = 0.2 \ldots 0.5$ (all values quoted here assume a composition of $X_C = X_O = 0.5$, Timmes & Woosley (1992)). The thermal expansion parameter $\mu$ reflects the partial lifting of electron degeneracy in the burning products, and is much lower than the typical value found in chemical, ideal gas systems (Williams 1985).
Observed on scales much larger than $\delta$, the internal reaction-diffusion structure can be neglected and the flame can be approximated as a density jump that propagates locally with the normal speed $S_l$. This “thin flame” approximation allows a linear stability analysis of the front with respect to spatial perturbations. The result shows that thin flames are linearly unstable on all wavelengths. It was discovered first by Landau (1944) and Darrieus (1944) and is hence called the “Landau-Darrieus” (LD) instability. Subject to the LD instability, perturbations grow until a web of cellular structures forms and stabilizes the front at finite perturbation amplitudes (Zeldovich 1966). The LD instability therefore does not, in general, lead to the production of turbulence. In the context of SN Ia models, the nonlinear LD instability was studied by Blinnikov & Sasorov (1996), using a statistical approach based on the Frankel equation, and by Niemeyer & Hillebrandt (1995a) employing 2D hydrodynamics and a one-step burning rate. Both groups concluded that the cellular stabilization mechanism precludes a strong acceleration of the burning front as a result of the LD instability. However, Blinnikov & Sasorov (1996) mention the possible breakdown of stabilization at low stellar densities (i.e., high $\mu$) which is also indicated by the lowest density run of Niemeyer & Hillebrandt (1995a) – this may be important in the framework of active turbulent combustion (see below). The linear growth rate of LD unstable thermonuclear flames with arbitrary equation of state was derived by Bychkov & Liberman (1995a). The same authors also found a one-dimensional, pulsational instability of degenerate C+O flames (Bychkov & Liberman 1995b) which was later disputed by Blinnikov (1996).

The best studied and probably most important hydrodynamical effect for modeling SN Ia explosions is the Rayleigh-Taylor (RT) instability (Rayleigh 1883; Chandrasekhar 1961) resulting from the buoyancy of hot, burned fluid with respect to the dense, unburned material. Several groups have investigated the RT instability of nuclear flames in SNe Ia by means of numerical hydrodynamical simulations (Müller & Arnett 1982, 1986; Livne 1993; Khokhlov 1994, 1995; Niemeyer & Hillebrandt 1995b). After more than five decades of experimental and numerical work, the basic phenomenology of nonlinear RT mixing is fairly well understood (Fermi 1951; Layzer 1955; Sharp 1984; Read 1984; Youngs 1984): Subject to the RT instability, small surface perturbations grow until they form bubbles (or “mushrooms”) that begin to float upward while spikes of dense fluid fall down. In the nonlinear regime, bubbles of various sizes interact and create a foamy RT mixing layer whose vertical extent $h_{RT}$ grows with time $t$ according to a self-similar growth law, $h_{RT} = \alpha g(\mu/2)t^2$, where $\alpha$ is a dimensionless constant ($\alpha \approx 0.05$) and $g$ is the background gravitational acceleration (Sharp 1984; Youngs 1984; Read 1984).

Secondary instabilities related to the velocity shear along the bubble surfaces (Niemeyer & Hillebrandt 1997) quickly lead to the production of turbulent velocity fluctuations that cascade from the size of the largest bubbles ($\approx 10^7$ cm) down to the microscopic Kolmogorov scale, $l_k \approx 10^{-4}$ cm where they are dissipated (Niemeyer & Hillebrandt 1995b; Khokhlov 1995). Since no computer is capable of resolving this range of scales, one has to resort to statistical or scaling approximations of those length scales that are not properly resolved. The most prominent scaling relation in turbulence research is Kolmogorov’s law for the cascade of velocity fluctuations, stating that in the case of isotropy and statistical stationarity, the mean velocity $v$ of turbulent eddies with size $l$ scales as $v \sim l^{1/3}$ (Kolmogorov 1941). Knowledge of the eddy velocity as a function of
length scale is important to classify the burning regime of the turbulent combustion front (Niemeyer & Woosley 1997; Niemeyer & Kerstein 1997; Khokhlov, Oran, & Wheeler 1997). The ratio of the laminar flame speed and the turbulent velocity on the scale of the flame thickness, \( K = S_l/v(\delta) \), plays an important role: if \( K \gg 1 \), the laminar flame structure is nearly unaffected by turbulent fluctuations. Turbulence does, however, wrinkle and deform the flame on scales \( l \) where \( S_l \ll v(l) \), i.e. above the Gibson scale \( l_g \) defined by \( S_l = v(l_g) \) (Peters 1988). These wrinkles increase the flame surface area and therefore the total energy generation rate of the turbulent front (Damkőhler 1940). In other words, the turbulent flame speed, \( S_t \), defined as the mean overall propagation velocity of the turbulent flame front, becomes larger than the laminar speed \( S_l \). If the turbulence is sufficiently strong, \( v(L) \gg S_l \), the turbulent flame speed becomes independent of the laminar speed, and therefore of the microphysics of burning and diffusion, and scales only with the velocity of the largest turbulent eddy (Damkőhler 1940; Clavin 1994):

\[
S_t \sim v(L) .
\] (2)

Because of the unperturbed laminar flame properties on very small scales, and the wrinkling of the flame on large scales, the burning regime where \( K \gg 1 \) is called the corrugated flamelet regime (Pope 1987; Clavin 1994).

As the density of the white dwarf material declines and the laminar flamelets become slower and thicker, it is plausible that at some point turbulence significantly alters the thermal flame structure (Khokhlov et al. 1997; Niemeyer & Woosley 1997). This marks the end of the flamelet regime and the beginning of the distributed burning, or distributed reaction zone, regime (e.g., Pope 1987). So far, modeling the distributed burning regime in exploding white dwarfs has not been attempted explicitly since neither nuclear burning and diffusion nor turbulent mixing can be properly described by simplified prescriptions. Phenomenologically, the laminar flame structure is believed to be disrupted by turbulence and to form a distribution of reaction zones with various lengths and thicknesses. In order to find the critical density for the transition between both regimes, we need to formulate a specific criterion for flamelet breakdown. A criterion for the transition between both regimes is discussed in Niemeyer & Woosley (1997); Niemeyer & Kerstein (1997) and Khokhlov et al. (1997):

\[
l_{\text{cutoff}} \leq \delta .
\] (3)

Inserting the results of Timmes & Woosley (1992) for \( S_l \) and \( \delta \) as functions of density, and using a typical turbulence velocity \( v(10^6 \text{cm}) \sim 10^7 \text{ cm s}^{-1} \), the transition from flamelet to distributed burning can be shown to occur at a density of \( \rho_{\text{dis}} \approx 10^7 \text{ g cm}^{-3} \) (Niemeyer & Kerstein 1997).

The close coincidence of \( \rho_{\text{dis}} \) and the preferred value for \( \rho_{\text{DDT}} \) (Höflich & Khokhlov 1996; Nomoto et al. 1996) inspired some authors (Niemeyer & Woosley 1997; Khokhlov et al. 1997) to suggest that both are related by local flame quenching and re-ignition via the Zeldovich induction time gradient mechanism (Zeldovich et al. 1970), whereby a macroscopic region with a uniform temperature gradient can give birth to a supersonic spontaneous combustion wave that steepens into a detonation (Woosley 1990, and references therein). In the context of the SN Ia explosion mechanism, this effect was first analyzed by Blinnikov & Khokhlov (1986, 1987). Whether or not the gradient mechanism can account for
DDTs in the delayed detonation scenario for SNe Ia is still controversial; while Khokhlov et al. (1997) conclude that it can, Niemeyer (1999) – using arguments based on incompressible computations of microscopic flame-turbulence interactions by Niemeyer et al. (1999) – states that thermonuclear flames may be too robust with respect to turbulent quenching to allow the formation of a sufficiently uniform temperature gradient.

Assuming that the nonlinear RT instability dominates the turbulent flow that advects the flame, the passive-surface description of the flame neglects the additional stirring caused by thermal expansion within the flame brush itself, accelerating the burnt material in random directions. Both the spectrum and cutoff scale may be affected by “active” turbulent combustion (Kerstein 1996; Niemeyer & Woosley 1997). Although the small expansion coefficient $\mu$ indicates that the effect is weak compared to chemical flames, a quantitative answer is still missing.

Finally, we note that some authors also studied the multidimensional instability of detonations in degenerate C+O matter (Boisseau et al. 1996; Gamezo et al. 1999), finding unsteady front propagation, the formation of a cellular front structure and locally incomplete burning in multidimensional C+O detonations. These effects may have interesting implications for SN Ia scenarios involving a detonation phase.

5.1.2 FLAME IGNITION

As the white dwarf grows close to the Chandrasekhar mass $M_{\text{Ch}} \approx 1.4M_\odot$, the energy budget near the core is governed by plasmon neutrino losses and compressional heating. The neutrino losses increase with growing central density until the latter reaches approximately $2 \times 10^9$ g cm$^{-3}$ (Woosley & Weaver 1986a). At this point, plasmon creation becomes strongly suppressed while electron screening of nuclear reactions enhances the energy generation rate until it begins to exceed the neutrino losses. This “smoldering” of the core region marks the beginning of the thermonuclear runaway (Arnett 1969; Arnett 1971; Woosley & Weaver 1986a). During the following $\sim 1000$ years, the core experiences internally heated convection with progressively smaller turnover time scales $\tau_c$. Simultaneously, the typical time scale for thermonuclear burning, $\tau_b$, drops even faster as a result of the rising core temperature and the steep temperature dependence of the nuclear reaction rates.

During this period, the entropy and temperature evolution of the core is affected by the convective URCA process, a convectively driven electron capture-beta decay cycle leading to neutrino-antineutrino losses. It was first described in this context by Paczynski (1972) who argued it would cause net cooling and therefore delay the runaway. Since then, the convective URCA process was revisited by several authors (e.g., Bruenn 1973; Iben 1982; Barkat & Wheeler 1990; Mochkovitch 1996) who alternately claimed that it results in overall heating or cooling. The most recent analysis (Stein, Barkat, & Wheeler 1999) concludes that while the URCA neutrinos carry away energy, they cannot cool the core globally but instead slow down the convective motions.

At $T \approx 7 \times 10^8$ K, $\tau_c$ and $\tau_b$ become comparable, indicating that convective plumes burn at the same rate as they circulate (Nomoto et al. 1984; Woosley & Weaver 1986a). Experimental or numerical data describing this regime of strong reactive convection is not available, but several groups are planning to conduct numerical experiments at the time this article is written. At $T \approx 1.5 \times 10^9$ K,
becomes extremely small compared with $\tau_c$, and carbon and oxygen virtually burn in place. A new equilibrium between energy generation and transport is found on much smaller length scales, $l \approx 10^{-4}$ cm, where thermal conduction by degenerate electrons balances nuclear energy input (Timmes & Woosley 1992). The flame is born.

The evolution of the runaway immediately prior to ignition of the flame is crucial for determining its initial location and shape. Using a simple toy model, Garcia-Senz & Woosley (1995) found that under certain conditions, burning bubbles subject to buoyancy and drag forces can rise a few hundred km before flame formation, suggesting a high probability for off-center ignition at multiple, unconnected points. As a consequence, more material burns at lower densities, thus producing higher amounts of intermediate mass elements than a centrally ignited explosion. In a parameter study, Niemeyer et al. (1996) and Reinecke et al. (1999a) demonstrated the significant influence of the location and number of initially ignited spots on the final explosion energetics and nucleosynthesis.

5.1.3 PROMPT DETONATION

The first hydrodynamical simulation of an exploding $M_{\text{chan}}$-white dwarf (Arnett 1969) assumed that the thermonuclear combustion commences as a detonation wave, consuming the entire star at the speed of sound. Given no time to expand prior to being burned, the C+O material in this scenario is transformed almost completely into iron-peaked nuclei and thus fails to produce significant amounts of intermediate mass elements, in contradiction to observations (Filippenko 1997a, 1997b). It is for this reason that prompt detonations are generally considered ruled out as viable candidates for the SN Ia explosion mechanism.

In addition to the empirical evidence, the ignition of a detonation in the high density medium of the white dwarf core was argued to be an unlikely event. In spite of the smallness of the critical mass for detonation at $\rho \approx 2 \times 10^9$ g cm$^{-3}$ (Niemeyer & Woosley 1997; Khokhlov et al. 1997) and the correspondingly large number of critical volumes in the core ($\sim 10^{18}$), the stringent uniformity condition for the temperature gradient of the runaway region (Blinnikov & Khokhlov 1986, 1987) was shown to be violated even by the minute amounts of heat dissipated by convective motions (Niemeyer & Woosley 1997). A different argument against the occurrence of a prompt detonation in C+O white dwarf cores was given by Kriminski et al. (1998), who found that C+O detonations may be subject to self-quenching at high material densities ($\rho > 2 \times 10^7$ g cm$^{-3}$) (see also Imshennik et al. 1999).

5.1.4 PURE TURBULENT DEFLAGRATION

Once ignited (Sec. (5.1.2)), the subsonic thermonuclear flame becomes highly convoluted as a result of turbulence produced by the various flame instabilities (Sec. (5.1.1)). It continues to burn through the star until it either transitions into a detonation or is quenched by expansion. The key questions with regard to explosion modeling are: a) What is the effective turbulent flame speed $S_t$ as a function of time, b) Is the total amount of energy released during the deflagration phase enough to unbind the star and produce a healthy explosion, and c) Does the resulting ejecta composition and velocity agree with observations?

By far the most work has been done on 1D models, ignoring the multidimen-
sionality of the flame physics and instead parameterizing $S_t$ in order to answer b) and c) above (see Woosley & Weaver 1986a; Nomoto et al. 1996, for reviews). One of the most successful examples, model W7 of Nomoto et al. (1984), clearly demonstrates the excellent agreement of “fast” deflagration models with SN Ia spectra and light curves. $S_t$ has been parameterized differently by different authors, for instance as a constant fraction of the local sound speed (Höflich & Khokhlov 1996; Iwamoto et al. 1999), using time-dependent convection theory (Nomoto et al. 1976; Buchler & Mazurek 1975; Nomoto et al. 1984; Woosley et al. 1984), or with a phenomenological fractal model describing the multiscale character of the wrinkled flame surface (Woosley 1990; Woosley 1997b). All of these studies essentially agree that very good agreement with the observations is obtained if $S_t$ accelerates up to roughly 30 % of the sound speed. There remains a problem with the overproduction of neutron rich iron-group isotopes in fast deflagration models (Woosley et al. 1984; Thielemann, Nomoto, & Yokoi 1986; Iwamoto et al. 1999), but this may be alleviated in multiple dimensions (see below). Turning this argument around, Woosley (1997a) argues that $^{48}$Ca can only be produced by carbon burning in the very high density regime of a $M_{\text{ch}}$ white dwarf core, providing a clue that a few SNe Ia need to be $M_{\text{ch}}$ explosions igniting at $\rho \geq 2 \times 10^9$ g cm$^{-3}$. A slightly different approach to 1D SN Ia modeling was taken by Niemeyer & Woosley (1997), employing the self-similar growth rate of RT mixing regions (Sec. (5.1.1)) to prescribe the turbulent flame speed. Here, all the free parameters are fixed by independent simulations or experiments. The result shows a successful explosion, albeit short on intermediate mass elements, suggesting that the employed flame model is still too simplistic.

A number of authors have studied multidimensional deflagrations in exploding $M_{\text{ch}}$ -white dwarfs using a variety of hydrodynamical methods (Livne 1993; Arnett & Livne 1994a; Khokhlov 1995; Niemeyer & Hillebrandt 1995b; Niemeyer et al. 1996; Reinecke et al. 1999a). The problem of simulating subsonic flames in large-scale simulations has two aspects: the representation of the thin, propagating surface separating hot and cold material with different densities, and the prescription of the local propagation velocity $S_t(\Delta)$ of this surface as a function of the hydrodynamical state of the large-scale calculation with numerical resolution $\Delta$. The former problem has been addressed with artificial reaction-diffusion fronts in PPM (Khokhlov 1995; Niemeyer & Hillebrandt 1995b; Niemeyer et al. 1996) and SPH (Garcia-Senz et al. 1998), a PPM-specific flame tracking technique (Arnett 1997), and a hybrid flame capturing/tracking method based on level sets (Reinecke et al. 1999b) (see Fig. 3). Regarding the flame speed prescription, some authors assigned the local front propagation velocity assuming that the flame is laminar on unresolved scales $l < \Delta$ (Arnett & Livne 1994a), by postulating that $S_t(\Delta)$ is dominated by the terminal rise velocity of $\Delta$-sized bubbles (Khokhlov 1995), or by using Eq. (2) together with a subgrid-scale model for the unresolved turbulent kinetic energy providing $v(\Delta)$ (Niemeyer & Hillebrandt 1995b; Niemeyer et al. 1996; Reinecke et al. 1999a).

In most multidimensional calculations on stellar scales to date, the effective turbulent flame speed stayed below the required 30 % of the sound speed. The detailed outcome of the explosion is controversial; while some calculations show that the star remains gravitationally bound after the deflagration phase has ceased (Khokhlov 1995), others indicate that $S_t$ may be large enough to produce a weak but definitely unbound explosion (Niemeyer et al. 1996). These discrepancies can probably be attributed to differences in the description of the turbulent flame and
Figure 3: Snapshots of the temperature and the front geometry in a Chandrasekhar-mass deflagration model at 1.05s (from Reinecke et al. (1999c)). Shown are a model with “low” resolution ($256^2$) (upper figure) and one with three times higher resolution, respectively. Due to the larger surface area of the better resolved model it exploded, whereas the other one remained marginally bound.
to numerical resolution effects that plague all multidimensional calculations.

Niemeyer & Woosley (1997) and Niemeyer (1999) speculate about additional physics that can increase the burning rate in turbulent deflagration models, in particular multipoint ignition and active turbulent combustion (ATC), i.e. the generation of additional turbulence by thermal expansion within the turbulent flame brush. ATC can, in principle, explain the acceleration of $S_t$ up to some fraction of the sound speed (Kerstein 1996), but its effectiveness is so far unknown. Multipoint ‘ignition, on the other hand, has already been shown to significantly increase the total energy release compared to single-point ignition models (Niemeyer et al. 1996; Reinecke et al. 1999a). Furthermore, it allows more material to burn at lower densities, thus alleviating the nucleosynthesis problem of 1D fast deflagration models (Niemeyer et al. 1996).

We conclude the discussion of the pure turbulent deflagration scenario with a checklist of the model requirements summarized in Sec. (2.4). Assuming that some combination of buoyancy, ATC, and multipoint ignition can drive the effective turbulent flame speed to $\sim 30\%$ of the sound speed – which is not evident from multidimensional simulations – one can conclude from 1D simulations that pure deflagration models readily comply with all observational constraints. Most authors agree that $S_t$ decouples from microphysics on large enough scales and becomes dominated by essentially universal hydrodynamical effects, making the scenario intrinsically robust. A noteworthy exception is the location and number of ignition points that can strongly influence the explosion outcome and may be a possible candidate for the mechanism giving rise to the explosion strength variability. Other possible sources of variations include the ignition density and the accretion rate of the progenitor system (Umeda et al. 1999; Iwamoto et al. 1999). All of these effects may potentially vary with composition and metallicity and can therefore account for the dependence on the progenitor stellar population.

5.1.5 DELAYED DETONATION

Turbulent deflagrations can sometimes be observed to undergo spontaneous transitions to detonations (deflagration-detonation transitions, DDTs) in terrestrial combustion experiments (e.g., Williams 1985, pp. 217–219). Thus inspired, it was suggested that DDTs may occur in the late phase of a $M_{\rm chan}$-explosion, providing an elegant explanation for the initial slow burning required to pre-expand the star, followed by a fast combustion mode that produces large amounts of high-velocity intermediate mass elements (Khokhlov 1991a; Woosley & Weaver 1994a). Many 1D simulations have meanwhile demonstrated the capability of the delayed detonation scenario to provide excellent fits to SN Ia spectra and light curves (Woosley 1990; H"oflich & Khokhlov 1996), as well as reasonable nucleosynthesis products with regard to solar abundances (Khokhlov 1991b; Iwamoto et al. 1999). In the best fit models, the initial flame phase has a rather slow velocity of roughly one percent of the sound speed and transitions to detonation at a density of $\rho_{\rm DDT} \approx 10^7$ g cm$^{-3}$ (H"oflich & Khokhlov 1996; Iwamoto et al. 1999). The transition density was also found to be a convenient parameter to explain the observed sequence of explosion strengths (H"oflich & Khokhlov 1996).

Various mechanisms for DDT were discussed in the early literature on delayed detonations (see Niemeyer & Woosley (1997) and references therein). Recent investigations have focussed on the induction time gradient mechanism (Zeldovich et al. 1970; Lee, Knystautas, & Yoshikawa 1978), analyzed in the context of
SNe Ia by Blinnikov & Khokhlov (1986) and Blinnikov & Khokhlov (1987). It was realized by Khokhlov et al. (1997) and Niemeyer & Woosley (1997) that a necessary criterion for this mechanism is the local disruption of the flame sheet by turbulent eddies, or, in other words, the transition of the burning regime from “flamelet” to “distributed” burning (Sec. (5.1.1)). Simple estimates (Niemeyer & Kerstein 1997) show that this transition should occur at roughly $10^7$ g cm$^{-3}$, providing a plausible explanation for the delay of the detonation.

The critical length (or mass) scale over which the temperature gradient must be held fixed in order to allow the spontaneous combustion wave to turn into a detonation was computed by Khokhlov et al. (1997) and Niemeyer & Woosley (1997); it is a few orders of magnitude thicker than the final detonation front and depends very sensitively on composition and density.

The virtues of the delayed detonation scenario can again be summarized by completing the checklist of Sec. (2.4). It is undisputed that suitably tuned delayed detonations satisfy all the constraints given by SN Ia spectra, light curves, and nucleosynthesis. If $\rho_{\text{DDT}}$ is indeed determined by the transition of burning regimes – which in turn might be composition dependent (Umeda et al. 1999) – the scenario is also fairly robust and $\rho_{\text{DDT}}$ may represent the explosion strength parameter. Note that in this case, the variability induced by multipoint ignition needs to be explained away. If, on the other hand, thermonuclear flames are confirmed to be almost unquenchable, the favorite mechanism for DDTs becomes questionable (Niemeyer 1999). Moreover, should the mechanism DDT rely on rare, strong turbulent fluctuations one must ask about those events that fail to ignite a detonation following the slow deflagration phase which, on its own, cannot give rise to a viable SN Ia explosion. They might end up as pulsational delayed detonations or as unobservably dim, as yet unclassified explosions. Multidimensional simulations of the turbulent flame phase may soon answer whether the turbulent flame speed is closer to 1 % or 30 % of the speed of sound and hence decide whether DDTs are a necessary ingredient of SN Ia explosion models.

5.1.6 PULSATIONAL DELAYED DETONATION

In this variety of the delayed detonation scenario, the first turbulent deflagration phase fails to release enough energy to unbind the star which subsequently pulses and triggers a detonation upon recollapse (Nomoto et al. 1976; Khokhlov 1991b). This model was studied in 1D by Höflich & Khokhlov (1996) and Woosley (1997b) (who calls it “pulsed detonation of the first type”) and in 2D by Arnett & Livne (1994b). Höflich & Khokhlov (1996) report that it produces little $^{56}$Ni but a substantial amount of Si and Ca and may therefore explain very subluminous events like SN 1991bg. Woosley (1997b), using a fractal flame parameterization, also considered “pulsed deflagrations”, i.e. re-ignition occurs as a deflagration rather than a detonation, and “pulsed detonations of the second type” in which the burning also re-ignites as a flame but later accelerates and touches off a detonation. This latter model closely resembles the standard delayed detonation, whereas the former may or may not produce a healthy explosion, depending on the prescribed speed of the rekindled flame (Woosley 1997b).

Obtaining a DDT by means of the gradient mechanism is considerably more plausible after one or several pulses than during the first expansion phase (Khokhlov et al. 1997) as the laminar flame thickness becomes macroscopically large during the expansion, allowing the fuel to be preheated, and turbulence is significantly
enhanced during the collapse.

The "checklist" for pulsational delayed detonations looks similar to that of simple delayed detonations (see above), with somewhat less emphasis on the improbability for DDT. Some fine-tuning of the initial flame speed is needed to obtain a large enough pulse in order to achieve a sufficient degree of mixing, while avoiding to unbind the star in a very weak explosion (Niemeyer & Woosley 1997). Again, these “fizzles” may be very subluminous and may have escaped discovery. We finally note that all pulsational models are in conflict with multidimensional simulations that predict an unbound star after the first deflagration phase.

5.2 Sub-Chandrasekhar Mass Models

C+O white dwarfs below the Chandrasekhar mass do not reach the critical density and temperature for explosive carbon burning by accretion, and therefore need to be ignited by an external trigger. Detonations in the accreted He layer were suggested to drive a strong enough shock into the C+O core to initiate a secondary carbon detonation (Weaver & Woosley 1980; Nomoto 1980, 1982a; Woosley, Weaver, & Taam 1980; Sutherland & Wheeler 1984; Iben & Tutukov 1984). The nucleosynthesis and light curves of Sub-\(M_{\text{Ch}}\) models, also known as “helium ignitors” or “edge-lit detonations”, were investigated in 1D (Woosley & Weaver 1994b; Höflich & Khokhlov 1996) and 2D (Livne & Arnett 1995) and found to be superficially consistent with SNe Ia, especially subluminous ones (Ruiz-Lapuente et al. 1993a). Their ejecta structure is characterized almost inevitably by an outer layer of high-velocity Ni and He above the intermediate mass elements and the inner Fe/Ni core.

These models are favored mostly by the statistics of possible SN Ia progenitor systems (Yungelson & Livio 1998; Livio 2000) and by the straightforward explanation of the one-parameter strength sequence in terms of the WD mass (Ruiz-Lapuente, Burkert, & Canal 1995). However, they appear to be severely challenged both photometrically and spectroscopically: Owing to the heating by radioactive \(^{56}\text{Ni}\) in the outer layer they are somewhat too blue at maximum brightness and their light curve rises and declines too steeply (Höflich & Khokhlov 1996; Nugent et al. 1997; Höflich et al. 1997). Perhaps even more stringent is the generic prediction of He-ignitors to exhibit signatures of high-velocity Ni and He, rather than Si and Ca, in the early and maximum spectra which is in strong disagreement with observations (Nugent et al. 1997; Höflich et al. 1997).

With respect to the explosion mechanism itself, the most crucial question is whether and where the He detonation manages to shock the C+O core sufficiently to create a carbon detonation. 1D models, by virtue of their built-in spherical symmetry, robustly (and unphysically) predict a perfect convergence of the inward propagating pressure wave and subsequent carbon ignition near the core (Woosley & Weaver 1994b). Some 2D simulations indicate that the C+O detonation is born off-center but still due to the convergence of the He-driven shock near the symmetry axis of the calculation (Livne 1990; Livne & Glasner 1991) while others find a direct initiation of the carbon detonation along the circle where the He detonation intersects the C+O core (Livne 1997; Arnett 1997; Wiggins & Falle 1997; Wiggins, Sharpe, & Falle 1998). Using 3D SPH simulations, Benz (1997) failed to see carbon ignition in all but the highest resolution calculations, where carbon was ignited directly at the interface rather than by shock convergence. Further, C ignition is facilitated if the He detonation starts...
at some distance above the interface, allowing the build-up of a fully developed pressure spike before it hits the carbon (Benz 1997). This result was confirmed by recent 3D SPH simulations (Garcia-Senz, Bravo, & Woosley 1999) that also examined the effect of multiple He ignition points, finding enhanced production of intermediate mass elements in this case. Hence, multidimensional SPH and PPM simulations presently confirm the validity of He-driven carbon detonations, in particular by direct ignition, but they also demonstrate the need for very high numerical resolution in order to obtain mutually consistent results (Arnett 1997; Benz 1997).

To summarize, sub-$M_{\text{chan}}$ models are most severely constrained by their prediction of an outer layer of high-velocity Ni and He. Should further research conclude that spectra, colors, and light curves are less contaminated by this layer than presently thought, they represent an attractive class of candidates for SNe Ia, especially subluminous ones, from the point of view of progenitor statistics and the one-parameter explosion strength family. Note, however, that the SN Ia luminosity function in this scenario is directly linked to the distribution of white dwarf masses, predicting a more gradual decline on the bright side of the luminosity function than indicated by observations (Vaughan et al. 1995; Livio 2000). The explosion mechanism itself appears realistic, at least in the direct carbon ignition mode, but more work is needed to firmly establish the conditions for ignition of the secondary carbon detonation.

5.3 Merging White Dwarfs

The most obvious strength of the merging white dwarfs, or double-degenerate, scenario for SNe Ia (Webbink 1984; Iben & Tutukov 1984; Paczynski 1985) is the natural explanation for the lack of hydrogen in SN Ia spectra (Livio 2000) (cf. Sec. (2.1)). Furthermore, in contrast to the elusive progenitor systems for single degenerate scenarios, there is meanwhile some evidence for the existence of double degenerate binary systems (Saffer et al. 1998) despite earlier suspicions to the contrary (e.g., Bragaglia 1997). These systems are bound to merge as a consequence of gravitational wave emission with about the right statistics (Livio 2000) and give rise to some extreme astrophysical event, albeit not necessarily a SN Ia.

Spherically symmetric models of detonating merged systems, parameterized as C+O white dwarfs with thick envelopes, were analyzed by Höflich, Khokhlov, & Müller (1992); Khokhlov et al. (1993) and Höflich & Khokhlov (1996), giving reasonable agreement with SN Ia light curves. 3D SPH simulations of white dwarfs mergers (Benz et al. 1990; Rasio & Shapiro 1995; Mochkovitch, Guerrero, & Segretain 1997) show the disruption of the less massive star in a matter of a few orbital times, followed by the formation of a thick hot accretion disk around the more massive companion. The further evolution hinges crucially on the effective accretion rate of the disk: In case $\dot{M}$ is larger than a few times $10^{-6} \, M_\odot \, \text{yr}^{-1}$, the most likely outcome is off-center carbon ignition leading to an inward propagating flame that converts the star into O+Ne+Mg (Nomoto & Iben 1985; Saio & Nomoto 1985; Kawai, Saio, & Nomoto 1987; Timmes, Woosley, & Taam 1994; Saio & Nomoto 1998). This configuration, in turn, is gravitationally unstable owing to electron capture onto $^{24}\text{Mg}$ and will undergo accretion-induced collapse (AIC) to form a neutron star (Saio & Nomoto 1985; Mochkovitch & Livio 1990; Nomoto & Kondo 1991). A recent re-examination of Coulomb corrections to the
equation of state of material in nuclear statistical equilibrium indicates that AIC in merged white dwarf systems is even more likely than previously anticipated (Bravo & Garcia-Senz 1999).

Dimensional analysis of the expected turbulent viscosity due to MHD instabilities (Balbus, Hawley, & Stone 1996) suggests that it is very difficult to avoid such high accretion rates (Mochkovitch & Livio 1990; Livio 2000). Even under the unphysical assumption that angular momentum transport is dominated entirely by microscopic electron-gas viscosity, the expected life time of \( \sim 10^9 \) yrs (Mochkovitch & Livio 1990; Mochkovitch et al. 1997) and high UV luminosity of these accretion systems would predict the existence of \( \sim 10^7 \) such objects in the Galaxy, none of which have been observed (Livio 2000).

A possible solution to the collapse problem is to ignite carbon burning as a detonation rather than a flame immediately during the merger event, either in the core of the more massive star (Shigeyama et al. 1992) or at the contact surface (D Arnett & PA Pinto, private communication). This alternative clearly warrants further study.

To summarize, the merging white dwarf scenario has to overcome the crucial problem of avoiding accretion-induced collapse before it can be seriously considered as a SN Ia candidate. Its key strengths are a plausible explanation for the progenitor history yielding reasonable predictions for SN Ia rates, the straightforward explanation of the absence of H and He in SN Ia spectra, and the existence of a simple parameter for the explosion strength family (i.e., the mass of the merged system).

6 SUMMARY

In this review we have outlined our present understanding of Type Ia supernovae, summarizing briefly the observational constraints, but putting more weight on models of the explosion. From the tremendous amount of work carried out over the last couple of years it has become obvious that the physics of SNe Ia is very complex, ranging from the possibility of very different progenitors to the complexity of the physics leading to the explosion and the complicated processes which couple the interior physics to observable quantities. None of these problems is fully understood yet, but what one is tempted to state is that, from a theorist’s point of view, it appears to be a miracle that all the complexity seems to average out in a mysterious way to make the class so homogeneous. In contrast, as it stands, a save prediction from theory seems to be that SNe Ia should get more divers with increasing observed sample sizes. If, however, homogeneity would continue to hold this would certainly add support to the Chandrasekhar-mass single-degenerate scenario. On the other hand, even an increasing diversity would not rule out Chandrasekhar-mass single-degenerate progenitors for most of them. In contrast, there are ways to explain how the diversity is absorbed in a one parameter family of transformations, such as the Phillips-relation or modifications of it. For example, we have argued that the size of the convective core of the white dwarf prior to the explosion might provide a physical reason for such a relation.

As far as the explosion/combustion physics and the numerical simulations are concerned significant recent progress has made the models more realistic (and reliable). Thanks to ever increasing computer resources 3-dimensional simulations
have become feasible which treat the full star with good spatial resolution and realistic input physics. Already the results of 2-dimensional simulations indicate that pure deflagrations waves in Chandrasekhar-mass C+O white dwarfs can lead to explosions, and one can expect that going to three dimensions, because of the increasing surface area of the nuclear flames, should add to the explosion energy. If confirmed, this would eliminate pulsational detonations from the list of potential models. On the side of the combustion physics, the burning in the distributed regime at low densities needs to be explored further, but it is not clear anymore whether a transition from a deflagration to a detonation in that regime is needed for successful models. In fact, according to recent studies such a transition appears to be rather unlikely.

Finally, sub-Chandrasekhar-mass models seem to face problems, both from the observations and from theory, leaving us with the conclusion that we seem to be lucky and Nature was kind to us and singled out from all possibilities the simplest solution, namely a Chandrasekhar-mass C+O white dwarf and a nuclear deflagration wave, to make a Type Ia supernova explosion.

Literature Cited


Lundmark K. 1921. *PASP* 33:234


Minkowski R. 1940. *PASP* 52:206


Popper DM. 1937. PASP 49:283
Read CI. 1984. Physica D 12:45
Type Ia Supernova Explosion Models