A Brane World Model with Intersecting Branes

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ABSTRACT

A brane world model is investigated, in which there are many branes that may intersect and self intersect. One of the branes, being a 3-brane, represents our spacetime, while the other branes, if they intersect our brane world, manifest themselves as matter in our 3-brane. It is shown that such a matter encompasses dust of point particles and higher dimensional \( p \)-branes, and all those objects follow "geodesics" in the world volume swept by our 3-brane. We also point point out that such a model can be formulated in a background independent way, and that the kinetic term for gravity arises from quantum fluctuation of the brane.

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1 Introduction

The idea that our world is a 3-brane embedded in a higher dimensional bulk space is not new [1]-[3]. Recently it attracted much attention, since Randall and Sundrum [4] have found that gravity can be localized on a brane. Such a property results as a solution to Einstein’s equations around a static 3-brane embedded in 5-dimensional space with negative cosmological constant. Matter including the fields of the standard model is confined to the brane, while gravity propagates in the bulk, but in the Randall-Sundrum scenario gravity turns out to be effectively localized on the brane too.

Besides the research exploring the properties of various ”brane world” scenarios based on the classical Einstein equations, there is also a lot of activity aiming at formulating a consistent theory of quantum gravity. In the works by Rovelli, Smolin and Baez [5], the need was stressed that a really fundamental theory should be background independent. This means that spacetime together with its metric should emerge from the properties of some more basic objects. The basic objects could be spin networks [6], spin foams, or perhaps strings and various branes. A background independent theory of $p$-branes should be formulated without using the concept of a preexisting embedding space and metric. A configuration of branes is all what exists in such an approach. There is no embedding space. If there are many such branes, then they are supposed to form, up to a good approximation, the embedding space. The latter space is in fact identified with such a configuration of many branes.

We shall gradually build up the model. First we shall assume that we have a brane, representing a world (a brane world for short), moving in a background space $V_N$. Then we shall assume that $V_N$ is conformally flat. We shall take a special conformally flat metric, such that it is singular on a set of branes. Then we shall observe that the intersections of all those branes with some chosen brane behave as matter on that brane. Such a matter consists of $p$-branes of various dimensionalities and their equations of motions turn out to be those of minimal surface (geodesic in particular, when $p = 0$) in the brane world metric. It is also possible that a brane world intersects itself. Quantum fluctuations of such a brane induce the Einstein-Hilbert action in the brane world and also quantum fluctuations of matter.
2 The brane in a bulk with a singular conformally flat metric

Let us consider a brane moving in a curved background embedding space \( V_N \), called \( \text{bulk} \). Such a brane sweeps an \( n \)-dimensional surface which I call \( \text{worldsheet} \). The dynamical principle governing motion of the brane requires that its worldsheet be a minimal surface. Hence the action is

\[
I[\eta^a] = \int \sqrt{f} \, d^n x
\]

where

\[
f \equiv \det f_{\mu\nu} , \quad f_{\mu\nu} \equiv \partial_\mu \eta^a \partial_\nu \eta^b \gamma_{ab}
\]

Here \( x^\mu, \mu = 0, 1, 2, ..., n - 1 \) are coordinates on the worldsheet \( V_n \), while \( \eta^a(x) \) are the embedding functions. The metric of the embedding space (from now on called also \( \text{bulk} \)) is \( \gamma_{ab} \), and the induced metric on \( V_N \) is \( \bar{f}_{\mu\nu} \).

Suppose now that the metric of \( V_N \) is conformally flat \([3, 7]\) (with \( \eta_{ab} \) being the Minkowski metric tensor):

\[
\gamma_{ab} = \phi \eta_{ab}
\]

Then from (2) we have

\[
\bar{f}_{\mu\nu} = \phi \partial_\mu \eta^a \partial_\nu \eta^b \eta_{ab} \equiv \phi f_{\mu\nu}
\]

\[
\bar{f} \equiv \det \bar{f}_{\mu\nu} = \phi^n \det f_{\mu\nu} \equiv \phi^n f
\]

\[
\sqrt{|\bar{f}|} = \omega |f| , \quad \omega \equiv \phi^{n/2}
\]

Hence the action (1) reads

\[
I[\eta^a] = \int d^n x \omega(\eta) \sqrt{\bar{f}} d^n x
\]

which looks like an action for a brane in a flat embedding space, except for a function \( \omega(\eta) \) which depends on position \( \eta^a \) in the embedding space \( V_N \).

Function \( \omega(\eta) \) is related to the fixed background metric \( \gamma_{ab} \) which is arbitrary in principle. Let us now assume that \( \omega(\eta) \) consists of a constant part \( \omega_0 \) plus a singular

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2 Usually, when \( n > 2 \), such a surface is called \( \text{world volume} \). Here I prefer to retain the name \( \text{worldsheet} \), by which we can vividly imagine a surface in an embedding space.

3 We use here the same symbol \( \eta^a \) either for position coordinates in \( V_N \) or for the embedding functions \( \eta^a(x) \).
part with support on a set of surfaces $V_{m_j}$, of dimension $m_j$ and described by embedding functions $\eta_j^a(x_j^\mu)$, denoted $\eta_j$ for short:

$$\omega(\eta) = \omega_0 + \sum_j \int \kappa_j \delta^N(\eta - \eta_j) \sqrt{|f_j|} \, dx_j$$  \hspace{1cm} (8)

Here $dx_j\sqrt{|f_j|} \equiv d^{m_j} x_j \sqrt{|f(x_j)|}$ is the invariant volume element on $V_{m_j}$.

Inserting (8) into (7) we obtain an action which contains the kinetic term for the worldsheet $V_n$ and an interactive term between $V_n$ and $V_{m_j}$:

$$I[\eta] = \int \omega_0 \sqrt{|f|} \, dx + \sum_j \int \kappa_j \delta^N(\eta - \eta_j) \sqrt{|f|} \sqrt{|f_j|} \, dx \, dx_j$$  \hspace{1cm} (9)

The set of worldsheets $V_{m_j}$ form a background with the singular conformally flat metric, given by (3),(6) and (8), in which the worldsheet $V_n$ lives.

3 A system of many intersecting branes

Now we shall assume that $V_{m_j}$ are dynamical too. Therefore we add a corresponding kinetic term to the action. So we obtain an action for a system of intersecting branes $\eta_i$, $i = 1, 2, \ldots$:

$$I[\eta_i] = \sum_i \int \omega_0 \sqrt{|f_i|} \, dx_i + \frac{1}{2} \sum_{ij} \int \omega_{ij} \delta^N(\eta_i - \eta_j) \sqrt{|f_i|} \sqrt{|f_j|} \, dx_i \, dx_j$$  \hspace{1cm} (10)

Besides the kinetic term for free branes, our action (10) contains also the interactive terms. The interactions result from the intersections of the branes.

The equations of motion for the $i$-th brane are

$$\partial_\mu \left[ \sqrt{|f_i|} \partial^\mu \eta_i^a \left( \omega_0 + \sum_{i \neq j} \int \omega_{ij} \delta^N(\eta_i - \eta_j) \sqrt{|f_j|} \, dx_j \right) \right] = 0$$  \hspace{1cm} (11)

The same equations (with the identification $\eta \equiv \eta_i$, $\kappa_j \equiv \omega_{ij}$) follow also from (9). However, with (10) we have a self consistent system, where each brane determines the motion of all the others.

Returning now to the action (9) experienced by one of the branes whose worldsheet is represented by $\eta_i^a(x_i) \equiv \eta^a(x)$, we find after integrating out $x_j$, $j \neq i$ that

$$I[\eta] = \omega_0 \int d^n x \sqrt{|f|} + \sum_j \kappa_j \int d^n x \, d^{n+1} \xi \left( \det \partial^A X^\mu_j \partial_B X^\nu_j f_{\mu\nu} \right)^{1/2} \delta^n(x - X_j(\xi))$$  \hspace{1cm} (12)
For various $p_j$, the latter expression is an action for a system of point particles ($p_j = 0$), strings ($p_j = 1$), and higher dimensional branes ($p_j = 2, 3, ...$), described by $X^\mu_j(\xi)$, moving in the background metric $f_{\mu\nu}$, which is the induced metric on our brane $V_n$ (see Fig.1)

Figure 1: The intersection between two different branes $V_n$ and $\hat{V}_m$ can be a $p$-brane $V_{p+1}$.

We see that the interactive term in (10) manifests itself in various ways, depending on how we look at it. It is a manifestation of the fact that the metric of the embedding space is curved (in particular, the metric is singular on the system of branes). From the point of view of a chosen brane $V_n$ the interactive term becomes the action for a system of $p$-branes (including point particles) moving on $V_n$. If we now adopt the brane world view, where $V_n$ is our spacetime, we see that matter on $V_n$ comes from other branes’s worldsheets which happen to intersect our worldsheet $V_n$. Those other branes, in turn, are responsible for the non trivial metric of the embedding space.

3.1 The brane interacting with itself

In (9) or (10) we have a description of a brane interacting with other branes. What about self interaction? In the second term of the action (9),(10) we have excluded self interaction. In principle we should not exclude self interaction, since there is no reason why a brane could not interact with itself.

Let us return to the action (9) and let us calculate $\omega(\eta)$, this time assuming for simplicity that there is only one brane $V_{m_j} \equiv \hat{V}_m$ which coincides with our brane $V_n$.  

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Hence the intersection is the brane \( V \) itself, and according to (8) we have

\[
\omega(\eta) = \omega + \kappa \int d^n \hat{x} \sqrt{|\hat{f}|} \delta^N (\eta - \hat{\eta}(\hat{x}))
\]

\[
= \omega + \kappa \int d^n \xi \sqrt{|\hat{f}|} \delta^N (x - X(\xi))
\]

\[
= \omega + \kappa \int d^n x \delta^N (x - X(x)) = \omega + \kappa
\]  

(13)

Here the coordinates \( \xi^A, A = 0, 1, 2, ..., n - 1 \) cover the manifold \( V_n \), and \( \hat{f}_{AB} \) is the metric of \( V_n \) in coordinates \( \xi^A \). The other coordinates are \( x^\mu, \mu = 0, 1, 2, ..., n - 1 \). In the last step in (13) we have used the property that the measure is invariant, \( d^n \xi \sqrt{|\hat{f}|} = d^n x \sqrt{|f|} \).

The result (13) demonstrates that we do not need to separate a constant term \( \omega_0 \) from the function \( \omega(\eta) \). For a brane moving in a background of many branes we can replace (8) with

\[
\omega(\eta) = \sum_j \kappa_j \delta^N (\eta - \eta_j) \sqrt{|f_j|} d\xi_j
\]  

(14)

where \( j \) runs over all the branes within the system. Any brane feels the same background, and its action for a fixed \( i \) is

\[
I[\eta_i] = \int \omega(\eta) \sqrt{|f_i|} d\xi_i = \sum_j \kappa_j \delta^N (\eta_i - \eta_j) \sqrt{|f_i| \sqrt{|f_j|}} d\xi_i d\xi_j
\]  

(15)

However the background is self consistent: it is a solution to the variational principle given by the action

\[
I[\eta_i] = \sum_{i \geq j} \omega_{ij} \delta^N (\eta_i - \eta_j) \sqrt{|f_i| \sqrt{|f_j|}} d\xi_i d\xi_j
\]  

(16)

where now also \( i \) runs over all the branes within the system; the case \( i = j \) is also allowed.

In (16) the self interaction or self coupling occurs whenever \( i = j \). The self coupling term of the action is \( (\kappa_i = \omega_{ii}) \)

\[
I_{\text{self}}[\eta_i] = \sum_i \kappa_i \int \delta^N (\eta(x_i) - \eta_i(x'_i)) \sqrt{|f_i(x_i)| \sqrt{|f_i(x'_i)|}} d\xi_i d\xi'_i
\]

\[
= \sum_i \kappa_i \int \delta^N (\eta - \eta_i(x_i)) \delta^N (\eta - \eta_i(x'_i)) \sqrt{|f_i(x_i)| \sqrt{|f_i(x'_i)|}} d\xi_i d\xi'_i d^N \eta
\]

\[
= \sum_i \kappa_i \int \delta^N (\eta - \eta_i(x_i)) \delta^N (x_i - x'_i) \sqrt{|f_i(x_i)|} d\xi_i d\xi'_i d^N \eta
\]

\[
= \sum_i \kappa_i \sqrt{|f_i(x_i)|} d^N x_i
\]  

(17)

where we have used the same procedure which led us to eq.(13). We see that the interactive action (16) automatically contains also the minimal surface terms, so that they do not need to be postulated separately.
3.2 A system of many branes creates bulk and its metric

We can now imagine that a system of branes (a brane configuration) can be identified with the embedding space in which a single brane moves. Here we have a concrete realization of that idea. We have a system of branes which intersect. The only interaction between the branes is due to intersection ("contact interaction"). The interaction at the intersection influences the motion of a (test) brane: it feels a potential because of the presence of other branes. If there are many branes and a test brane moves in the midst of them, then on average it feels a metric field which is approximately continuous. Our test brane moves in an effective metric of the embedding space.

A single brane or several branes give the singular conformal metric. Many branes are expected to give, on average, an arbitrary metric.

Figure 2: A system of many intersecting branes creates the bulk metric. In the absence of the branes there is no bulk (no embedding space).

There is a close interrelationship between the presence of branes and the bulk metric. In the model we discuss here the bulk metric is singular on the branes, and zero elsewhere. Without the branes there is no metric and no bulk. Actually the bulk consists of the branes which determine its metric.

Something quite analogous occurs in string theory, more precisely, in the theory of closed superstrings. Although classically the string theory is formulated in a background spacetime with a fixed (Minkowski) metric, it turns out after quantization that the background metric in which the quantum string moves cannot be fixed. The string itself
determines what are the equations of motion for the metric of the embedding space (the target space in the string theoretic jargon). This could be intuitively understood by noticing that the quantum string automatically involves many strings. A generic quantum state is a many strings state and effectively it leads to gravity in target space. What is still not quite satisfactory in string theory is its background dependent starting point, namely use of the Minkowski metric. I believe that the many intersecting brane model (which, of course, includes also strings) resolves the issue of background independence at the classical level, since the action (16), which includes also self interaction, contains no metric of a background embedding space. It is true that in (16) we have the quantity \( \sqrt{f_i} \), \( f_i \equiv \det f_{\mu\nu} \), where \( f_{\mu\nu} \equiv \partial_{\mu} \eta^a_i \partial_{\nu} \eta^b_i \eta_{ab} \). The latter quantity is the metric on the \( i \)-th brane worldsheet, but now it can not be considered as a metric induced from an embedding space metric, since according to (14), (3) and (6) the latter metric vanishes outside the branes. Hence in effect there is no embedding space, apart from the system of branes itself. The fixed quantity \( \eta_{ab} \) can even less be interpreted as a metric of an embedding space. It is the Minkowski metric of the flat space to which there corresponds a conformally flat space which, because of the singular conformal factor, is identified with the system of branes. The latter conformally flat space is our dynamical system, but the former flat space (with metric \( \eta_{ab} \)) is not, and therefore can hardly be considered as a background space for our dynamical system of branes.

4 The origin of matter in brane world

Our principal idea is that we have a system of branes (a brane configuration). With all the branes in the system we associate the embedding space (bulk). One of the branes (more precisely, its worldsheet) represents our spacetime. Interactions between the branes (occurring at the intersections) represent matter in spacetime.

4.1 Matter from the intersection of our brane with other branes

We have seen that matter in \( V_n \) naturally occurs as a result of the intersection of our worldsheet \( V_n \) with other worldsheets. We obtain exactly the stress-energy tensor for dust of point-particles, or \( p \)-branes in general. Namely, varying the action (12) with
With respect to $\eta^a(x)$ we obtain

$$\omega_0 D_\mu D^\mu \eta_a + D_\mu ( T^{\mu\nu} \partial_\nu \eta_a ) = 0 \quad (18)$$

with

$$T^{\mu\nu} = \sum_j \int \int d^{p_j+1}\xi \left( \det \partial_A X_j^\mu \partial_B X_j^\nu f_{\mu\nu} \right)^{1/2} \frac{\delta^n(x - X_j(\xi))}{\sqrt{|f|}} \quad (19)$$

being the stress-energy tensor for a system of $p$-branes (which are the intersections of $V_n$ with the other worldsheets). By $D_\mu$ we denote the covariant derivative with respect to the world sheet metric $f_{\mu\nu} \equiv \partial_\mu \eta^a \partial_\nu \eta_a$. The above expression for $T^{\mu\nu}$ holds if the extended objects have any dimensions $p_j$. In particular, when all objects have $p_j = 0$ (point-particles) eq.(19) becomes

$$T^{\mu\nu} = \sum_j \kappa_j \int d\tau \frac{\dot{X}_\mu \dot{X}_\nu \delta(x - X(\tau))}{\sqrt{X^2}} \frac{1}{\sqrt{|f|}} \quad (20)$$

From the equations of motion (18) we obtain

$$D_\mu T^{\mu\nu} = 0 \quad (21)$$

which implies that any of the objects sweeps a minimal surface $V_{p+1}$ in $V_n$. When $p_j = 0$ we have a geodesic in $V_n$.

### 4.2 Matter from the intersection of our brane with itself

Our model of intersecting branes allows for the possibility that a brane intersects itself, as schematically illustrated in Fig. 3.

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4We contract (18) by $\partial_\nu \eta^a$ and take into account the identity $D_\alpha \eta_a \partial_\nu \eta^a = 0$. 

Figure 3: Illustration of a self-intersecting brane. At the intersection $V_{p+1}$, because of the contact interaction, the stress-energy tensor on the brane $V_{n}$ is singular, and it manifests itself as matter on $V_{n}$. The manifold $V_{p+1}$ is a worldsheet swept by a $p$-brane and it is a minimal surface (e.g. a geodesic, when $p = 0$) in $V_{n}$.

The analysis used so far is valid also for the situations like the one in Fig. 3, if we divide the worldsheet $V_{n}$ in two pieces which are glued together at a submanifold $C$, situated somewhere within the "loop" region.

There is a variety of ways a worldsheet can self intersect. For instance, It may intersect itself many times to form a sort of helix or spiral. Instead of the intersection with a single loop, like in Fig. 4, the intersection may form a double or triple loop (Fig. 4).
In this respect some interesting new possibilities occur, waiting to be explored in detail. For instance, it is difficult to imagine how the three particles entangled in the topology of the situation in Fig. 4 could be separated to become asymptotically free. Hence this might be a possible classical model for hadrons composed of quarks; the extra dimensions of $V_n$ would bring, via Kaluza-Klein mechanism, the chromodynamic force into the action. Moreover, the topology of the model appears to be chiral.

In summary, it is obvious that a self intersecting brane can provide a variety of matter configurations on the brane. This is a fascinating and intuitively clear mechanism for the origin of matter in a brane world.
5 Discussion and conclusion

The curvature scalar does not occur in the brane world action (10). In previous publications [7, 11] we have noticed that the Einstein-Hilbert action on the brane’s worldsheet can be induced from quantum fluctuations of the brane. A similar model had also been considered within the idea of Sakharov’s induced gravity [8] in refs.[9]. Instead of the action (10) it is convenient to take another, classically equivalent action, which is not only a functional of the embedding functions \( \eta^a(x) \), but also of the induced metric \( g_{\mu\nu} \). Such an action is known under the name sigma model action or the Howe-Tucker action [10]. The quantization of the latter action enables one to express an effective action as a functional of \( g_{\mu\nu} \). The effective action is obtained in the Feynman functional integral in which we functionally integrate over \( \eta^a(x) \), so that what remains is a functional dependence on the induced metric \( g_{\mu\nu} \). This effective action contains the Ricci scalar \( R \) and its higher orders. Therefore the field equations contain the Einstein equations on the brane worldsheet and the terms arriving from higher orders of \( R \). In other words, in the effective theory obtained after performing the quantum average over various branes with the same induced metric \( g_{\mu\nu} \), the latter approximately satisfies the Einstein equations. If having not a single brane action but an action, like (10), for a system of many branes which can intersect, and self intersect, then we obtain on a chosen brane the matter term for point-particle and higher \( p \)-brane sources. Quantum fluctuations of the 3-brane render the state of those matter sources to behave as quantum sources. In the case of (bosonic) point particles the latter source turns out to be just the usual action for a scalar field [7, 11]. A generalization to fermionic branes and hence to fermionic sources on the world brane has not yet been explicitly constructed, but I expect it should be straightforward, starting from the existing knowledge of superstrings and supersymmetry.

Nowadays there is a lot of activity in the so called ”brane world” scenario, but its full power seems not yet been entirely appreciated. Conventionally, strings and higher \( p \)-branes are considered as extended relativistic objects in spacetime which necessarily has more than 4 dimensions (e.g. 26 for bosonic strings). Then there arises a problem of how to compactify all those extra dimensions. Various ingenious models and methods are being investigated. I prefer to adopt an alternative view, namely, that a 4-dimensional worldsheet swept by a 3-brane already represents spacetime. Hence, no compactification
of dimensions of the embedding space (called also the target space or bulk) $V_N$ is needed, since the latter space is not our spacetime. Moreover, in effect an embedding space was shown to be identical with the system of many branes. One of those branes is our world (brane world), while the other branes, if they intersect our brane, are manifested as matter in our world. So the other branes can have a physical influence on our world as well.

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References


