The one-pion-exchange three-nucleon force and the \( A_y \) puzzle

L. Canton\(^1,2\) and W. Schadow\(^3\)

\(^1\)Istituto Nazionale di Fisica Nucleare, Sez. di Padova, Via F. Marzolo 8, Padova I-35131, Italy
\(^2\)Physics Department, University of Manitoba, Winnipeg, MB, Canada R3T 2N2
\(^3\)TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

(March 30, 2001)

We consider a new three-nucleon force generated by the exchange of one pion in the presence of a 2N correlation. The underlying irreducible diagram has been recently suggested by the authors as a possible candidate to explain the puzzle of the vector analyzing powers \( A_y \) and \( iT_1 \) for nucleon-deuteron scattering. Herein, we have calculated the elastic neutron-deuteron differential cross section, \( A_y, iT_{11}, T_{20}, T_{21}, \) and \( T_{22} \) below break-up threshold by accurately solving the Alt-Grassberger-Sandhas equations with realistic interactions. We have also studied how \( A_y \) evolves below 30 MeV. The results indicate that this new 3NF diagram provides one possible additional contribution, with the correct spin-isospin structure, for the explanation of the origin of this puzzle.

PACS numbers: 24.70+s, 21.30.Ch, 25.10.+s, 25.40.Dn, 21.45.+v, and 13.75.Cs

The \( A_y \) puzzle (or, more appropriately, the puzzle of the vector analyzing powers) is probably the most famous of the open problems in 3N scattering at low energy. The problem with this observable has been observed quite early at the Tokio & Sendai Conference (Few-Body XI, 1986) \([1]\), since at that time the first reliable Faddeev calculations with realistic 2N potentials were becoming available, thanks in particular to the employment of separable expansion methods which transform the 3N scattering equations of Alt, Grassberger, and Sandhas (AGS) \([2]\) into an effective, multichannel Lippmann-Schwinger equation. This method of calculation has been pushed forward to obtain accurate results particularly by the Graz group \([3]\). Since then, various alternative methods of solution of the 3N scattering equation have been developed and tested \([4]\), and outstanding progresses have been made in the computational techniques in order to: 1) include three-nucleon forces (3NF) in the 3N scattering equations \([4-6]\); 2) treat explicitly the \( \Delta \) dynamics in the 3N system \([7]\); 3) provide a combined description of the 3N dynamics with Coulomb, realistic 2N, and phenomenological 3N forces \([8]\).

The puzzle was confirmed by these new approaches, and it turned out that the existing 2\(\pi\)-3NF \([9-11]\) provided a too small effect for \( A_y \) \([4,5,8]\), and not always in the right direction. In absence of new 3NF’s that could explain the puzzle, and since the 3N \( A_y \) is rather sensitive to the \( ^3P_1 \) \( NN \) phase shifts, it was concluded in Ref. \([12]\) (see also references therein) that such phases and the associated \( NN \) potentials derived from modern phase-shift analysis must be modified at low energy. These modifications can be achieved without affecting appreciably the 2N data because the low-energy 2N observables cannot resolve the \( ^3P_1 \) phases uniquely due to the Fermi-Yang ambiguities. However, as has been argued in Ref. \([13]\), it is not possible to increase the 3N \( A_y \) with reasonable changes in the \( NN \) potential, hence additional 3NF’s of new structure have to be considered. Recently, an attempt has been made \([14]\) using a purely phenomenological 3NF of spin-orbit type, constructed \textit{ad hoc} to affect only the triplet-odd states of 2N subsystem. 3NF terms of pion-range/short-range form \([15]\) have been reconsidered lately from the point of view of Chiral Perturbation Theory \([16] (\chiPT)\), which predicts for these terms a non-negligible role. Qualitatively, it was found that these terms somewhat affect \( A_y \), but a quantitative conclusion could not be derived.

It is our intention to indicate here a possible solution of this puzzle in terms of a new 3NF proposed recently by the authors \([17]\). This force is generated by the one-pion-exchange diagram when one of the two nucleons involved in the exchange process rescatters with a third one while the pion is “in flight”. The underlying diagram has been derived starting from a formalism \([18]\) devoted to the explicit treatment of the pion dynamics in the 3N system. The resulting dynamical equation resembles a Faddeev-AGS equation, but entails in its inner structure the full complexity of the underlying four-body \((\pi NNN)\) system. A gradual procedure to project out the pion degrees of freedom has been discussed in Ref. \([19]\), where it has been shown that this formalism leads to irreducible 3NF diagrams, and includes in particular the 3NF we will evaluate here.

3NF diagrams of the type derived in Ref. \([17]\) and analyzed here are not new in the literature \([20-22]\), but they have been discarded in modern few-nucleon calculations because of the presence of a cancellation effect which has been observed in Refs. \([23,24]\) and discussed later from the point of view of effective chiral Lagrangians \([25]\). This cancellation involves meson retardation effects of the iterated Born term, and the irreducible diagrams generated by sub-summing all time orderings involving the combined exchange of two mesons amongst the three nucleons. However, this cancellation is incomplete \([17]\) and generates a three-nucleon force through a subtraction term of the type “\(t-v\)” with the 2N \( t\)-matrix being
The full 3NF is given by summing over the cyclic permutations of the nucleons, \( V_{3N} = V_{3N}^1 + V_{3N}^2 + V_{3N}^3 \). The momenta \( p, q \) represent respectively the Jacobi coordinates of the pair “12”, and spectator “3”, while \( E \) is the 3N energy. We have set the pion-nucleon coupling constant to the traditional value \( f_{\pi NN}^2/(4\pi) = 0.078 \), and have employed standard form-factors of monopole type to describe the effective, composite nature of the meson-baryon coupling:

\[
V_{3N}(p, q, p', q'; E) = \frac{f_{\pi NN}^2(Q)}{m_\pi^2} \frac{1}{(2\pi)^3} \left( (\sigma_1 \cdot Q)(\sigma_3 \cdot Q)(\tau_1 \cdot \tau_3) + (\sigma_2 \cdot Q)(\sigma_3 \cdot Q)(\tau_2 \cdot \tau_3) \right)
\]

The explicit expression of this force \[17\] is

\[
V_{3N}(p, q, p', q'; E) = \frac{f_{\pi NN}^2(Q)}{m_\pi^2} \frac{1}{(2\pi)^3} \left( (\sigma_1 \cdot Q)(\sigma_3 \cdot Q)(\tau_1 \cdot \tau_3) + (\sigma_2 \cdot Q)(\sigma_3 \cdot Q)(\tau_2 \cdot \tau_3) \right)
\]

\[
+ \frac{f_{\pi NN}^2(Q)}{m_\pi^2} \frac{1}{(2\pi)^3} \left( \tilde{t}_{12}(p, p'; E - \frac{Q^2}{2\pi} - m_\pi) \right)
\]

\[
\times \left( (\sigma_1 \cdot Q)(\sigma_3 \cdot Q)(\tau_1 \cdot \tau_3) + (\sigma_2 \cdot Q)(\sigma_3 \cdot Q)(\tau_2 \cdot \tau_3) \right).
\]

The chosen 2N model interaction implicitly determines the value of pion-nucleon cut-off in \( V_{3N} \), e.g. \( \Lambda_\pi = 1.7 \) GeV for the Bonn B potential. The transferred momentum \( Q = q' - q \) enters also in \( \omega_\pi = \sqrt{m_\pi^2 + Q^2} \). \( \tilde{t}_{ij} \) denotes the subtracted \( t \)-matrix between nucleons 1 and 2, defined according to the prescription

\[
\tilde{t}_{12}(p, p'; Z) = c(Z) t_{12}(p, p'; Z) - v_{12}(p, p').
\]

FIG. 1. The \( A_y \) puzzle, for \( nd \) scattering at 3 MeV (Lab). Calculations with the Bonn B potential (dots), and with the high-rank BBEST potential (lines). With the BBEST potential there are two calculations, one obtained using the (non separable) 2-dim approach, the other with the separable 1-dim algorithm. The curves are not distinguishable. Data (triangles) from Ref. [29].

The three-nucleon forces can be incorporated in the scattering equation in a relatively simple way if the 2N input potential is of finite rank. We sketch the procedure for a rank one case: once the separable 2N \( t \)-matrix is given, \( t = |g_1\rangle \tau(g_1), \) and the (anti)symmetrized AGS equation have been rewritten in the Lovelace form,

\[
X_{11} = Z_{11} + Z_{11} \tau X_{11},
\]

of the existing 2N \( t \)-matrices can guarantee such extrapolation since they are all constrained by experiments at the deuteron pole and at \( Z \geq 0 \). Furthermore, \( c(Z) \) might also correct for possible model dependencies on how this imperfect cancellation manifest itself, as already observed at the begining of this communication. On general grounds, one expects that with increasing energy in \( nd \) scattering, the factor \( c(Z) \) should shift towards one, since the off-shell 2N \( t \)-matrix in \( V_{3N} \) approaches gradually the energy region with experimental constrains.

To calculate the \( nd \) scattering observables below threshold, we have used the high-rank BBEST potential as two body input [3,26]. With this separable representations of the Bonn B potential, it is possible to solve accurately the Faddeev-AGS scattering equations, and obtain results comparable (with errors less than 1%) to those obtained from a direct solution of the Faddeev equations using the original potentials as input. As an example, Fig. 1 shows the results we have obtained for \( A_y \) at 3 MeV with the BBEST potential and with the original Bonn interaction. There are three curves since the calculations have been performed using both the separable (1-dimensional) algorithm and the non-separable (2-dimensional) method based on spline interpolation and Padé approximants, but the lines are practically indistinguishable. Similar tests have been made also before [27,28] on various occasions. Triangles represent the \( nd \) experimental data from Ref. [29].
integral equation with the driving term calculated as follows

\[ Z_{11} = \langle g_1 | G_0 P | g_1 \rangle + \langle g_1 | G_0 V_{1}^{3NF} G_0 | g_1 \rangle. \]  

(5)

The first contribution is the standard 2N driving term, with \( G_0 \) and \( P \) being the free Green’s function and the cyclic/anti-cyclic permutator, respectively, while the second takes into account the effects of the 3N force. This procedure is consistent with the formalism developed in Ref. [19] to include irreducible pionic effects in the 3N dynamics, and at the same time it corresponds to the established perturbative procedure to include 3NF effects in the separable AGS equation [30].

The results obtained are shown in Fig. 2, for the differential cross-section, and the analyzing powers \( A_y, iT_{11}, T_{20}, T_{21}, \) and \( T_{22} \). The panel exhibits the results obtained with the BBEST potential, where a rank-4 expansion has been used for all states with \( j \leq 2 \), aside for the states \( {}^{1}S_0 \) (rank 5), \( {}^{3}S_1 \) (rank 6), and \( {}^{3}P_2 \) (rank 5).

The results without the 3NF are shown by the dashed lines, and are practically indistinguishable from the corresponding results for the Bonn B potential. The solid curve considers the additional contribution of the 3N force expressed in Eq. (1). We have used the subtraction method of Eq. (3), with \( t \) and \( v \) given by the Bonn B interaction while the parameter \( c(Z) \) has been set to 0.73 for this energy. Results obtained with the PEST/Paris potential are similar.

In Fig. 3 we show how the puzzle evolves above the break-up threshold, up to 30 MeV. In the top panel we compare our theoretical calculations with the experimental data at 10 MeV [31], obtaining the correct reproduction for \( A_y \) with the 3NF when \( c(Z) = 0.735 \). In the middle panel comparison is made with \( pd \) data at 18 MeV [32]. Comparison of \( nd \) calculations with \( pd \) data is somewhat questionable because of the perturbations introduced by the Coulomb field. However it is known that, aside for the angles in forward direction, one of the main effects of the Coulomb field in \( A_y \) can be approximately reproduced by comparing the charged data with calculations performed at energies lowered of about 0.5-0.7 MeV. For this reason the data are compared with calculations at 17.3 MeV. (However, care must be exercised in interpreting this fact as a Coulomb slow-down effect [33]) At this energy we obtain the reproduction of the observable with \( c(Z) = 0.773 \). The bottom panel shows the results obtained at 29.6 MeV, compared with the corresponding data at the same energy [34] (triangles), and with \( pd \) data at 30.2 MeV [35] (squares). The solid line has been obtained with \( c(Z) = 0.81 \). It is evident that as the energy increases, the \( c(Z) \) parameter shifts slowly towards one, as expected.

We checked also how the triton binding energy is affected, and found that here the effects are relatively small. With the highest possible rank, \( i.e., \) with a rank 5 representation in all states with \( j \leq 2 \), except the coupled states \( {}^{3}S_1 \) (rank 6), and \( {}^{3}P_2 \) (rank 7), the BBEST+3NF result is \(-8.137 \) while the corresponding 2NF result is \(-8.090 \) (MeV).

We observe that some unavoidable approximations entered in this study. For instance, in obtaining Eq. (1)
we have ignored nucleonic recoil effects and have divided the subtracted $t$-matrix by the pion mass, instead of $\omega_\pi$, to get simpler expressions in partial waves. In addition, there might be a possible 3$N \pi$ contribution of shorter range for the exchange of a $\rho$-meson; this term would then counteract the one-pion-exchange 3$N \pi$, at least in the tensor part. Other uncertainties are related to the perturbative treatment of the pion dynamics in the AGS equation [19]. Finally, uncertainties about the fully off-the-energy-shell extrapolation of the NN $t$-matrix entering in this 3$N \pi$ forced us to introduce the effective parameter $c(Z)$.

As discussed in Refs. [17,19], the 3$N \pi$ contribution analyzed in this study is just one class of irreducible diagrams generated by the pion dynamics, and in a more complete analysis also other classes of 3$N \pi$ diagrams should be taken into account. Forces of the type TM [9], Brasil [10], Urbana [11], belong to another class and they complement the 3$N \pi$ whose effects have been calculated here separately. Some of these 3$N \pi$’s give small corrections to $A_y$, but not always in the right direction [8,16]; their relevance, however, appears to be greater elsewhere (e.g., the binding energy of the triton). There are also additional 3$N \pi$ terms of shorter range that might contribute, and in particular those obtained from $\chi$PT [16] appear very promising in providing an additional correction to $A_y$, possibly in the right direction.

To summarize and conclude: we have evaluated here for the first time a new “pionic” effect in the 3$N$ system. The effect is a natural consequence of a recently developed theory [18,19] for the combined $\pi$-3$N$ dynamics in the 3$N$ system, and has been recast into a 3$N \pi$ term of new structure by the authors [17]. The underlying 3$N \pi$ diagram complements the extensively discussed 2$\pi$-3$N \pi$ diagrams, and this complementarity shows up in the way this force affects the 3$N$ observables: while the 2$\pi$-3$N \pi$ terms have a large contribution on the 3$N$ binding energy and little effects (in the considered energy range) on the vector analyzing powers, we have shown here that this new 3$N \pi$ term greatly modifies in particular these two spin observables, and has the potential to provide in full the solution of the $A_y$ puzzle. Conversely, we checked also that the same force produces smaller changes for the triton binding energy. Since both effects are clearly needed for describing the low-energy behaviour of the 3$N$ system, it will be important to investigate at this point what will be the effect of the combined treatment of these two forces.

ACKNOWLEDGMENTS

This work is supported by the Italian MURST-PRIN Project “Fisica Teorica del Nucleo e dei Sistemi a Più Corpi”. W. Sch. thanks INFN and the University of Padova for hospitality and acknowledges support from the Natural Science and Engineering Research Council of Canada. L. C. thanks TRIUMF for hospitality.

$T_{20}$
$T_{21}$
$d\sigma/d\Omega$ [mb/sr]