HIGH-ENERGY PERFORMANCE OF LEP

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Abstract

The performance of LEP at energies well beyond Phase 1 is estimated. It is shown that the performance limit arises from the dynamic aperture determined by chromatic effects, rather than the physical aperture given by the vacuum chamber. It is shown that detuned low-$\beta$ insertions with higher than the nominal values of $\beta_y$ perform better in this energy range.

1. Introduction

It is the declared intention to operate LEP at energies up to 125 GeV and to cover the whole high-energy regime with a single machine lattice, specifically with the lattice described in the Version 11 parameter list\(^1\). The possibility is left open to operate LEP in Phase 1 with a lower tune but the same arrangement of magnetic elements.

It therefore seems appropriate to check now what needs to be done in order to make LEP work well at these high energies. It is first shown that the physical aperture is large enough to accommodate the circulating beam up to the highest energies contemplated. Next, some results for the dynamic aperture obtained after chromaticity correction are reviewed. Since the dynamic aperture is smaller than the physical one, it represents the real performance limitation. It is proposed to operate LEP at the limit of the dynamic aperture by varying the damping partition numbers with the energy. The optimum luminosity which can be reached in this manner is calculated.
2. **Beam size and physical aperture.**

The beam size and momentum spread are the result of balancing the effects of quantum excitation and synchrotron radiation damping. The rms momentum spread $\sigma_e$ (in relative units) is given by

$$\sigma_e = \frac{P E}{J_e}^{\frac{1}{2}}$$  \hspace{1cm} (1)

Here $P$ is a constant for a given lattice, $E$ is the energy and $J_e$ is the damping partition number for synchrotron oscillations. For LEP Version 11 we have $P = 1.539 \times 10^{-5}$ GeV$^{-1}$. The horizontal betatron emittance $E_x$ is given by

$$E_x = B^2 E^2 / J_x$$  \hspace{1cm} (2)

Here $B$ is another constant for a given lattice, $B = 5.743 \times 10^{-12}$ m GeV$^{-2}$ for LEP Version 11, and $J_x$ is the damping partition number for horizontal betatron oscillations. In a machine with a median plane the vertical emittance $E_y$ is largely determined by errors in the magnet alignment and by the beam-beam effect. With complete equipartition of energy between horizontal and vertical betatron oscillations, for "fully coupled beams", the vertical emittance becomes

$$E_y J_y \leq \frac{1}{2} E_x J_x$$  \hspace{1cm} (3)

Since the distribution functions for betatron and synchrotron oscillations are Gaussian, the beam sizes follow from quadratic addition of their contributions. At a position with betatron amplitude functions $\beta_x$ and $\beta_y$ and horizontal dispersion $D_x$, the rms beam sizes $\sigma_x$ and $\sigma_y$ are given by

$$\sigma_x^2 = E_x \beta_x + (D_x \sigma_e)^2$$  \hspace{1cm} (4)

$$\sigma_y^2 = E_y \beta_y$$  \hspace{1cm} (5)

In a machine with a median plane, the vertical dispersion $D_y$ vanishes, therefore this term does not occur in (5). In such a machine, the sum of $J_x$ and $J_e$ is a constant.
\[ J_X + J_e = 3 \]  \hspace{1cm} (6)

On the design orbit of a separated function lattice \( J_X = J_Y = 1 \) and \( J_e = 2 \). Since the derivative \( \frac{dJ_X}{d\delta} = -\frac{dJ_e}{d\delta} \) with \( \delta = \Delta E/\bar{E} \) is quite large in a separated - function lattice with short quadrupoles and long bending magnets, \( J_X \) and \( J_e \) can be adjusted by small changes in \( \delta \), the relative difference between the design energy of the machine at which electrons circulate along the design orbit, and their equilibrium energy. This is achieved by even smaller relative changes in the RF frequency \( f \), given by

\[ \frac{\Delta f}{f} = \frac{\alpha \Delta J_X}{\frac{dJ_X}{d\delta}} \]  \hspace{1cm} (7)

Here \( \alpha \) is the momentum compaction (\( \alpha = 1.928 \cdot 10^{-4} \) in LEP Version 11) and \( \Delta J_X \) is the desired change in \( J_X \). In a simple FODO lattice with quadrupoles of length \( l_Q \) and bending magnets of total length \( l_B \) in a half cell, and phase advance \( \mu \) per cell, the variation of \( J_X \) is:

\[ \frac{dJ_X}{d\delta} = \frac{4l_B}{l_Q} \frac{2 + \frac{1}{2} \sin^2 \frac{\mu}{2}}{\sin^2 \frac{\mu}{2}} \]  \hspace{1cm} (8)

For LEP Version 11, with \( l_Q = 1.6 \text{ m} \), \( l_B = 35.03 \text{ m} \) and \( \mu = 90^\circ \), (8) yields \( \frac{dJ_X}{d\delta} = -394.8 \), in good agreement with the computed figure \( \frac{dJ_X}{d\delta} = -393.9 \).

Because of (1), (2) and (6), (4) has a minimum when \( J_X \) is given by\(^2\)

\[ J_X = \frac{3}{3 + \frac{D_X P/B \sqrt{\Delta_X}}{l_B}} \]  \hspace{1cm} (9)

For LEP Version 11, this becomes \( J_X = 1.54 \).

It is customary in \( e^+e^- \) storage ring design to determine the physical half apertures \( a_X \) and \( a_Y \) by imposing the conditions
\[ a_x \geq F_x \sigma_x \] (10)

\[ a_y \geq F_y \sigma_y \] (11)

where the factors \( F_x \) and \( F_y \) are typically 10. Conversely, for given physical apertures, (10) and (11) can be used to obtain the maximum operating energy.

Table I shows the required apertures \( a_x \) and \( a_y \) for LEP Version 11 as a function of the energy \( E \), for several values of \( J_x \). They are always smaller than the physical apertures. It follows that from this point of view LEP Version 11 can indeed be operated up to 125 GeV over the whole range of values of \( J_x \) from 0.5 to 2.0.

Table I. Physical half apertures in normal LEP 11 lattice for various energies \( E \) and \( J_x \), assuming \( F_x = F_y = 10 \).

<table>
<thead>
<tr>
<th>E/GeV</th>
<th>( J_x = 0.5 )</th>
<th>( J_x = 1 )</th>
<th>( J_x = 1.54 )</th>
<th>( J_x = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>85</td>
<td>36.0</td>
<td>28.2</td>
<td>26.3</td>
<td>27.7</td>
</tr>
<tr>
<td>95</td>
<td>40.2</td>
<td>31.5</td>
<td>29.4</td>
<td>31.0</td>
</tr>
<tr>
<td>105</td>
<td>44.4</td>
<td>34.8</td>
<td>32.5</td>
<td>34.2</td>
</tr>
<tr>
<td>115</td>
<td>48.6</td>
<td>38.1</td>
<td>35.6</td>
<td>37.5</td>
</tr>
<tr>
<td>125</td>
<td>52.9</td>
<td>41.4</td>
<td>38.7</td>
<td>40.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>16.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>18.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>22.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>24.3</td>
</tr>
</tbody>
</table>

It might be useful to compare the assumed values of \( F_x \) and \( F_y \) with those actually necessary in operating machines.

3. **Dynamic aperture**

The dynamic aperture \( d_x \) and \( d_y \), and the dynamic acceptances \( A_x = d_x^2/\beta_x \) and \( A_y = d_y^2/\beta_y \) describe the betatron amplitudes at which electrons can circulate in a machine indefinitely. They are determined by nonlinear phenomena arising from the sextupoles used in correcting chromatic effects in general and the chromaticity in particular.
In an operating machine, the dynamic acceptance can be obtained experimentally by partly obstructing the physical aperture with scrapers. In PETRA this experiment is done by first blowing up the beam by kicking until a reduction of its lifetime is observed, and by then moving in scrapers until a further reduction in lifetime occurs. The aperture left by the scrapers is the dynamic aperture.

In a future machine like LEP, the dynamic acceptance can only be found by programs which track the particle orbits for many revolutions and check whether they remain within the physical aperture, or at least bounded.

Although much work has been done on chromaticity correction by appropriate arrangement of sextupoles there are no universal rules which are known to yield the largest dynamic acceptances. There are not even rules which indicate the properties which a linear lattice ought to have in order for its chromatic effects to be corrected well. The only option remaining under these circumstances is to make deductions from the chromaticity correction schemes which are available now. If better schemes happen to be developed in the future, the results can only be better than those derived from the presently known schemes.

In principle, the stable region is a volume in \((E_x, E_y, \delta)\) space centered around the origin. The parameters \(E_x\) and \(E_y\) are the emittances related to the amplitudes of betatron oscillations, and \(\delta\) is the relative amplitude of synchrotron oscillations. One would like to know the largest surface of some reasonably simple shape, e.g. an ellipsoid, which falls entirely inside the stable region. This procedure eliminates "islands" of stability away from the origin which are of no practical use.

To simplify the presentation and to reduce the amount of computation, only cuts of the stable volume are shown and used later on. Fig. 1 shows the stability limit as a function of the synchrotron oscillation amplitude \(\delta\), with the nominal \(\beta\)'s at the crossing points, and with the \(\beta\)'s increased by factors 2, 3 and 5. On each curve, the lower point is stable and the upper one unstable. Here the cut is taken such that \(E_y = \frac{3}{7}E_x\). Fig. 1 is obtained with the PATRICIA program\(^3\) following particles for 400 turns. Another cut is shown in Fig. 2. Here the synchrotron oscillation amplitude is fixed at \(\delta = 1.06194 \%\) and the stability limit for \(E_y\) is plotted as a function of \(E_x\), when the \(\beta\)'s are increased by a factor of two. Circles mark stable orbits and crosses unstable ones.
The sextupole arrangement is that of the parameter list\(^1\). Their excitation was calculated using the HARMON program\(^4\) in the following way: The 4 sextupole strengths in each half octant were computed by imposing low values for \(d\beta_x/d\delta\) and \(d\beta_y/d\delta\) at one crossing point and the centre of the arc. In the cases marked (*) these values were used as starting values for a further HARMON computation for a full octant, imposing the \(\beta\)-conditions on the two crossing points. In the other cases the sextupole strengths were not changed. The sextupole strength thus arrived at are shown in Table II.

Table II. Sextupole strengths used for chromaticity correction.

<table>
<thead>
<tr>
<th>(\beta)-factor</th>
<th>1*</th>
<th>2</th>
<th>3</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Length/m</td>
<td>Strength/m(^3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SF</td>
<td>0.4</td>
<td>-.158428</td>
<td>-.1567254</td>
<td>-.1547379</td>
</tr>
<tr>
<td>SD</td>
<td>0.76</td>
<td>+.120240</td>
<td>+.1377195</td>
<td>+.1245564</td>
</tr>
<tr>
<td>SF1</td>
<td>0.4</td>
<td>-.131290</td>
<td>-.1284963</td>
<td>-.1117196</td>
</tr>
<tr>
<td>SD1</td>
<td>0.76</td>
<td>+.263279</td>
<td>+.1892917</td>
<td>+.1730543</td>
</tr>
<tr>
<td>SF3</td>
<td>0.4</td>
<td>-.209173</td>
<td>-.1428462</td>
<td>-.1367287</td>
</tr>
<tr>
<td>SD3</td>
<td>0.76</td>
<td>+.156250</td>
<td>+.1310474</td>
<td>+.1176168</td>
</tr>
<tr>
<td>SF4</td>
<td>0.4</td>
<td>-.152496</td>
<td>-.1208274</td>
<td>-.1139653</td>
</tr>
<tr>
<td>SD4</td>
<td>0.76</td>
<td>+.310124</td>
<td>+.2007587</td>
<td>+.1805877</td>
</tr>
</tbody>
</table>

4. **Optimum use of the dynamic aperture**

How can the data about the beam size in chapter 2 and the data about the dynamic aperture in chapter 3 be used together to arrive at a conclusion about the optimum way for operating LEP at high energy, keeping the beam at the limit of the dynamic aperture over a whole range of energies?

Consider first the simple case where \(J_x = 1\) and is independent of energy. In this case, the square root of the horizontal emittance \(E_x\) is proportional to the energy \(E\). This is shown by the rising straight line in Fig. 3, where \(F_x \sqrt{E_x} = 10 \sqrt{E_x}\) is plotted against the energy \(E\). The energy spread \(\sigma_e\) is also proportional to the energy. We can use Fig. 1 to obtain a value for the dynamic acceptance by reading the ordinate at \(F_e \sigma_e = 10 \sigma_e\).
Repeating this for several energies yields the decreasing straight line in Fig. 3. The crossing point marks the energy where the beam just fills the available aperture. For higher energies the dynamic aperture is too small, for lower energies it is not filled.

If the calculation is repeated for a higher value of \( J_x \), e.g. \( J_x = 2 \), the following things change. The square root of the horizontal emittance is reduced by a factor \( \sqrt{2} \). The energy spread \( \sigma_e \) increases by a factor of \( \sqrt{2} \), and hence the dynamic aperture is reduced. In this case, the two lines cross at a higher energy.

This example suggests that the optimum way of operating LEP is to adjust \( J_x \) as a function of the energy \( E \) such that the dynamic aperture limit is just reached at the operating energy. This law of variation takes into account that the dynamic aperture depends on the momentum spread in the beam and is not a constant like the physical aperture. The appropriate relation between \( E \) and \( J_x \) will now be derived.

The square root of the dynamic acceptance \( \sqrt{A_x} \) obtained by tracking and shown in Fig. 1 is fitted by a straight line

\[
\frac{1}{2} A_x = a_o - a_1 \delta \tag{12}
\]

The momentum error \( \delta \) is obtained by multiplying (1) by a factor \( F_e \)

\[
\delta = F_e p E / J_e^\frac{1}{2} \tag{13}
\]

The required acceptance is obtained from the beam emittance \( E_x \) by multiplying (3) by a factor \( F_b \)

\[
A_x = F_b E E / J_x^\frac{1}{2} \tag{14}
\]

Substituting (13) into (12) and equating (12) and (14) yields the desired relation between \( E \) and \( J_x \)

\[
\left( \frac{F_b B}{J_x^\frac{1}{2}} + \frac{a_1 \sigma E}{(3 - J_x)^\frac{1}{2}} \right) E = a_o \tag{15}
\]
The easiest way to continue the analysis is computing $E$ as a function of $J_x$ for a range of values $0.5 \leq J_x \leq 2$ previously suggested as a reasonable operating range. It may be observed that the square bracket in (14) has a minimum, and therefore $E$ a maximum, at

$$J_x = \frac{3}{1 + (F_e a_1 P/F_b)^{\frac{3}{2}}} \quad (16)$$

In order to perform the calculation, values for $F_e$ and $F_b$ must be chosen. Following the same conventional wisdom as for the choice of the physical aperture suggests $F_e = F_b = 10$. The pairs of values $(J_x, E)$ thus obtained may then be used in the BEAMPARAM program to estimate the LEP performance.

The results of such calculations are shown in Fig. 4. The variation of $J_x$ shows the vertical tangent related to the minimum (16). Below some energy $J_x$ is held constant at its lower limit $J_x = 0.5$. It is implied that in this case the beam is blown up to the limit of the dynamic aperture by wiggler magnets, keeping $E_x$ and $a_e$ constant. This is the only energy range where wiggles can be usefully employed. They always increase the energy spread and therefore always reduce the available dynamic aperture. In most cases they also increase the beam emittance thus filling the dynamic aperture at a lower energy. Varying $J_x$ permits combining increases in emittance and reductions in momentum spread and vice versa. Therefore varying $J_x$ is much more attractive than wiggles, and exploited to the extreme in the above calculation. This is different from discussions only invoking the physical aperture when wiggles and varying $J_x$ were found to be interchangeable to a larger extent, the "unnecessary" energy losses in the wiggles tipping the balance in favour of $J_x$.

The choice of the multiplication factors $F_e$ and $F_b$ has a profound effect on the results. In Fig. 4, $F_b = F_e = 10$ is used. Eq. (15) shows that a change in $F_b$ and $F_e$ leads immediately to a change in the energy $E$ for a given $E_x$, such that for $F_b = F_e = F$, the product $PE$ is a constant. A change in $F$ by a factor $(1 + \varepsilon)$ leads to a change in the energy scale in Fig. 4 by a factor $1/(1 + \varepsilon)$ and to a change in luminosity at the scaled energy by a factor $(1 + \varepsilon)^{-4} \approx 1 - 4\varepsilon$. It is known from SPEAR that the beam-beam limit $\Delta Q$ becomes smaller when the aperture is reduced. Experimental data from the more recent machines on the aperture factors $F_b$ and $F_e$ necessary for operation with colliding beams would help in determining appropriate values for $F_b$ and
\( F_e \) to be used for LEP. Another handle are computer simulations giving histograms of density distributions\(^8\) combined with analytical estimates of the beam lifetime. Since the various cases are all computed with the same values of \( F_e \) and \( F_b \) their relative comparison is quite accurate although there is considerable uncertainty in the absolute energy scale.

In the luminosity estimates it is assumed that the beam sizes and currents are adjusted such that \( \Delta Q_x = \Delta Q_y = 0.03 \), and that the geometrical layout of the interaction regions is as given in the LEP Parameter List\(^1\). This is both optimistic and pessimistic. It is optimistic because the vertical beam blowup in collision is ignored. It occurs in all existing machines and is also observed in computer simulations. However, at the beam-beam limit assumed, \( \Delta Q = 0.03 \), the blowup is quite small\(^9\), about 20%. The calculation is pessimistic because the possible replacement of the standard insertions by mini-1\( \chi \) insertions (called mini-\( \beta \) insertions elsewhere) hold the promise of having the same dynamic apertures at smaller values of \( \beta \) and hence at a higher luminosity. The calculation is also pessimistic because it ignores the possibility to operate at twice the current with \( \Delta Q_x \approx 2\Delta Q_y \) and with a vertical beam blowup by a factor of about 2, which would increase the luminosity by a factor of about 2.

The ratio \( E_y/E_x \) depends on \( J_x \) even when the beams are fully coupled, because for \( J_x \neq 1 \) the horizontal and vertical damping rates are different. The dynamic aperture depends on \( E_y/E_x \) as shown in Fig. 2. A more complicated calculation taking this into account has been done with answers not too different from those shown here.

5. **Conclusions**

The dynamic aperture has been calculated for the same LEP Version 11 lattice in the arcs and the same style of chromaticity convection, with the low-\( \beta \) insertions detuned to \( \beta \)-values up to five times the nominal ones. It is found that the dynamic aperture increases with these \( \beta \)-values. When the damping partition number is varied with the operating energy, the LEP performance can be optimised by keeping the beams at the dynamic aperture limit at all energies above a certain limit. It can then be shown that the best performance at the highest energies is achieved by using higher \( \beta \)-values which have higher dynamic apertures although at first sight they would lead to a smaller luminosity.
These qualitative conclusions remain true for all machines whose dynamic aperture is smaller than their physical aperture. The quantitative conclusions apply to the chromaticity correction scheme and to the amplitude allowance factors $F_b$ and $F_e$ used. They may have to be revised when new chromaticity correction schemes are developed and/or when experimental data for $F_b$ and $F_e$ become available, which yield very different dynamic apertures.

6. Acknowledgement

I should like to thank J. Jowett for helping with the computations.

7. References

3). H. Wiedemann, PEP-220 (1976)
4). M. H. R. Donald, PEP note 311 (1979)
8). S. Myers, LEP note 327 (1981)
9). S. Myers, LEP note 310, Fig. 7 (1981)
Fig. 1. Dynamic aperture $\sqrt{E_x} = \sqrt{E_y}$ vs. amplitude $\delta$ of synchrotron oscillations at $Q_s = 0.07$. The lower points show an unstable (steady) case. The labels on the lines are the detuning factors from the nominal values $\delta$ at the crossing points ($E_x = 1.6 \text{ m}$, $E_y = 0.4 \text{ m}$).
Fig. 2. Dynamic aperture $\sqrt{E_Y}$ vs. $\sqrt{E_X}$ at constant synchrotron oscillation amplitude $\delta = 1.06\%$ and at $Q_s = 0.07$. The upper (lower) points show an unstable (stable) case. The lines indicate the fully coupled emittance ratios for several values of $J_x$. 
Fig. 3. Beam size (full line) and dynamic aperture (dashed line) vs. energy. The damping partition numbers are $J_X = 1$, $J_e = 2$. The arrow marks the highest operating energy.
Fig. 4. Luminosity $L$ (full lines) and circulating current $I$ in one beam (dashed line) vs. energy when the damping partition numbers are optimised to fill the dynamic aperture. The labels on the curves are the detuning factors from the nominal values of $\beta$ at the crossing points ($\beta_x = 1.6$ m, $\beta_y = 0.1$ m).