Triggers star formation in expanding shells

S. Ehlerová, J. Palouš

Astronomical Institute, Academy of Sciences of the Czech Republic, Boční II 1401, 141 31 Prague 4, Czech Republic

Received 28 June 2000/ Accepted yy

Abstract. We discuss the induced star formation in dense walls of expanding shells. The fragmentation process is studied using the linear perturbation theory. The influence of the energy input, the ISM distribution and the ISM speed of sound is examined analytically and by numerical simulations. We formulate the universal condition for the gravitational fragmentation of expanding shells: if the total surface density of the disk is higher than a certain critical value, shells are unstable. The value of the critical density depends on the energy of the shell and the sound speed in the ISM.

Stars: formation – ISM: bubbles – ISM: supernova remnants

1. Introduction

HI shells (or supershells) are structures identified in the distribution of the neutral hydrogen (HI) in the Milky Way and in many nearby galaxies. The typical shell consists of a rather rarefied medium surrounded by a dense thin wall, which in some cases expands supersonically to the ambient interstellar medium (ISM). Dimensions of shells lie in the range from a few 10 pc to a few kpc. Energies involved in their creation are of the order of $10^{50} - 10^{54}$ erg. Shells may have miscellaneous shapes, but many of them are nearly spherical, elliptical or cylindrical. Observations of shells are reviewed in Brinks & Walter (1998).

The energy needed for the creation of a shell originates either in the combined effect of massive stars in an OB association (the intense radiation, stellar winds and supernova explosions), in the very energetic event possibly connected to the gamma-ray burst (a hypernova or merging of compact companions in a binary system) or in the kinetic energy of a high velocity cloud (HVC) infalling to the galactic HI disk. In this paper we do not investigate the HVC scenario, which is rather distinct from the others. Remaining two mechanisms release a comparable amount of energy, the main difference is the duration of the energy supply. The energy flux from massive stars in OB associations lasts for a certain time interval, depending on the star formation history, on the IMF and metallicity, typically it may be 10 to 20 Myr. On the contrary, in the second case the energy is released in a few seconds. In this paper we shall designate the shells created by the energy released from massive stars in OB associations the “SNR” shells, the others will be “GRB” shells.

The density of the swept-up gas in walls of shells is higher than the average density of the unperturbed ambient medium, increasing the probability of the star formation there. The star formation in dense walls of HI shells, the star formation triggered by the previous energy input, is observed in some galaxies, e.g. in IC 2574 (Walter & Brinks, 1999). Its significance for the evolution of galaxies was studied by Palouš et al. (1994) and others.

In Ehlerová et al., 1997 (Paper I) we studied properties of gravitationally unstable shells in giant spirals and dwarf galaxies. In the present paper we continue in this study and try to find general conditions for the gravitational fragmentation and the following star formation. Namely, we are interested in the significance of the density stratification of the ISM, the sound speed in the ISM and the type of the energy source.

2. The thin shell approximation

The energy input from an OB association or other sources creates a blastwave, which propagates into the ambient medium (Ostriker & McKee 1988; Bisnovatyi-Kogan & Silich 1995). Since during the majority of the evolution its radius is much larger than its thickness, the thin shell approximation can be used. The blastwave is considered as an expanding infinitesimally thin layer surrounding the hot medium inside.

The analytical solution of the expansion in the thin shell approximation was derived by Sedov (1959). In a static, homogeneous medium without the external or internal gravitational field, the blastwave is always spheri-
Neglecting the external pressure, the radius of the shell $R$ grows with time as

$$R(t) = 53.1 \left( \frac{L}{10^{51} \text{erg} \text{Myr}^{-1}} \right)^{\frac{1}{3}} \times \left( \frac{\mu n}{1.3 \text{ cm}^{-3}} \right)^{-\frac{1}{3}} \times \left( \frac{t}{\text{Myr}} \right)^{\frac{1}{3}} \text{ pc}$$

(1)

where $L$ is the energy input from the source, $n$ and $\mu$ are the volume particle density of the ambient medium and the relative weight of one particle. Alternatively, if the energy from the source is released abruptly (as in one supernova or hypernova explosion), the equation looks like

$$R(t) = 72.2 \left( \frac{E_{\text{tot}}}{10^{51} \text{erg}} \right)^{\frac{1}{3}} \times \left( \frac{\mu n}{1.3 \text{ cm}^{-3}} \right)^{-\frac{1}{3}} \times \left( \frac{t}{\text{Myr}} \right)^{\frac{1}{3}} \text{ pc}$$

(2)

where $E_{\text{tot}}$ is the total energy released from the source.

Similarly we could write relations for the expansion velocity of the shell $v_{\text{exp}}$, its mass, column density $\Sigma$ and other quantities of interest; see Paper I or Ehlerová (2000).

### 2.1. Fragmentation of the thin shell

The growth of fragments in the expanding thin shell was analyzed in the linear approximation by Elmegreen (1994) and Vishniac (1994). Their results were applied in Paper I and they are used in this paper as well.

The blastwave sweeps the ambient medium forming the thin shell, while its surface increases. The net result of these two counteracting processes is in the homogeneous medium the growth of the column density of the shell:

$$\Sigma = \frac{1}{3} R n.$$  

Any density perturbation on the shell surface is stretched by the expansion, while its self-gravity supports the growth of the perturbation. The instantaneous maximum growth rate of the perturbation is

$$\omega = \frac{3 v_{\text{exp}}}{R} + \sqrt{\frac{v_{\text{exp}}^2}{R^2} + \left( \frac{\pi G \Sigma}{c_{\text{sh}}} \right)^2},$$

(3)

where $c_{\text{sh}}$ is the speed of sound within the shell, $G$ is the constant of gravity. The perturbation grows only if $\omega > 0$.

The wavelength of the fastest transversal perturbation, $\lambda$, is

$$\lambda = \frac{2 v_{\text{exp}}^2}{G \Sigma}.$$  

(4)

This wavelength must be smaller than the radius of the shell $R$, otherwise the perturbed region would not fit into $\omega > 0$.

The equation (3) shows clearly, that at early stages of the evolution the shell is stable, as the fast expansion stretches all perturbations which might appear. Large values of $v_{\text{exp}}$ and small values of $R$ make $\omega$ negative. Only later, when $v_{\text{exp}}$ is small, $R$ large and $\Sigma$ high, the self-gravity begins to play a role and fragments may form. At the time $t_b$ the growth rate $\omega$ becomes positive. This is the first moment, when the shell starts to be unstable and the fragmentation process may begin. To estimate the rate by which fragments grow, we calculate the fragmentation integral $I_f(t)$:

$$I_f(t) = \int_{t_b}^{t} \omega(t')dt'.$$

(5)

At the time $t = t_f$, when $I_f(t) = 1$, we consider fragments to be well developed and we say, that the fragmentation is successful. This time is rather arbitrary and also the growth rate $\omega$ is influenced by nonlinear effects at later stages. However, our findings are derived mostly from the time $t_b$, which is well defined.

Successful formation of fragments does not yet guarantee the formation of new molecular clouds. Molecules can form, if the column density in a fragment surpasses the critical value for the self-shielding against Ly$\alpha$ photons (Franco & Cox, 1986):

$$\Sigma_{\text{shield}} = 5 \times 10^{20} \frac{Z^2}{Z} \text{ cm}^{-2}.$$  

(6)

### 2.2. The analytical solution

Inserting the analytical solution 1 (or 2) and relations for $v_{\text{exp}}$ and $\Sigma$ to the fragmentation conditions (3) and (4) we get relations for the time $t_b$, the radius and the expansion velocity at this time. Here we show relations for the continuous energy input:

$$t_b = 28.8 \left( \frac{c_{\text{sh}}}{\text{kms}^{-1}} \right)^{\frac{1}{3}} \times \left( \frac{L}{10^{51} \text{erg} \text{Myr}^{-1}} \right)^{-\frac{1}{3}} \times \left( \frac{\mu n}{1.3 \text{ cm}^{-3}} \right)^{-\frac{1}{3}} \text{ Myr}$$

(7)

$$R(t_b) = 399 \left( \frac{c_{\text{sh}}}{\text{kms}^{-1}} \right)^{\frac{1}{3}} \times \left( \frac{L}{10^{51} \text{erg} \text{Myr}^{-1}} \right)^{\frac{1}{3}} \times \left( \frac{\mu n}{1.3 \text{ cm}^{-3}} \right)^{-\frac{1}{3}} \text{ pc}$$

(8)

$$v_{\text{exp}}(t_b) = 8.13 \left( \frac{c_{\text{sh}}}{\text{kms}^{-1}} \right)^{-\frac{1}{3}} \times \left( \frac{L}{10^{51} \text{erg} \text{Myr}^{-1}} \right)^{\frac{1}{3}} \text{ kms}^{-1}.$$  

(9)
Fig. 1. The analytical solution: a continuous energy input (left panel) and an abrupt energy input (right panel). Solid lines are curves of the condition $\Sigma = \Sigma_{\text{shield}}$. Dotted and dashed lines show critical fluxes and energies (see equations 10, 11): dotted lines are for $c_{\text{ext}} = 5 \text{ km s}^{-1}$, dashed for $c_{\text{ext}} = 10 \text{ km s}^{-1}$.

(similarly for the abrupt energy input). The radius $R(t_b)$ is a lower limit to the distance on which the fragmentation (and the triggered star formation) may happen: the expansion velocity $v_{\exp}(t_b)$ is an upper limit to the random component of the velocity of newly created clouds (or stars). The time $t_f$ was calculated analytically in Paper I, the ratio $t_f/t_b \sim 2.03$.

Relations (7), (8) and (9) are derived under the assumption of the supersonic motion, i.e. $v_{\exp}(t) > c_{\text{ext}}$, where $c_{\text{ext}}$ is the sound speed in the ambient medium. If the shell becomes a sound wave before it starts to be unstable (i.e. before the time $t_b$), it will always remain stable (the sound wave does not sweep up the ambient medium any more). If the expansion is supersonic till the time $t_f$ or longer, we consider the fragmentation as successful. If the expansion decelerates to $c_{\text{ext}}$ between times $t_b$ and $t_f$, then either the self-gravity of fragments is sufficient to overcome the stabilizing effect of the stretching, or not. Fig. 1 shows stable and unstable regions in the $L - n$ and $E_{\text{tot}} - n$ planes. Curves, separating regions, where the condition (6) is fulfilled, are also drawn. However, as the column density of fragments is higher than the average value in the shell, the condition (6) is too strict. The linear perturbation theory does not estimate the value of the increased density very well, its calculations with the inclusion of nonlinear terms will be the subject of the separate communication (Wünsch & Palouš, 2000).

Assuming that the expansion velocity $v_{\exp}$ has to be greater than or equal to the sound speed in the ambient medium $c_{\text{ext}}$, we can derive from the equation (9) the critical luminosity of the energy source $L_{\text{crit}}$; for $L_{\text{crit}}$ the expansion velocity at the time $t_b$ equals the sound speed $c_{\text{ext}}$. If the energy input is greater than $L_{\text{crit}}$, the shell starts to fragment; if the input is smaller, the shell is stable.

$$L_{\text{crit}} = \left( \frac{c_{\text{ext}}}{8.13 \text{ km s}^{-1}} \right)^4 \left( \frac{c_{\text{sh}}}{\text{km s}^{-1}} \right)^{10^5 \text{ erg Myr}^{-1}}$$  (10)

$L_{\text{crit}}$ does not depend on the density of the ambient medium but it is a strong function of its speed of sound.

Similarly, we can calculate the critical energy $E_{\text{crit}}$ for the abrupt energy input (shells with the lower energy are stable, shells with the higher one start to fragment):

$$E_{\text{crit}} = \left( \frac{c_{\text{ext}}}{3.80 \text{ km s}^{-1}} \right)^{\frac{2}{7}} \left( \frac{c_{\text{sh}}}{\text{km s}^{-1}} \right)^{\frac{2}{7}} \times \left( \frac{n}{\text{cm}^{-3}} \right)^{-\frac{1}{2}} 10^{51} \text{ erg}$$  (11)

which depends both on the density and sound speed of the ambient medium. The last equation also means, that for each total energy, there is a critical ISM density $n_{\text{crit}}$ depending on the speeds of sound in the ISM and in the shell; shells evolving in the lower density are always stable (see also section 5).

Analogically, $v_{\exp}$ decreases to $c_{\text{ext}}$ at the time $t_f$ for the energy flux $L_{\text{frag}}$ or the total energy $E_{\text{frag}}$. For higher values of $L$ or $E_{\text{tot}}$ the fragmentation process is finished during the supersonic expansion and there is not an easy way to dissolve perturbations by stretching.
in the ambient medium $c_{\text{ext}}$ and the speed of sound in the shell $c_{\text{sh}}$ as constants. If not said otherwise, $c_{\text{ext}} = 5 \, \text{km s}^{-1}$, $c_{\text{sh}} = 1 \, \text{km s}^{-1}$. The value of $c_{\text{ext}}$ in normal galaxies lies in the interval $(3 - 12) \, \text{km s}^{-1}$, with the lower limit associated with quiet regions with no star formation, while the upper one is connected to disturbed, star-forming places. The value of $c_{\text{sh}}$ is not known, arguments for its low $(1 - 3 \, \text{km s}^{-1})$ value can be found in e.g. Jechumtal (1999) for the Holmberg II galaxy, or Ehlerová (2000) for the Milky Way.

3. Simulations of expanding shells

The thin shell approximation has been applied in numerical simulations in one and two dimensions by many authors (see Ikeuchi, 1998, for a review on this subject). The models have been further extended into three dimensions by Palouš (1990) and Silich et al. (1996).

The code, which we use in this paper, is an improved and extended successor to the code of Palouš (1990), described in Paper I and in Efremov et al. (1999). The thin shell is divided into a number of elements; a system of equations of motion, mass and energy for each element is solved.

The equation of motion is

$$\frac{d}{dt} (m_{\text{sh}} v_{\text{exp}}) = \Delta S_{\text{sh}} \left[ (P_{\text{int}} - P_{\text{ext}}) + n v_{\text{ext}} (v_{\text{exp}} - v_{\text{ext}}) \right] + m_{\text{sh}} g, \quad (12)$$

where $m_{\text{sh}}$, $v_{\text{exp}}$, $\Delta S_{\text{sh}}$ are the mass, the expansion velocity and the surface of an element of the shell, $P_{\text{int}}$ and $P_{\text{ext}}$ are pressures inside and outside the bubble, $n$ and $v_{\text{ext}}$ are the density and velocity of the ambient medium, and $g$ is the gravitational acceleration.

The equation of mass conservation is

$$\frac{d}{dt} m_{\text{sh}} = (v_{\text{exp}} - v_{\text{ext}}) \perp n \Delta S_{\text{sh}} \quad (13)$$

The term on the right hand side (rhs), which gives the increase of mass $m_{\text{sh}}$, is used as long as the normal component of the velocity, $(v_{\text{exp}} - v_{\text{ext}}) \perp$, exceeds the speed of sound in the ambient medium. After the expansion becomes subsonic, the mass accumulation stops.

The total energy $E_{\text{tot}}$ is

$$E_{\text{tot}} = E_{\text{pot}} + E_{\text{th}} + E_{\text{kin}}, \quad (14)$$

where $E_{\text{pot}}$ is the gravitationnal potential energy, $E_{\text{th}}$ and $E_{\text{kin}}$ are the thermal and kinetic energies:

$$E_{\text{th}} = E_{\text{th, int}} + E_{\text{th, sh}} + E_{\text{th, ext}}, \quad (15)$$

$$E_{\text{kin}} = E_{\text{kin, int}} + E_{\text{kin, sh}} + E_{\text{kin, ext}}, \quad (16)$$

where the second parts of subscripts on the rhs refer respectively to the medium inside the bubble, in the shell, disregard the thermal energy of the unperturbed ambient medium $E_{\text{th, ext}}$ and the kinetic energy of the medium inside and outside the bubble $E_{\text{kin, int}}$ and $E_{\text{kin, ext}}$.

The change of the total energy is given as

$$\frac{d}{dt} E_{\text{tot}} = L - \Lambda \quad (17)$$

where $L$ is the energy input rate from the energy source and

$$\Lambda = \Lambda_{\text{int}} (n_{\text{int}}, T_{\text{int}}) + \Lambda_{\text{sh}} (n_{\text{sh}}, T_{\text{sh}}) \quad (18)$$

$\Lambda_{\text{int}} (n_{\text{int}}, T_{\text{int}})$ is the radiative cooling rate from the bubble interior, $n_{\text{int}}$ is the internal volume density, $T_{\text{int}}$ is the internal temperature; $\Lambda_{\text{sh}} (n_{\text{sh}}, T_{\text{sh}})$ is the radiative cooling rate from the dense shell, $n_{\text{sh}}$ and $T_{\text{sh}}$ are the density and temperature in the shell.

The balance equation of the thermal energy in the shell is

$$\frac{d}{dt} E_{\text{th, sh}} = \frac{1}{2} \frac{d m_{\text{sh}}}{dt} \left( v_{\text{exp}} - v_{\text{ext}} \right) \perp^2 - \Lambda_{\text{sh}} \quad (19)$$

where the first term on the rhs gives the fraction of the kinetic energy of the shell which is converted into the shell thermal energy due to the compression of the ambient medium. $\Lambda_{\text{sh}} (n_{\text{sh}}, T_{\text{sh}})$ is the cooling rate of the shell, the volume density $n_{\text{sh}}$ is estimated assuming that the thickness of the shell is a small fraction (~ 0.1) of the shell radius.

The internal thermal energy is calculated from the total energy of the structure (equations (14), (17)):

$$E_{\text{th, int}} = E_{\text{tot}} - E_{\text{kin}} - E_{\text{pot}} - E_{\text{th, sh}} \quad (20)$$

The internal pressure $P_{\text{int}}$ is derived from the internal thermal energy $E_{\text{th, int}}$ and the volume of the bubble $V_{\text{int}}$

$$P_{\text{int}} = \frac{2}{3} \frac{E_{\text{th, int}}}{V_{\text{int}}}. \quad (21)$$

$T_{\text{int}}$ and $T_{\text{sh}}$ are given by the thermal energies and numbers of particles in the bubble $N_{\text{int}}$ and in the shell $N_{\text{sh}}$: $T_{\text{int}} = \frac{E_{\text{th, int}}}{\frac{2}{3} N_{\text{int}}}$ and $T_{\text{sh}} = \frac{E_{\text{th, sh}}}{\frac{2}{3} N_{\text{sh}}}$. The radiative cooling rate from both the bubble and the shell is calculated using the cooling functions of Böhringer & Hensler (1989) for $T \in (10^{5.5} - 10^9) \, \text{K}$, and Schmutzler & Tscharnuter (1993) for $T \in (10^8 - 10^{10}) \, \text{K}$.

At the beginning of calculations the initial energy is divided into the kinetic and thermal energies of the shell and of the bubble. We usually choose ratios corresponding to the solution of Sedov (1959) or Weaver et al. (1977), however different ratios between initial energies (and masses in the shell and in the bubble) do not influence results substantially.

In the version of the code described in Efremov et al. (1999) we introduced the mixing of the mass and energy
created by the abrupt energy input — the thermal energy solution). This has comparatively large influence on shells. Expansion cannot be treated as isothermal (as in the Sedov theory (section 2.1)). Conditions for the fragmentation (3 and 4) are evaluated separately for each element of the shell. In the case of non-spherical shells, fragmentation properties vary with the position on the shell. Typically, for shells growing in a smooth distribution of gas with the \(E\)-\(n\) gradient, a dense ring is created in the region of the maximum density, which is the most unstable part of the shell, while large lobes of the shell in low-density regions are stable. But also the gravitational field of the galaxy (especially the differential rotation) may change the fragmentation properties. In the following we present results for the most unstable parts of shells.

3.1. Fragmentation of expanding shells

To describe the gravitational instability of expanding shells, the code uses results of the linear perturbation theory (section 2.1). Conditions for the fragmentation (3 and 4) are evaluated separately for each element of the shell. In the case of non-spherical shells, fragmentation properties vary with the position on the shell. Typically, for shells growing in a smooth distribution of gas with the \(E\)-\(n\) gradient, a dense ring is created in the region of the maximum density, which is the most unstable part of the shell, while large lobes of the shell in low-density regions are stable. But also the gravitational field of the galaxy (especially the differential rotation) may change the fragmentation properties. In the following we present results for the most unstable parts of shells.

4. Numerical experiments

Numerical simulations include some effects, which are neglected in the analytical solution. The most important ones are: 1) the pressure of the ambient medium and radiative cooling; 2) the finite, time-limited energy input from an OB association; and perhaps the most obvious one 3) the stratification of the ISM in galaxies. Including the pressure of the ambient medium means, that the expansion cannot be treated as isothermal (as in the Sedov solution). This has comparatively large influence on shells created by the abrupt energy input — the thermal energy \(E_{th, \text{int}}\) is decreasing quite rapidly; but only a minor influence on shells produced by the continuous energy input. Effects of the external pressure and radiative cooling on the fragmentation will not be discussed here separately.

The ISM in galaxies is not perfectly smooth, but contains a lot of small-scale density fluctuations (see Silich et al., 1996, for the partial analysis of their influence on shells), is turbulent etc. However, coherent shells and bubbles exist even in a rather turbulent ISM, our experiments in a medium with a hierarchy of density fluctuations show, that shells are mostly influenced by the large-scale gradients in the ISM (with the exception of the very perturbed regions, where the coherent shock does not form). Therefore we think, that the discussion about properties of shells in the smooth medium does not lose its relevance to the real situation.

4.1. The time-limited energy input

The energy input from an OB association is distributed over some time, but timescales, on which the fragmentation starts, usually exceed the estimated lifetime of the association. This means, that we should take into account the finite amount of energy and stop the energy input after some time (typically about 10–20 Myr).

An example of the influence of the finite lifetime of the OB association is shown in Fig. 2. There the total energy was released in 15 Myr (obviously, higher total energies imply higher fluxes — corresponding fluxes are indicated on the right axes). Dashed lines give the boundary between the stable and unstable regions in the \(L - n\) plane for the unlimited energy input (shells above the line are gravitationally unstable). We see, that (especially) for lower energies and lower \(c_{\text{ext}}\), the finite lifetime of an OB association stabilizes the shell, the boundary between stable (circles) and unstable (triangles) shells moves to higher values of \(L\).

Fig. 3 shows critical densities \(n_{\text{crit, hom}}\) of the homogeneous ambient medium (= “the numerical equivalent of the equation 11”) — the density \(n_{\text{crit, hom}}\) is the lowest density for which the shell with the given energy input starts to fragment. We do not follow a further evolution of the shell, so it is possible, that the fragmentation will not succeed. However, for lower densities the fragmentation cannot even start. “SNR” and “GRB” energy types differ in the way how the energy is released to the shell, which means different expansion velocities at the same radius. Thus it is not surprising, that \(n_{\text{crit, hom}}\) differ for these two types. The higher is the energy, the lower is the density (for “SNR” \(n_{\text{crit, hom}} \propto E_{\text{tot}}^{-1.6}\); for “GRB” \(n_{\text{crit, hom}} \propto E_{\text{tot}}^{-2.1}\)). The source, which releases the energy continuously, may trigger the star formation in a lower density than the same total energy released abruptly. To trigger the star formation in a given density of the ISM, the energy released abruptly must be higher than the one released continuously.

\[
E_{\text{th, int}} = \frac{E_{\text{tot}}}{c_{\text{ext}}} \leq \frac{1}{2} \rho_{\text{crit}} \frac{L}{c_{\text{ext}}} \left( \frac{c_{\text{ext}}}{c_{\text{crit, hom}}} \right)^{2} \left( \frac{n_{\text{crit, hom}}}{n_{\text{crit, hom}}} \right)^{-1} \frac{L}{c_{\text{crit, hom}}} \left( \frac{c_{\text{crit, hom}}}{c_{\text{ext}}} \right)^{2} \left( \frac{n_{\text{crit, hom}}}{n_{\text{crit, hom}}} \right)^{-1}
\]

\[
E_{\text{ext}} = E_{\text{tot}} - E_{\text{th, int}} \leq \frac{1}{2} \rho_{\text{crit}} \frac{L}{c_{\text{ext}}} \left( \frac{c_{\text{ext}}}{c_{\text{crit, hom}}} \right)^{2} \left( \frac{n_{\text{crit, hom}}}{n_{\text{crit, hom}}} \right)^{-1} \frac{L}{c_{\text{crit, hom}}} \left( \frac{c_{\text{crit, hom}}}{c_{\text{ext}}} \right)^{2} \left( \frac{n_{\text{crit, hom}}}{n_{\text{crit, hom}}} \right)^{-1}
\]
Fig. 2. The influence of the time-limited energy input: simulations of “SNR” shells in a homogeneous medium with $c_{\text{ext}} = 5 \text{ km/s}$ (left panel) and $c_{\text{ext}} = 10 \text{ km/s}$ (right panel). Circles denote cases, where shells are gravitationally stable; empty triangles are shells which start to fragment but unsuccessfully (i.e. they do not reach $I_f = 1$); filled triangles are unstable shells. Lines show $L_{\text{crit}}$ in the time-unlimited case.

Fig. 4. The evolution of a “SNR” shell with $E_{\text{tot}} = 5 \times 10^{52} \text{ erg}$ in stratified disks. $n_0$ and $\sigma$ are parameters of the Gaussian distribution (22). The meaning of symbols is the same as in Fig. 2. Lines are curves of constant surface densities of the disk. The speed of sound in the ISM is 5 $\text{ km/s}$ (left panel) and 10 $\text{ km/s}$ (right panel).
4.2. The stratification of the ISM

The ISM in galaxies is not homogeneous, but disk-like. The wavelengths of instabilities are comparable to the scales of the gas distribution. Consequently the density gradients must be taken into account.

As an example we show the fragmentation of a “SNR” shell created by $E_{\text{tot}} = 5 \times 10^{52} \text{erg}$ evolving in a disk with the Gaussian stratification

$$n(z) = n_0 e^{-\frac{z^2}{\sigma^2}}$$

\[(22)\]

(Fig. 4), $z$ is the distance from the plany of the symmetry.

To see the influence of the $z$-stratification, we made a set of experiments in three types of the ISM distribution (see Figure 5) — for the same surface density $N$ the three disks differ in thicknesses.

$$n_1(z) = n_0 e^{-\frac{z^2}{\sigma^2}}$$

\[(23)\]

$$n_2(z) = n_0 e^{-\frac{z^2}{\sigma^2}} + 0.5n_0 e^{-\frac{4z^2}{\sigma^2}}$$

\[(24)\]

$$n_3(z) = n_0 e^{-\frac{z^2}{\sigma^2}} + 0.25n_0 e^{-\frac{4z^2}{\sigma^2}}$$

\[(25)\]

We focus on the time $t_b$ when the fragmentation starts. Results are shown in Fig. 6 and 7. We see, that in some cases (e.g. the lower energy case — the left panel in Fig. 6) shells are unstable only if the maximum volume density in the disk equals or surpasses the value $n_{\text{crit, hom}}$. In other cases (e.g. the higher energy case — the right panel in Fig. 6) the maximum volume density of the first unstable shell depends on the thickness of the disk, but in such a way, that the total surface density of the disk is constant. Further, the value of this surface density varies with the energy, but it is not dependent on the profile of the disk (see Fig. 7).

These results lead us to a hypothesis, that the most important quantity, which decides if the shell fragments, is the total surface density of the disk, regardless of the surface density higher than the critical value $N_{\text{crit}}$ start to fragment, shells in lower densities are stable. However, in some cases (especially for thick disks and low energies), $N_{\text{crit}}$ would predict very low volume densities in the $z = 0$ plane — lower than $n_{\text{crit, hom}}$. In such a case shells are unstable only when the maximum volume density exceeds $n_{\text{crit, hom}}$ (“nothing can be better than the homogeneous case”).

In thin disks ($\sigma \leq 200 \text{ pc}$) $N_{\text{crit}}$ increases to higher values for a continuous energy input (“SNR” shells) (see Fig. 4 and 7). The explanation is, that in this case the shells are very likely to undergo a blow-out. The blow-out enables a fraction of the hot medium inside the shell to escape to the halo, decreasing the effective energy and pressure pushing the high density part of the shell. This leads to the increase of the surface density needed for the fragmentation in the case of thin disks.

Critical densities $n_{\text{crit, hom}}$ change in the range of several orders. Regardless of the energy, the higher is the density, the smaller is the radius $R(t_b)$, at which the fragmentation appears for the first time (see Eq. 8 or Tables 1 and 2). For radii lower than $\approx 50 \text{ pc}$ the large scale stratification of the disk does not play a role and the situation can be studied as a homogeneous case (this is for densities higher than about 50 or 100 $\text{cm}^{-3}$). On the opposite side, shells in a very low density ISM reach very large dimensions (and also $R(t_b)$ are enormous). The effect of the gravitational field of the galaxy on shells is not discussed here, but in our opinion large shells (with a radius $\sim 2 \text{ kpc}$ and more), which are relatively old, are probably destroyed by the shear preventing their fragmentation.

<table>
<thead>
<tr>
<th>$E_{\text{tot}}$</th>
<th>disk</th>
<th>$n_{\text{crit, hom}}$</th>
<th>$R(t_b)$</th>
<th>$N_{\text{crit}}$</th>
<th>rem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{45}$</td>
<td>cold</td>
<td>$0.77$</td>
<td>$0.36$</td>
<td>16</td>
<td>vd</td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>$&gt; 1000$</td>
<td>$-$</td>
<td>$-$</td>
<td>$t_b &lt; \tau_{\text{OB}}$</td>
</tr>
<tr>
<td>$5 \times 10^{44}$</td>
<td>cold</td>
<td>$0.064$</td>
<td>$1.2$</td>
<td>3.1</td>
<td>$t_b &lt; \tau_{\text{OB}}$</td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>$&gt; 1000$</td>
<td>$-$</td>
<td>$-$</td>
<td></td>
</tr>
<tr>
<td>$10^{43}$</td>
<td>cold</td>
<td>$0.023$</td>
<td>$2.0$</td>
<td>1.5</td>
<td>$t_b &lt; \tau_{\text{OB}}$</td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>$0.78$</td>
<td>$0.50$</td>
<td>27</td>
<td>$vd$</td>
</tr>
<tr>
<td>$5 \times 10^{42}$</td>
<td>cold</td>
<td>$0.0017$</td>
<td>$6.0$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>$0.059$</td>
<td>$1.8$</td>
<td>4.7</td>
<td>$-$</td>
</tr>
<tr>
<td>$10^{42}$</td>
<td>cold</td>
<td>$&lt; 0.001$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>$0.022$</td>
<td>$2.4$</td>
<td>2.9</td>
<td>$-$</td>
</tr>
</tbody>
</table>

Table 1. Critical densities $n_{\text{crit, hom}}$ and $N_{\text{crit}}$ for the “SNR” shells in cold ($c_{\text{ext}} = 5 \text{ km}^{-1}$) and hot ($c_{\text{ext}} = 10 \text{ km}^{-1}$) disks. $R(t_b)$ is the radius of the shell at $t_b$ for the homogeneous case. “vd” means, that the criterion of a minimum $n_{\text{crit, hom}}$ applies (not necessarily in the whole range of thicknesses). The energy is released during the period $\tau_{\text{OB}} = 15 \text{ Myr}$.

Tables (1) and (2) give values of the critical density $n_{\text{crit, hom}}$ and the critical surface density $N_{\text{crit}}$ for different input energies.
Fig. 6. “GRB” shells in Gaussian disks (the distribution (23)). Solid lines show the minimum density \( n_0 \) needed for the onset of the fragmentation. Dotted lines are values of \( n_{\text{crit, hom}} \), the critical volume density for the homogeneous case. Dashed lines show the in-plane density \( n_0 \) calculated from the condition of the constant surface density of the disk. The energy of the shell is \( 5 \times 10^{52} \text{erg} \) (left panel) and \( 10^{54} \text{erg} \) (right panel).

Table 2. Critical densities \( n_{\text{crit, hom}} \) and \( N_{\text{crit}} \) for the “GRB” shells in cold \( (\rho_{\text{ext}} = 5 \text{ km s}^{-1}) \) and hot \( (\rho_{\text{ext}} = 10 \text{ km s}^{-1}) \) disks. \( R(t_b) \) is the radius of the shell at \( t_b \) for the homogeneous case. “vd” means, that the condition of the minimum \( n_{\text{crit, hom}} \) applies.

<table>
<thead>
<tr>
<th>( E_{\text{tot}} ) (erg)</th>
<th>disk</th>
<th>( n_{\text{crit, hom}} ) (cm(^{-3}))</th>
<th>( R(t_b) ) (kpc)</th>
<th>( N_{\text{crit}} ) (( 10^{20} \text{ cm}^{-2} ))</th>
<th>rem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{52} )</td>
<td>cold</td>
<td>&gt; 100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>&gt; 1000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 5 \times 10^{52} )</td>
<td>cold</td>
<td>5.9</td>
<td>0.13</td>
<td>30</td>
<td>vd</td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>310</td>
<td>0.025</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 10^{53} )</td>
<td>cold</td>
<td>1.3</td>
<td>0.27</td>
<td>14</td>
<td>vd</td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>60</td>
<td>0.057</td>
<td>( \approx 200 )</td>
<td>vd</td>
</tr>
<tr>
<td>( 5 \times 10^{54} )</td>
<td>cold</td>
<td>0.047</td>
<td>1.4</td>
<td>2.3</td>
<td>vd</td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>2.5</td>
<td>0.28</td>
<td>26</td>
<td>vd</td>
</tr>
<tr>
<td>( 10^{54} )</td>
<td>cold</td>
<td>0.012</td>
<td>2.8</td>
<td>1.0</td>
<td>vd</td>
</tr>
<tr>
<td></td>
<td>hot</td>
<td>0.62</td>
<td>0.56</td>
<td>12</td>
<td>vd</td>
</tr>
</tbody>
</table>

5. The total surface density of the disk as the main parameter

The star formation in galaxies is quite well described by a Schmidt law with a threshold, i.e. above a certain critical disk surface density \( N_{\text{SF} \text{crit}} \) the star formation rate per surface unit \( (\Sigma_{\text{SFR}}) \) depends on the local gas surface density \( N_{\text{gas}} \):

\[
\Sigma_{\text{SFR}} = AN_{\text{gas}}^a
\]  

where \( a \in (1 - 2) \) (Kennicutt, 1998); and below \( N_{\text{SF} \text{crit}} \) the star formation rate falls down sharply. The critical density varies among different galaxies, it lies in the range \( N_{\text{SF} \text{crit}} \in (10^{20}, 10^{21}) \text{ cm}^{-2} \) (Kennicutt, 1989). This interval corresponds nicely to the interval of \( N_{\text{crit}} \) for reasonably large input energies (see Fig 8). It means, that the star formation triggered by expanding shells probably plays a significant role in the SF history in galaxies. Maybe it governs the SFR, especially in regions near to the threshold: places with the density near to the thresh-
which gives $128$ for the ratio of $c_{\text{hot}}$ to $c_{\text{cold}}$, $n_{\text{crit,hom}}$. The ratio between the fragmentation in hot disks is much less probable than in cold ones. The ratio between $n_{\text{crit,hom}}$ for hot and cold disks ($\frac{n_{\text{crit,hom,hot}}}{n_{\text{crit,hom,cold}}}$) does not depend on the energy. From the analytical solution for GRB shells (equation 11) we get a very sharp dependence of $n_{\text{crit}}$ on the speed of sound

$$\frac{n_{\text{crit,hom,hot}}}{n_{\text{crit,hom,cold}}} = \left(\frac{c_{\text{ext,hot}}}{c_{\text{ext,cold}}}\right)^7$$

which gives 128 for the ratio of $c_{\text{ext,hot}}$ to $c_{\text{ext,cold}}$. Simulations lead to a less steep dependence, $\sim 34$ for “SNR” shells and $\sim 50$ for “GRB” shells, corresponding to the power-law indices $\sim 5.6$ and $\sim 5.1$ respectively. Ratios $\frac{N_{\text{crit,hot}}}{N_{\text{crit,cold}}}$ have the power law indices around 4 (slightly different for SNR and GRB shells).

5.1. The gravitational field of the galaxy and off–plane explosions

The gravitational field of the galaxy may change the shape and the velocity field of the shell and thus the conditions for the fragmentation. However, simulations show, that the differential rotation “only” changes the shape of the unstable regions: instead of a ring, where the fragmentation properties are the same everywhere, two regions with the maximum value of the fragmentation integral appear on tips of the ellipse. Galaxies with a weak gravitational field (e.g. dwarfs) do not influence the shells even to that degree (see Eldorová, 2000).

Energy sources, which are not located in the symmetry plane of the disk, produce asymmetrical shells. However, as the fragmentation takes place in the densest regions, it is not influenced very much by the asymmetry of the shell. The blow-out effects (and the subsequent decrease of the “effective” energy) are enhanced in this case.

6. Conclusions

We have studied the fragmentation processes in expanding shells using the analytical solution and numerical simulations. The influence of the total energy, the kind of the energy input (“SNR” vs “GRB”), the sound speed in the ISM, a thickness and a profile of the gaseous disk were analyzed with the following result: for each type of the source and a total energy of the shell there exist a critical surface density of the disk $N_{\text{crit}}$: shells evolving in disks with a lower density than this are stable against the fragmentation. There is one exception to this rule: the maximum volume density deduced from $N_{\text{crit}}$ can never be lower than the $n_{\text{crit,hom}}$ — the density of the homogeneous medium in which the spherical shell with the same

![Fig. 8. Critical surface densities $N_{\text{crit}}$ for different energies, types of an energy input and sounds of speed.](image)

energy starts to be unstable — otherwise the shell does not fragment. Both densities ($N_{\text{crit}}$ and $n_{\text{crit,hom}}$) depend on the speed of sound in the disk, for warmer disks the fragmentation is more difficult.

Both studied kinds of energy sources (“GRB” = the abrupt energy input, and “SNR” = the continuous energy input) are able to create gravitationally unstable shells; shells formed by the abrupt sources are more stable.

Values of $N_{\text{crit}}$ for reasonably large values of energy are of the order of $\left(10^{20} - 10^{21}\right) \text{cm}^{-2}$, which coincides with the value of observed threshold surface densities for the star formation in galaxies (Kennicutt, 1989). The agreement of these two quantities may indicate that the contribution of the star formation induced by shells to the total star formation is important.

The very steep dependence of critical densities $n_{\text{crit,hom}}$ and $N_{\text{crit}}$ on the speed of sound in the ISM (see equation 27 and a discussion below it) indicates (confirms) the importance of the self-regulation in the triggered star formation. Young stars in OB associations release the energy and compress the ambient ISM, creating shells. In dense walls of shells, the star formation may be triggered, if the disk surface density surpasses a critical value $N_{\text{crit}}$. The star formation is accompanied by the heating of the ISM, i.e. increasing random motions and the speed of sound. This leads to the increase of the needed $N_{\text{crit}}$ and a subsequent reduction of the star formation rate. The cooling of the ISM by the dissipation of energy in supersonic shocks, shock-shock collisions, etc. leads to the decrease of $N_{\text{crit}}$ and to the increase of the SFR, closing the self-regulating cycle.

Acknowledgements. Authors gratefully acknowledge financial support by the Grant Agency of the Academy of Sciences of the Czech Republic under the grant No. A300305/1997 and support by the grant project of the Academy of Sciences of the Czech Republic No. K1-003-601/4.