A Small Cosmological Constant, Grand Unification and Warped Geometry

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Abstract

We will show that it is possible to obtain the observed value for the four-dimensional cosmological constant $\Lambda_4$ by an exponential suppression of an initially Planck-sized constant. It is a surprising feature that the suppression-length corresponds to the GUT-scale. To make the relationship explicit, we consider a set-up consisting of two positive tension domain-walls, separated by a distance corresponding to the inverse of the GUT-scale and show how a realistic $\Lambda_4$ is obtained through a warp-factor of the five-dimensional theory. It is then pointed out, that the embedding of the set-up into IIB string-theory naturally describes a spontaneously broken $SU(6)$ SUSY GUT with interesting consequences for the doublet-triplet splitting problem of GUT-theories. Furthermore, the various hypercharge assignments of the SM matter fields arise in a simple way from a consistency argument and composition of two sorts of “basic” open string states.

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1 Introduction

The enormous smallness of the four-dimensional cosmological constant as constrained from cosmological and astronomical measurements by [1]

$$|\Lambda_4| \lesssim 10^{-47}\text{GeV}^4 \approx (1.8\text{ meV})^4,$$

is still not understood in a satisfactory way from a theoretical point of view. The energy-regime of the upper bound of some meV is rather unnatural in particle physics and a more common characteristic of condensed matter phenomena. However, it has to be noticed that the upper bound on the electron neutrino mass can be as low as 1meV [2], which comes strikingly close to this value. If experiment will eventually show that both numbers are indeed of the same order, this would be an intriguing hint to some deeper relation between the Standard Model and Gravity.

The hope that eventually a consistent theory of quantum gravity might be able to explain the vexing smallness has not been fulfilled yet, as the only consistent candidate, string- or M-theory, relies so heavily on exact supersymmetry. Since the tininess of the cosmological constant is measured at energies where Bose-Fermi degeneracy is seen to be violated, a supersymmetry-breaking mechanism would be needed which nonetheless should not give rise to a large $\Lambda_4$. An interesting M-theory inspired proposal has been made in [3]. The idea is that in three dimensions, supersymmetry enforces a zero cosmological constant but can exist without matching bosonic and fermionic degrees of freedom. If such a three-dimensional theory contains a modulus similar to the dilaton of string-theory, one could expect that at strong coupling a new dimension will open up. The hope would be that during the transition from weak to strong coupling the properties of a zero cosmological constant and in addition Bose-Fermi non-degeneracy are conserved.

Whereas in the very early universe a non-vanishing cosmological constant is welcome during the phase of inflation, we face the problem to understand the smallness of the cosmological constant in our low-energy world, nowadays. Therefore, we shall take the point of view in this paper, that there should also exist a rationale to understand the adjustment of the cosmological constant to tiny values not only by taking refuge to a Quantum Gravity description valid at Planck-energies but also by employing merely degrees of freedom which are available at low energies.

Furthermore, we shall adopt the view that our four-dimensional world consists of a thick wall, embedded in some higher-dimensional partially compactified space. Conceiving
our world as being located on a type IIB string-theory D3-brane in a ten-dimensional ambient space allowed to attack such fundamental problems as gauge and gravitational coupling unification or the Standard Model hierarchy problem from completely different point of views (see [21] and references therein) than the traditional technicolor or low-energy supersymmetry approaches. In a T-dualized type I string scenario, where two to six internal compact dimensions orthogonal to the 3-brane are chosen much larger than the remaining compact dimensions, one is able to lower the fundamental higher-dimensional Planck scale down to the TeV-scale [22]. This necessitates the large internal dimensions to be as large as 1mm resp. 1 fermi for two resp. six large internal dimensions. Most pronounced in the case of two large dimensions, this leads to another hierarchy between the new fundamental TeV-scale and the compactification scale $\mu \equiv h c/1 \text{mm} \approx 10^{-4}\text{eV}$. This drawback could be overcome by considering not a direct product structure for the background space-time but a warped metric instead. In particular, the warped metric of a slice of an AdS-space suspended between two four-dimensional domain walls offers a solution to the strong part of the hierarchy problem [23].

In [24] it has been shown how to stabilize the modulus, which describes the distance between the two walls, at a value of 10-50 Planck lengths. This is just the value which is compatible with the mentioned solution of the hierarchy problem. It remains to relax the fine-tuning condition between the bulk cosmological constant and the brane-tensions. Attempts in this direction have been undertaken recently [9],[10],[11]. However, the solution to the hierarchy problem cannot be maintained in these approaches as the solutions exhibit metrics that do have polynomial instead of exponential behaviour. The metrics vanish at two finite points in the extra dimension, thereby cutting off the infinite range through singularities. However, the nature of these singularities remains obscure.

A general review of the cosmological constant problem can be found in [4]. See [5],[6] for more recent reviews on the topic. [7] provides a recent discussion of the cosmological constant problem from the point of view of String-Theory). Apparently, lately there has been a noticeable increase in the efforts to solve the cosmological constant problem [8],[9],[10],[11],[12],[13],[14],[15],[16],[17],[18],[19],[20].

The outline of this paper is as follows. In the next section, we lay the framework for determining the effective four-dimensional cosmological constant $\Lambda_4$ by analyzing the Randall-Sundrum (RS) set-up [23]. In the following section, we start with the observation that to obtain the observed (upper bound of) $\Lambda_4$ by an exponential suppression from the Planck-scale enforces the introduction of a length which corresponds to the GUT-scale. A
geometric realization of such an exponentially suppressed $\Lambda_4$ is then given in terms of two positive tension domain walls. The next section deals with the effect that five-dimensional scalars have on $\Lambda_4$, by analyzing their effective four-dimensional potential. That they do not reintroduce a Planck-sized $\Lambda_4$ is a prerequisite for the embedding of the suppression mechanism of the five-dimensional wall picture into IIB string-theory, which is described in section 5. Finally, section 6 treats some generic features which result from the D3-brane description of the former wall set-up. Among then is a natural emergence of an SU(6) SUSY Grand Unified Theory (GUT) with gauge-group broken to the Standard-Model (SM) group SU(3)$_c \times$ SU(2)$_L \times$ U(1)$_Y$ and a natural occurrence of the doublet-triplet Higgs-boson mass-splitting. In two appendices, we first deal with generalizations of the two wall set-up to walls with unequal tensions and second analyze the influence of bulk scalars on $\Lambda_4$ for the unequal tension case.

2 The Effective Cosmological Constant

Let us start by recapitulating how the vanishing of the effective four-dimensional cosmological constant comes about in the RS scenario [23]. Whenever we are given a four-dimensional Poincaré-invariant flat metric $ds^2_4 = \eta_{\mu\nu}dx^\mu dx^\nu$, we deduce from the D=4 Einstein field equation

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi G\Lambda_4 g_{\mu\nu},$$

that this implies $\Lambda_4 = 0$. Since upon integrating out the fifth dimension in the RS set-up, we are left precisely with a Poincaré-invariant flat metric, we conclude that the effective $\Lambda_4$ in this scenario has to vanish. Subsequently, we will analyze in more detail how this is achieved precisely.

The RS-model [23] consists of two walls located at the fixed points of an $S^1/\mathbb{Z}_2$ orbifold in the fifth direction and a bulk gravitational plus cosmological constant part in between. The Planck-brane, on which the four-dimensional graviton is localized, sits at the first fixed-point, $x^5 = 0$ of the $\mathbb{Z}_2$-action, whereas our four-dimensional world is supposed to be placed on the SM-wall at $x^5 = \pi r$, the second fixed-point. It is only this latter wall on which the hierarchy problem can be solved by means of the exponential warp-factor in the anti-de Sitter bulk geometry. Concerning the interaction between the walls and bulk gravity, the dominant contribution comes from the wall tension term in the effective field theory on the wall [25]. Hence, if one is interested in a situation where the walls are
close to their ground states, it is reasonable to neglect gauge-fields, fermions and scalars on them. Taking this into account the RS-Lagrangean\(^2\) reads \[23\]

\[
S_{RS} = - \int d^4x \int_0^{\pi r} dx^5 \left\{ \sqrt{G} \left( M^3 R + \Lambda \right) + \sqrt{g_{Pl}} T_{Pl} \delta(x^5) + \sqrt{g_{SM}} T_{SM} \delta(x^5 - \pi r) \right\}.
\] (3)

As we will see in the later computation, it is important to just integrate the bulk piece over the interval\(^3\) \([0, \pi r]\) (or rather from \(-\epsilon\) to \(\pi r + \epsilon\) with \(\epsilon\) infinitesimal to incorporate the delta-function sources on the boundaries properly) in order to find a vanishing \(\Lambda_4\). The four-dimensional metrics \(g_{SM}^{(4)}, g_{Pl}^{(4)}\) are the pullbacks of the bulk metric to the two domain-wall world-volumes. Adopting the Ansatz

\[
ds^2 = e^{-A(x^5)} \eta_{\mu\nu} dx^\mu dx^\nu + (dx^5)^2,
\] (4)

the Einstein equation results in

\[(A')^2 = - \frac{1}{3M^3} \Lambda, \quad A'' = \frac{1}{3M^3} \left( T_{Pl} \delta(x^5) + T_{SM} \delta(x^5 - \pi r) \right). \] (5)

The solution to the first equation of (5) is given by

\[
A(x^5) = \pm k x^5, \quad k \equiv \sqrt{\frac{-\Lambda}{3M^3}},
\] (6)

where the integration constant has been set to zero for it can be absorbed into a rescaling of the \(x^\mu\) coordinates. To respect the imposed \(\mathbb{Z}_2\) symmetry, we have to take

\[
A(x^5) = \pm k |x^5|.
\] (7)

In the following, we will choose the plus-sign which allows for a solution of the hierarchy problem on the SM-wall. Noting that \(|x^5|'' = 2\delta(x^5)\), we rewrite our solution in an expanded form as

\[
A(x^5) = \frac{1}{2} k \left( |x^5| - |x^5 - \pi| \right) + \frac{1}{2} k \pi, \quad 0 \leq x^5 \leq \pi r
\] (8)

\(^2\)Subsequently, we will adopt General Relativity conventions as, e.g. used in [26].

\(^3\)This is analogous to the downstairs approach in heterotic M-theory [27], in which there is an analogous orbifold-procedure for the eleventh direction. In the alternative upstairs approach, one would integrate the Lagrangean density over the full circle but in addition has to place a factor of 1/2 in front of the integral due to the \(\mathbb{Z}_2\) symmetry of the action.
in order to satisfy the second equation of (5) with

\[ T_{Pl} = -T_{SM} = 3M^3k. \]  

Let us now determine the four-dimensional effective action by integration over the fifth dimension and start with the Einstein-Hilbert term of the bulk action. For this purpose, consider first the general \( D \)-dimensional case with metric

\[ ds^2 = G_{MN}dx^M dx^N = g^{(D-1)}_{\mu\nu}dx^\mu dx^\nu + (dx^D)^2 = f(x^D)g_{\mu\nu}(x^\rho) dx^\mu dx^\nu + (dx^D)^2, \]  

where \( \mu, \nu \) run over 1, \ldots, \( D-1 \) and \( M, N \) over 1, \ldots, \( D-1, D \). The \( D \)-dimensional curvature scalar can then be decomposed in the following way into the \( (D-1) \)-dimensional curvature scalar plus additional terms depending exclusively on \( x^D \)

\[ R(G) = \frac{1}{f} R(g) + \frac{1}{4} (D-1) \left( (D-2) [\ln f']^2 + 2(\ln f)'' + 2 \frac{f''}{f} \right). \]  

In addition, we have to take into account a factor \( \sqrt{G} = f^{(D-1)/2} \sqrt{g} \) in the measure of the action integral. Specializing now to the RS case with \( D = 5 \) we take the metric

\[ ds^2 = G_{MN}dx^M dx^N = e^{-A(x^5)}g_{\mu\nu}(x^\rho)dx^\mu dx^\nu + (dx^5)^2, \]  

with \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \), where \( h_{\mu\nu} \) describes the four-dimensional graviton propagating on the flat background. This has to be plugged into the RS-action and integrated over the fifth dimension. Using (11) with \( f(x^5) = e^{-A(x^5)} \), we get

\[ S_{EH} = - \int d^4x \int_0^{\pi r} dx^5 \sqrt{G}M^3 R(G) \]

\[ = - \int d^4x \int_0^{\pi r} dx^5 f^2 \sqrt{g}M^3 \left\{ \frac{R(g)}{f} + 3 [\ln f']^2 + 2(\ln f)'' + 2 \frac{f''}{f} \right\} \]

\[ = - \int d^4x \sqrt{g}M^3 \int_0^{\pi r} dx^5 \left\{ e^{-A}R(g) + e^{-2A} [5(A')^2 - 4A''] \right\}. \]  

Since we will come back to this formula afterwards, we note that up to this point it is valid for any metric which is of the form (12). Choosing the RS-metric we receive

\[ S_{EH} = - \int d^4x \sqrt{g}M^3 \left\{ R(g) \int_0^{\pi r} dx^5 e^{-kx^5} + \int_0^{\pi r} dx^5 e^{-2kx^5} [5k^2 \]

\[ - 4k (\delta(x^5) - \delta(x^5 - \pi r))] \right\}. \]  

\[ \text{By } f' \text{ we mean } df/dx^D. \]
Concerning the delta-function integration, we imagine performing the integration actually over the interval \([-\epsilon, \pi r + \epsilon]\) with \(\epsilon\) infinitesimal. This full consideration of all the fixed-point sources will be important to arrive at \(\Lambda_4 = 0\), finally. Thus the Einstein-Hilbert part gives

\[
S_{\text{EH}} = -\int d^4x \sqrt{g} \left\{ M_{\text{Pl}}^2 R(g) - \frac{3}{2} M^3 k (1-e^{-2k\pi r}) \right\},
\]

where \(M_{\text{Pl}}^2 = 2M^3(1-e^{-k\pi r})/k\) denotes the effective four-dimensional Planck-scale squared. The second part of the reduction comprises the wall sources and the bulk cosmological constant term

\[
S_{\text{Pl}} + S_{\text{SM}} + S_{\Lambda} = -\int d^4x e^{-2A(\pi r)} \sqrt{g} T_{\text{SM}} - \int d^4x e^{-2A(0)} \sqrt{g} T_{\text{Pl}}
- \int d^4x \int_0^{\pi r} dx' e^{-2A(x')} \sqrt{g} \Lambda \\
= -\int d^4x \sqrt{g} \left\{ e^{-2k\pi r} T_{\text{SM}} + T_{\text{Pl}} + \Lambda \int_0^{\pi r} dx' e^{-2kx'} \right\}
- \int d^4x \sqrt{g} \left\{ e^{-2k\pi r} T_{\text{SM}} + T_{\text{Pl}} - \frac{3}{2} M^3 k \left(1-e^{-2k\pi r}\right) \right\}
- \frac{3}{2} \int d^4x \sqrt{g} M^3 k \left(1-e^{-2k\pi r}\right).
\]

In conclusion, we see that due to the fine-tuned values of the Planck- and SM-wall tensions in terms of the bulk cosmological constant, both contributions to \(\Lambda_4\) add up to zero as expected.

Suppose that the brane-tensions had not been fine-tuned to their RS-values but were merely chosen to be equal up to a minus sign. In other words suppose, that the bulk cosmological constant takes any non-positive value. Then, according to the above calculation we expect a residual four-dimensional cosmological constant of the order of

\[
\Lambda_4 \sim (\sqrt{-3M^4\Lambda} - T_{\text{Pl}})(1-e^{-2k\pi r}).
\]

Such an effective \(\Lambda_4\) constitutes a potential for the hitherto unconstrained modulus \(r\). Its minimum lies at \(r = 0\) if we assume that \(\sqrt{-3M^4\Lambda} > T_{\text{Pl}}\) and would indeed drive \(\Lambda_4\) to zero\(^5\). (If \(\sqrt{-3M^4\Lambda} < T_{\text{Pl}}\), the minimum would lie at \(r = \infty\) which implies a runaway behaviour.) However, an estimation how close \(r\) has to come to zero to actually solve the

\(^5\)Note that in this limit the hierarchy-problem cannot be solved any longer.
cosmological constant problem is rather disenchanting. If we take \(\sqrt{-3M^3}\Lambda - T_{Pl} \simeq M_{Pl}^4\), \(k \simeq M_{Pl}\) and demand that \(\Lambda_4 \simeq (1\text{meV})^4\), we find an incredibly small \(r \simeq 10^{-125}l_{Pl}\), with \(l_{Pl} = M_{Pl}^{-1}\) and the Planck-mass \(M_{Pl} = 1.2 \times 10^{19}\text{GeV}\). This, however, is a region, where we surely cannot trust classical gravity as a reliable description.

Basically, the problem lies in the contribution of the 1 in the expression (16) for \(\Lambda_4\). Without it, we could solve the cosmological constant problem in a way analogous to the RS-mechanism of solving the hierarchy-problem, namely through the suppression by an exponential factor. Removing the Planck-brane does not help, since with a single wall only the disfavoured \(r\)-independent contribution survives. Because it has its origin in the integration of the RS-action over the region around \(x^5 = 0\), where the warp-factor becomes 1, we have to find a configuration of at least two walls, where the warp-factor can never become 1 but instead keeps exponentially small throughout the whole \(x^5\) integration region.

In general, \(\Lambda_4\) will be expressed by a product of a function containing \(\Lambda\) and the wall-tensions times a geometrical factor reflecting the geometry, in which the walls are embedded. Thence we perceive that a small \(\Lambda_4\) can either be obtained by using an appropriate ambient space geometry or alternatively by finding some mechanism which guarantees a small higher-dimensional \(\Lambda\) plus wall tensions. In this paper we will address the first approach and leave the second to [28].

3 Our World as a Two Wall System with Small \(\Lambda_4\)

We have seen in the last section that one should avoid placing a wall at the origin, the one fixed point of the \(\mathbb{Z}_2\) symmetry (since this gives rise to a Planck-scale \(\Lambda_4\)). Instead, we will place two walls at the \(\mathbb{Z}_2\) mirror-points \(x^5 = -l, l\). As we will see soon, the warp-factor, which in essence determines the degree of suppression of \(\Lambda_4\), decreases exponentially with \(l\) only if the two walls are located at \(\mathbb{Z}_2\) mirror-points.\(^6\) It is remarkable that already a

\(^6\)Note that in [18] time-dependent models with two walls were considered. However, due to the location of one of the walls at the fixed-point \(x^5 = 0\), one has to distinguish between a visible and a hidden world much as in the original RS-model. On the visible world the authors of [18] find an exponentially small cosmological constant in five dimensions. However, upon deriving the effective four-dimensional \(\Lambda_4\) of the whole set-up by integrating out the fifth dimension the huge cosmological constant of the hidden world reappears and spoils the smallness of \(\Lambda_4\). It is the very property of gravity to penetrate the whole bulk, which requires a small warp-factor throughout spacetime to obtain a small effective \(\Lambda_4\). This stands in contrast to the discussion of the hierarchy problem along the lines of [23], which only relies on the local
length of $l = 284 l_{Pl}$ can yield, in combination with an exponential suppression factor, the observed value (upper bound) for $\Lambda_4$ given in (1)

$$e^{-M_{Pl}l} M_{Pl}^4 = 10^{-47} \text{GeV}^4.$$ (17)

The intriguing observation is, that the double of this length, $2l = 568 l_{Pl}$, which will play later on the role of the distance between the two walls, corresponds to the GUT-unification scale

$$2l = 568 l_{Pl} \leftrightarrow M_{\text{GUT}} = 2 \times 10^{16} \text{GeV}.$$ (18)

This already indicates some deeper relationship between GUT-theories and the cosmological constant or in other words gravity than traditionally assumed. The connection with GUT-theories will be made more precise in section 5 and 6. Let us now visualize our world as possessing a thickness $2l$ in the $x^5$ direction, which when zoomed in consists of two walls located at $l$ and $-l$ respectively (see fig.1). Both walls can only communicate via gravity to each other. In order to save our world from precarious instabilities, we will choose both wall-tensions as positive. Because the inter-wall distance is larger than the Planck-length and even the string-length $l_s = \sqrt{\alpha'} \sim 10 l_{Pl}$, it is justified to describe the whole set-up by the low-energy action

$$S = -\int d^4x \int dx^5 \sqrt{G} \left( M^3 R(G) + \Lambda \right) - \int d^4x \int dx^5 \left( \sqrt{g_1^{(4)} T_1} \delta (x^5 + l) + \sqrt{g_2^{(4)} T_2} \delta (x^5 - l) \right).$$ (19)

Again $g_1^{(4),\mu\nu}$ and $g_2^{(4),\mu\nu}$ are the induced metrics arising from the pullback of $G_{MN}$ to the two wall world-volumes. Choosing again the Ansatz (4), the Einstein field equations reduce warp-factor at the position of the wall.
to (5) with the tension on the right-hand-side of the $A''$ equation given this time by

$$T(x^5) = T_1 \delta (x^5 + l) + T_2 \delta (x^5 - l) .$$

(20)

In order to concentrate on the essential aspects of the suppression mechanism, we will keep things as simple as possible in the following by choosing equal tensions $T_1 = T_2 = T$ for the walls. The case with unequal tensions will be treated in appendix A.

The solution\footnote{In case that we work on a circle and identify $-l \sim l$, the solution will only be given by the restriction to $-l \leq x^5 \leq l$.} to the Einstein equation is given by

$$A(x^5) = \frac{k}{2} |x^5 + l| + \frac{k}{2} |x^5 - l| = \begin{cases} x^5 \leq -l : & -kx^5 \\ -l \leq x^5 \leq l : & kl \\ x^5 \geq l : & kx^5 \end{cases} ,$$

(21)

together with the bulk cosmological constant

$$\Lambda(x^5) = \begin{cases} \Lambda_e , & |x^5| \geq l \\ \Lambda_i , & |x^5| < l \end{cases} = \begin{cases} -3M^3k^2 , & |x^5| \geq l \\ 0 , & |x^5| < l \end{cases}$$

(22)

and the wall-tension

$$T = 3M^3k .$$

(23)

Note that the two terms of (21) are dictated by the choice of sources in (20) and the symmetry of the set-up. Any further integration constant which could be added to (21) is immaterial, since it can be absorbed into a redefinition of the $x^\mu$ coordinates describing the four-dimensional section of the five-dimensional configuration. Again, the flat four-dimensional metric in the above Ansatz implies, as usual, a fine-tuning between the parameters $\Lambda$ and $T$

$$\Lambda_e = -\frac{1}{3} \frac{T^2}{M^3} , \quad \Lambda_i = 0 .$$

(24)

The function $A(x^5)$, which determines the warp-factor is displayed in fig.2. The corresponding warp-factor $e^{-A(x^5)}$ is upper-bounded by $e^{-kl}$ throughout the whole fifth dimension. This is the basic reason why, in view of (17), the warp-factor will be capable of suppressing any (induced from the higher-dimensional Riemann curvature scalar or actually bulk) Planck-size cosmological constant down to the observed value of $10^{-47}$GeV$^4$, if $k$ takes its natural value at $M_{Pl}$. From a low-energy (energy far below $M_{GUT}$) point
of view, we would regard the distance $2l$ between both walls as too small to recognize them as two separate walls. Such a low-energy observer would realize one wall with tiny thickness and a geometry which consists of two slices of Anti-de Sitter spacetime directly glued together. For her/him the graviton would appear localized on a single thick wall as described in [29].

The next task is again the determination of the effective four-dimensional action by integrating out the $x^5$ coordinate for the metric background

$$ds^2 = e^{-A(x^5)}g_{\mu\nu}(x^\nu)dx^\mu dx^\nu + (dx^5)^2.$$  \hfill (25)

Along the same lines as above for the RS-case and by employing (13), we get

$$S_{EH} = -\int d^4x\sqrt{g}M^3 \left\{ R(g) \int_{-\infty}^{\infty} dx^5 e^{-A} + \int_{-\infty}^{\infty} dx^5 e^{-2A} \left[ 5(A')^2 - 4A' \right] \right\}$$

$$= -e^{-kl} \int d^4x\sqrt{g}M^3 \left\{ 2R(g) \left[ \frac{1}{k} + \frac{l}{l} \right] - 3e^{-kl} \right\}. \hfill (26)$$

For the other terms we arrive at

$$S_{SM_1} + S_{SM_2} + S_A = -e^{-2kl} \int d^4x\sqrt{g} \left\{ 2T + \frac{\Lambda_c}{k} \right\}. \hfill (27)$$

Pulling out an overall factor of $e^{-kl}$ in front, the final effective action reads

$$S_{EH} + S_{SM_1} + S_{SM_2} + S_A$$

$$= -e^{-kl} \int d^4x\sqrt{g} \left\{ 2M^3 R(g) \left[ \frac{1}{k} + \frac{l}{l} \right] + e^{-kl} \left[ -3M^3k + 2T + \frac{\Lambda_c}{k} \right] \right\}. \hfill (28)$$
At the classical level an overall constant multiplying the whole action is irrelevant – it drops out of the classical equations. If we regard the cosmological constant problem as a low-energy one (since it is here where experiments contradicting theoretical expectations are carried out), quantum gravitational effects should not play a role and a classical description of gravity suffices. Unlike quantum gravitational effects, quantum effects of the strongly, weakly or electromagnetically interacting fields located on the walls may become important already at low-energies\textsuperscript{8}. They can be included and merely result in a renormalization of the wall-tension $T$. With this in mind, let us drop the overall scale-factor and finally arrive at the effective action

$$S_{D=4} = - \int d^4x \sqrt{g} \left\{ M^2_{\text{eff}} R(g) + \Lambda_4 \right\} ,$$

with the effective four-dimensional Planck-scale $M_{\text{eff}}$ and the four-dimensional cosmological constant $\Lambda_4$ given by\textsuperscript{9}

$$M^2_{\text{eff}} = 2M^3 \left[ \frac{1}{k} + l \right] \quad (31)$$

$$\Lambda_4 = e^{-kl} \left[ -3M^3k + 2T + \frac{\Lambda_e}{k} \right]. \quad (32)$$

Note the huge suppression-factor $e^{-kl}$ multiplying the whole effective cosmological constant, which is essential for our proposition of achieving the observed value for $\Lambda_4$. One can now show easily, that when the above obtained values (22),(23) for $T,\Lambda_e$, which correspond to the special flat solution $g_{\mu\nu}(x^0) = \eta_{\mu\nu}$ for the four-dimensional metric, are substituted in the derived effective action, we arrive at a zero $\Lambda_4$. This serves as a check on the derivation, since a flat four-dimensional metric, caused by the fine-tuned parameters, must necessarily entail a vanishing four-dimensional cosmological constant.

Let us now refrain from the restriction of fine-tuning the parameters and allow for generic values of the wall-tension and the cosmological constant, i.e. we want to suspend the fine-tuning constraints given by (24). Lifting the fine-tuning, corresponds to a non-trivial four-dimensional metric $g_{\mu\nu} \neq \eta_{\mu\nu}$ in the Ansatz

$$ds^2 = e^{-A(x^5)} g_{\mu\nu} dx^\mu dx^\nu + (dx^5)^2 .$$

\textsuperscript{8}The most drastic example is furnished by the mass of the light CP-even Higgs-boson $h^0$ of the MSSM. Its mass gets shifted by 1-loop corrections from its tree-value $m_Z$ up to 100-150GeV depending on the value of the mixing-parameter $\tan \beta$.

\textsuperscript{9}If we worked on a circle $x^5 \in [-l, l]$ with $-l \sim l$, we would obtain

$$M^2_{\text{eff}} = 2M^3 l , \quad \Lambda_4 = e^{-kl}(2T + 2l\Lambda_4) \quad (30)$$

with likewise exponentially suppressed $\Lambda_4$. 

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Generically we want to choose \( k \simeq M_{Pl}, \ T \simeq M_{Pl}^5, \ \Lambda_e \simeq M_{Pl}^5 \) and furthermore the fundamental five dimensional Planck-scale \( M \simeq M_{Pl} \). Furthermore, we have to remember to reintroduce \( \Lambda_i \). Since \( \Lambda_i \) has been absent in the above solution, we have to take refuge to the more general case with unequal tensions, where a non-vanishing \( \Lambda_i \) shows up. The corresponding expression for the four-dimensional effective cosmological constant (97) can be found in appendix A. If we take of it the limit of coinciding tensions (or equivalently \( k_{12} = k_1 - k_2 = 0 \)) but leave \( \Lambda_i \) free, we obtain

\[
\Lambda_4 = e^{-kl} \left[ -3M^3 k + 2T + \frac{\Lambda_e}{k} + 2l\Lambda_i \right]. \tag{34}
\]

To guarantee that \( 2l\Lambda_i \leq M_{Pl}^4 \), we have to choose \( \Lambda_i \leq (3 \times 10^{18}\text{GeV})^5 \), which still seems to be quite generic. We then recognize from (34), that the suppression through the exponential factor is sufficient to bring the various contributions to the four-dimensional cosmological constant down to its observed value (1) by means of (17).

The effective four-dimensional Planck-scale \( M_{eff} \simeq 24M_{Pl} \) comes out slightly too high. It can however be easily brought down, e.g. to \( M_{eff} \simeq M_{Pl} \), if we choose the fundamental scale as \( M \simeq 1.5 \times 10^{18}\text{GeV} \), which is close to the traditional string-scale \( M_s = 1/\sqrt{\alpha'} \) and may be considered as a hint to a stringy origin of the set-up. To summarize, we can choose the values of \( T, \Lambda_e, \Lambda_i \) without fine-tuning or taking refuge to supersymmetry but nonetheless attain the observed (upper bound) of the four-dimensional cosmological constant \( \Lambda_4 \).

4 The Effective Potential due to Bulk Scalars

The embedding of our five-dimensional set-up into IIB string-theory or F-theory along the lines of [30] gives a host of bulk-fields in addition. They arise from the usual dimensional reduction procedure from ten to five dimensions. It is important to check that these further fields do not reintroduce huge contributions to the effective four-dimensional cosmological constant upon further reduction from five to four dimensions. Otherwise the embedding of our set-up together with the above mechanism to exponentially suppress the cosmological constant would be immediately spoiled.

To this aim, we will examine in this section the four-dimensional effective potential which is engendered by a generic five-dimensional bulk scalar \( \Phi \). Let us assume a bulk scalar \( \Phi \) with quartic couplings to the two walls as in the Goldberger-Wise mechanism
[24] which stabilizes the RS-scenario. For the action of the scalar with mass $m$, let us take

$$S_\Phi = - \int d^4x \int_{-\infty}^{\infty} dx_5 \sqrt{G} \left\{ \frac{1}{2} G_{MN} \partial_M \Phi \partial_N \Phi + \frac{1}{2} m^2 \Phi^2 \right\}$$

$$- \int d^4x \int_{-\infty}^{\infty} dx_5 \left\{ \sqrt{g_4^{(4)}} \lambda_1 (\Phi^2 - v_1^2)^2 \delta(x^5 + l) + \sqrt{g_2^{(4)}} \lambda_2 (\Phi^2 - v_2^2)^2 \delta(x^5 - l) \right\}. \quad (35)$$

Assuming that $\Phi$ does only depend on $x^5$ and employing the five-dimensional metric

$$G_{MN} = e^{-A(x^5)} \eta_{\mu \nu} dx^\mu dx^\nu + (dx^5)^2 \quad (36)$$

together with (21) as the gravitational background, we arrive at the following field equation

$$(e^{-2A} \Phi')' - e^{-2A} m^2 \Phi = 4 \left[ e^{-2A(-l)} \lambda_1 (\Phi^2 - v_1^2) \Phi \delta(x^5 + l) + e^{-2A(l)} \lambda_2 (\Phi^2 - v_2^2) \Phi \delta(x^5 - l) \right]. \quad (37)$$

Away from the walls it has the solution

$$\Phi(x^5) = \begin{cases} 
  ae^{(1+\Gamma)A} + be^{(1-\Gamma)A}, & x^5 < -l \\
  ce^{mx^5} + de^{-mx^5}, & |x^5| \leq l \\
  ee^{(1+\Gamma)A} + fe^{(1-\Gamma)A}, & x^5 > l 
\end{cases} \quad (38)$$

with

$$\Gamma = \sqrt{1 + m^2/k^2} \quad (39)$$

and arbitrary coefficients $a, b, c, d, e, f$. In order to obtain a normalizable solution for $\Phi$, we are forced to set $a = e = 0$. Furthermore, demanding continuity of $\Phi$ at the walls determines $b$ and $f$ in terms of $c, d$

$$b = e^{(\Gamma-1)k}\tilde{b}, \quad \tilde{b} = ce^{ml} + de^{-ml} \quad (40)$$

$$f = e^{(\Gamma-1)k}\tilde{f}, \quad \tilde{f} = ce^{ml} + de^{-ml}. \quad (41)$$

To fix the remaining coefficients $c$ and $d$ one would have to plug the above bulk solution in the field equation and integrate over the fifth dimension to incorporate the wall boundary conditions. However, this leads to a complicated cubic equation in the unknowns $c, d$. An easier way [24] to arrive at a determination of the coefficients $c, d$ is to put the above
bulk solution into the scalar action and integrate over $x^5$ to arrive at an effective potential for the distance-parameter $l$. From the couplings of $\Phi$ to the walls the effective potential receives the contributions

$$
\int d^4x \left\{ \sqrt{g_1^{(4)}} \lambda_1 \left( \Phi^2(-l) - v_1^2 \right)^2 + \sqrt{g_2^{(4)}} \lambda_2 \left( \Phi^2(l) - v_2^2 \right)^2 \right\} .
$$

Hence, to minimize the potential for positive couplings $\lambda_1, \lambda_2$, we must set $\Phi(-l) = v_1$ and $\Phi(l) = v_2$. These two further conditions then allow for a determination of $c, d$ in terms of the parameters $v_1, v_2$

$$
c = \frac{-v_1 e^{-ml} + v_2 e^{ml}}{2 \sinh(2ml)} , \quad d = \frac{v_1 e^{ml} - v_2 e^{-ml}}{2 \sinh(2ml)} .
$$

Finally, the effective four-dimensional potential $V_\Phi$, defined by $S_\Phi = -\int d^4x \sqrt{\mathcal{g}} V_\Phi(l)$, becomes

$$
V_\Phi(l) = \frac{e^{-2kl}}{2l} \left\{ (v_1^2 + v_2^2) \left[ (1 - \Gamma)k + m \coth(2ml) \right] - 2v_1v_2 \frac{m}{\sinh(2ml)} \right\} ,
$$

where the identity $(1 - \Gamma)^2 k^2 + m^2 = 2\Gamma(1 - \Gamma)k^2$ has been utilized. For the special case of a massless, $m = 0$, bulk scalar $\Phi$, the effective potential simply reads

$$
V_\Phi(l) = \frac{e^{-2kl}}{4l} (v_1 - v_2)^2 .
$$

The first conclusion is, that regardless of the mass $m$ chosen, the important exponential suppression-factor shows up again, thus guaranteeing that for generical values of $v_1, v_2, m$ these effective potentials cannot generate a cosmological constant larger than its experimental value (1).

The second observation pertains to the possibility of achieving in addition stability with these tiny potentials. From (45) it is immediately recognizable, that no minimum at finite $l$ exists. For the case with $m \neq 0$, setting $\partial V_\Phi/\partial l$ equal to zero, amounts to solving the equation

$$
(w^2 + 1) \left[ \left( \frac{1}{r} + \cosh(2ml) \right) \sinh(2ml) + r \right] = 2w \left[ r \cosh(2ml) + \sinh(2ml) \right] ,
$$

where we have used the dimensionless variables

$$
w = \frac{v_1}{v_2} , \quad r = \frac{m}{k} .
$$
in terms of which we find $\Gamma = \sqrt{1 + r^2}$. It is now easy to analyze the last equation numerically with the result that there are no real and positive solutions for the distance-parameter $l$ for generic values of $v_1, v_2, m$. Therefore we conclude that in the general case with $m \neq 0$, the effective potential exhibits no minimum either.

The general case with different tensions $T_1 \neq T_2$ will be covered in appendix B. We thus learn that a generic bulk scalar, with couplings to the walls, leads to an effective potential, which is likewise exponentially suppressed. The addition of further bulk scalar fields therefore seems to present no obstruction to obtain the right value for $\Lambda_4$. This is an essential ingredient for an embedding of the suppression mechanism into IIB string- or F-theory, where one faces a host of extra bulk scalar fields from dimensional reduction to five dimensions.

We have seen that roughly the complete effective potential ($= \Lambda_4$) goes like $e^{-M_{Pl}d}$ with the inter-wall distance $d$. For $d = l = 284l_{Pl}$ nowadays this potential is very tiny, namely of the order of $(1\text{meV})^4$. Likewise the repelling force driving the walls apart is exceedingly small such that we can regard this scenario as quasi-static. In the very early universe, however, during the period of inflation, actually a nonvanishing four-dimensional cosmological constant $\Lambda_4 = V(\phi)$ is needed. Here $V(\phi)$ denotes the potential or vacuum energy density of the inflaton $\phi$. For example in the scenario of chaotic inflation [38] one imposes the initial condition $V(\phi) \approx M_{Pl}^4$, which implies $\Lambda_4 \approx M_{Pl}^4$. Thus in the very early universe, we face a situation, where the two walls have to be much closer together. At the same time this means that the repelling forces which drive the walls apart are significantly larger since the suppression-factor $e^{-M_{Pl}d}$ becomes larger. Therefore, this period witnessed (if there are no other stabilizing contributions to the potential at those early times) a rapid expansion in the $x^5$ direction.

5 IIB-String Embedding of the Set-Up

To better understand the appearance of the GUT-scale in the above low-energy set-up, we will now indicate how it can be embedded into IIB string-theory resp. F-theory. Moreover, we will point out a natural connection to an SU(6) SUSY Grand Unification (GUT), whose gauge symmetry has to be broken if we want to obtain a realistically small $\Lambda_4$. To this aim we are going to describe in this and the next section some rather generic features of such an embedding, which may serve as a guideline for the actual model building, e.g. in terms of an orbifold construction [34].

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Figure 3: The two walls resolved as stacks of D3-branes in a microscopic IIB string description.

Since the low-energy five-dimensional geometry consists of two half infinite AdS$_5$ patches divided by two four-dimensional positive tension walls with an interpolating $x^5$-finite flat spacetime interval in between, one may think of two stacks of D3-branes at opposite values of $X^5$ in the IIB-string setting$^{10}$. For the low-energy situation with two walls of equal tension, we should place the same amount of D3-branes in either stack. If we furthermore want to embrace the minimal supersymmetric extension of the Standard Model (MSSM), we have to group the branes in such a way, that the low-energy gauge-group SU(3)$_c \times$ SU(2)$_L \times$ U(1)$_Y$ arises. Together, this requires 3 D3-branes in one stack and another 3 making up the other stack. To accomodate for the SU(2)$_L \times$ U(1)$_Y$ part, we imagine a tiny split between the 3 branes of the second stack into 2 giving rise to SU(2)$_L$ and a single one responsible for U(1)$_Y$ (see fig.3). Actually, the D3-brane gauge-group will be U(1)$\times$ U(1)$\times$ U(1)$_Y$ with two further local U(1) symmetries. However, usually we have to project out some massless states (e.g. the unphysical four-dimensional gauge-field $A^{ij}_\mu$ in the last section), which is conveniently done through an orbifold procedure. Then typically only one of the three abelian gauge groups stays anomaly-free [34],[35], while the others become anomalous. Since we will arrive at the correct hypercharge assignments for the light MSSM matter in the next section, U(1)$_Y$ will be the anomaly-free factor. The anomalies of the two further abelian factors are cancelled in string-theory by a Green-Schwarz mechanism, which renders them massive. They remain as global symmetries with mass of the U(1) gauge-bosons at the string-scale.

$^{10}$X$^5$ is that coordinate which describes a variation in the IIB string-frame if we vary $x^5$ in the low-energy-frame.
For a $\mathbb{Z}_2$ symmetric ($\mathbb{Z}_2 : X^5 \to -X^5$) D3-brane configuration as depicted in fig.3, it has been argued in [30], that in the low-energy five-dimensional description the D3-brane sources lead to half-infinite throats which describe exactly two AdS$_5$ half-slices. Suppose there are no other gravitational sources located in between the two D3-brane stacks. Then due to the $\mathbb{Z}_2$ reflection symmetry, the low-energy warp-factor in the intermediate interval cannot develop a kink but has to be constant. Thus it seems indeed possible to arrive at the above low-energy geometry, given by (12),(21), through a simple set-up of oppositely placed D3-brane stacks in the context of IIB- or F-theory.

Because the D3-branes are localized in the internal six- (for IIB) or eight-dimensional (for F-theory) space, they are not sensitive to the global properties of the compactification space. However, there is a link between both which comes from tadpole cancellation [36],[37], or conservation of the RR 5-form flux, and states in our case with 6 D3-branes that the Euler-characteristic $\chi$ and the background fluxes have to obey

$$6 = \frac{\chi(K_8)}{24} - \frac{1}{4\pi^2} \int_{K_6 \cap 2} H \wedge \tilde{H}. \quad (48)$$

Here, $K_6$ denotes the base of the underlying F-theory compactification on an elliptically fibered Calabi-Yau fourfold $K_8$. Furthermore, the 3-forms $H, \tilde{H}$ are given by

$$H = H^R - \tau H^{NS}, \quad \tilde{H} = H^R - \bar{\tau} H^{NS}, \quad (49)$$

with $\tau = \tau_1 + i\tau_2 = a + ie^{-\phi}$ the modulus of the elliptic fibration composed out of the axion $a$ and the dilaton $\phi$.

The connection between the IIB string-frame metric $G^\sigma_{AB}; A,B = 1, \ldots, 10$ and the low-energy metric $G_{AB}$, which is used to measure length in the above five-dimensional set-up is as follows [30]. String-frame and Einstein-frame metric are related by

$$G^E_{AB} = e^{-\frac{4}{3}\phi} G^\sigma_{AB}, \quad (50)$$

whereas the low-energy metric

$$ds^2 = G_{AB} dx^A dx^B$$

$$= e^{-A(x^5)} g_{\mu\nu}(x^\rho) dx^\mu dx^\nu + (dx^5)^2 + h_{mn}(x^5,y^k) dy^m dy^n, \quad m,n = 6, \ldots, 10 \quad (51)$$

is related to $G^E_{AB}$ by a further Weyl-rescaling

$$G_{AB} = V_5^{1/4} G^E_{AB}, \quad V_5 = \frac{\int_{K_5} d^5 y \sqrt{h}}{L_{Pl}^5}. \quad (52)$$
Here, $L_{Pl} = g_s^{1/4} \sqrt{\alpha'}(2\pi)^{7/8}$ denotes the ten-dimensional (reduced) Planck-length and $g_s = e^{(\phi)}$. $K_5$ stands for those five-dimensional sections of the base-manifold $K_6$, for which $x^5$ is constant. The effect of these rescalings is a simple determination of $M$ in terms of $L_{Pl}$, which can be read off from the Einstein-Hilbert term

$$-\frac{1}{(2\pi)^7(\alpha')^4} \int d^{10}x \sqrt{G} R^\sigma = -\frac{1}{L_{Pl}^7} \int d^{10}x \sqrt{G^E} R^E = -\frac{1}{L_{Pl}^7} \int d^{10}x \sqrt{G^E} R^E ,$$

and upon dimensional reduction to five dimensions leads to the identification

$$M = \frac{1}{L_{Pl}} .$$

Utilizing (31), we end up with the following restriction on the ten-dimensional Planck-length

$$L_{Pl}^3 = \frac{2(1+kl)}{kM^2_{red}} \simeq \frac{1}{M_{GUT}M^2_{red}} = \frac{1}{(4 \times 10^{17} \text{GeV})^3} ,$$

if we choose generically $10 \lesssim kl$ and identify $M_{eff}$ with the four-dimensional reduced Planck-scale $M_{red} = M_{Pl}/\sqrt{16\pi} = 1.7 \times 10^{18} \text{GeV}$. In terms of the string-scale $M_s = 1/\sqrt{\alpha'}$ and the string-coupling $g_s$, we are led to the restriction

$$M_s = g_s^{1/4}(2\pi)^{7/8} 4 \times 10^{17} \text{GeV} .$$

The inter-wall distance $2l$ in the effective description and the length $2l_{\sigma}$ in the stringy description are related by

$$2l = V_5^{1/8} e^{-\frac{\pi}{2}2l_{\sigma}} .$$

Usually, in Calabi-Yau compactifications the compactification radius is chosen at the GUT-length $1/M_{GUT}$. Let us now see, what this assumption together with the information that $2l = 1/M_{GUT}$, amounts to for the ground state masses of the open strings. To this aim consider a U(6) oriented open string with its end points carrying the representations $6$ and $\bar{6}$. Compactification of the $X^5$ coordinate on a circle with radius $R \simeq 1/M_{GUT}$ allows for a Wilson-line around this circle generated by the gauge-field background

$$A^5 = \text{diag}(\theta_3, \theta_3, \theta_3, \theta_1, \theta_2, \theta_2)/(2\pi R) .$$

It breaks the $U(6)$ symmetry down to $U(3) \times U(2) \times U(1)$. In the T-dual picture this translates into D-branes placed at

$$\theta_3 R', \quad \theta_1 R', \quad \theta_2 R' ,$$

$$18$$
with the T-dual radius given by \( R' = \alpha' / R \). To cope with the cosmological constant problem, we have learned before that it is necessary to place the D-branes at two stacks (see fig. 3)

\[
\theta_3 R' = -l_\sigma, \quad \theta_1 R' - \theta_2 R' = l_\sigma .
\]

In the next section, we will face the problem of incorporating the massless MSSM fields into this D-brane set-up. One natural way to do this (see below) is to set the length of the interval \([-l_\sigma, l_\sigma]\) equal to the circumference \(2\pi R'\) by identifying \(-l_\sigma \sim l_\sigma\). Hence, the brane positions lie at

\[
\theta_3 = -\pi , \quad \theta_1 \simeq \theta_2 = \pi .
\]

After this spadework, let us now come to the lowest level string masses. An open string stretching from one brane-stack to the other gives rise to a ground state vector-multiplet with mass \(2l_\sigma T\), where \( T = 1/(2\pi \alpha')\) is the string-tension (and we have assumed that the two brane-stacks coincide in all other internal position coordinates except for \(X^5\)). Evaluated in the low-energy frame this amounts to a mass of

\[
M_{\text{open}} = V_5^{1/8} e^{-\frac{\phi}{R}} 2l_\sigma T = 2lT = \frac{T}{M_{\text{GUT}}} .
\]

Using the identification \(2l_\sigma = 2\pi R'\), this lowest-level mass becomes

\[
M_{\text{open}} = V_5^{1/8} e^{-\frac{\phi}{R}} \simeq V_5^{1/8} e^{-\frac{\phi}{M_{\text{GUT}}}} M_{\text{GUT}} .
\]

If we set \(e^\phi = g_s\) constant, (62) together with (63) impose a relation, which determines the string-scale in terms of \(g_s\) and the compactification parameter \(V_5\)

\[
M_s = \sqrt{\frac{2\pi V_5^{1/8}}{g_s^{1/4}}} M_{\text{GUT}} .
\]

If we furthermore assume that naturally \(\int_{K_5} d^5y \sqrt{h} = 1/M_{\text{GUT}}^5\), we obtain one further relation from (52)

\[
V_5^{1/5} = \frac{1}{g_s^{1/4} (2\pi)^{7/8}} \times \frac{M_s}{M_{\text{GUT}}} .
\]

Altogether (56),(64),(65) constitute three equations in the three unknowns \(M_s, g_s, V_5^{1/5}\) with the solution

\[
M_s = 3 \times 10^{17}\text{GeV} , \quad g_s = 7 \times 10^{-4} , \quad V_5^{1/5} = 20 .
\]
We therefore conclude that a description in terms of perturbative string-theory is adequate. Moreover in the low-energy frame the lowest level open string excitations are given by

\[ M_{\text{open}} = 40 M_{\text{GUT}}. \]  

(67)

Though our estimate relied on the assumption that \( 1/R = M_{\text{GUT}} \) and \( \int_{K_5} d^5 y \sqrt{h} = 1/M_{\text{GUT}}^3 \), it appears to be quite generic. For example directly identifying \( M_{\text{open}} \) with \( M_{\text{GUT}} \) would imply via (56) a \( g_s = 4 \times 10^{-7} \), which seems less natural than the above obtained value. In the string-frame the open string masses are of course situated precisely at the GUT-scale, since there we have \( M_{\text{open}} = 1/R = M_{\text{GUT}} \). Subsequently, when we speak of GUT-masses, we have (67) in mind.

Above the GUT-scale, one may expect the full restoration of the SU(6) gauge symmetry by moving all six branes on top of each other. However, it has to be noticed that this goes hand in hand with the creation of a huge Planck-scale cosmological constant. But, as already pointed out, for cosmological purposes this may be just fine, though usually a de Sitter inflationary expansion is considered below GUT-energies in order to dilute the density of topological defects which are relics from the spontaneous breaking of the GUT gauge-group. The precise connection between string states and massive GUT-states will be made explicit in the coming section.
U(1)-charge positive \( \rightarrow \bar{n} \) of SU(n)

U(1)-charge negative \( \rightarrow n \) of SU(n)

Figure 5: Fixing the orientation of the open strings.

6 Brane-Description and Broken SU(6) SUSY Grand Unification

6.1 String-Theory Perspective

From the above brane set-up, we immediately get the open string-states which are depicted in fig.4 together with their transformation properties under SU(3) and SU(2). Since we will identify later on the single U(1) with the \( U(1)_Y \) hypercharge, every open string describing a hypercharged state of the corresponding field theory has to connect the U(1)-brane. This is the reason why we consider in the sequel just the two types of “basic” open strings\(^{11}\) (plus their orientation-reversed partners) and not those which directly connect the SU(3) branes with the SU(2) branes\(^{12}\). Fixing the sign of the U(1)-charge normalization, we can assume that \( x, y > 0 \).

Let us now have a closer look at the orientation of the open strings starting or ending on the single U(1)-brane. Our convention is made clear in fig.5. In physical terms this orientation convention can be thought of as indicating the direction of the U(1)-flux originating from the U(1)-charge placed at one end of the open string. Let us regard the situation in which two of the strings of fig.4 meet at a common point on the U(1)-brane (see fig.6). The question arises, whether such a situation could lead to a further string state, composed out of the two “basic” ones. Let us therefore conceive a situation where

\(^{11}\)Note that for SU(2) the fundamental 2 is pseudoreal.

\(^{12}\)It is however also possible to extract the \( U(1)_Y \) as a linear combination of the three U(1)-factors of the D-branes [34],[35]. Indeed this is the only choice if one assigns only \( \pm 1 \) values to the open strings beginning or ending on the single U(1)-brane. In contrast to this, we will not impose in this paper this restriction and allow \textit{a priori} for arbitrary values.
an open string with weighted U(1)-charge $u$ and another with weighted U(1)-charge $v$ meet. By weighted we mean that the individual open string U(1)-charges are multiplied by the multiplicity originating from the Chan-Paton label of the other string end. If we assume that the charges do not cancel, $u + v \neq 0$, then the junction bears a net U(1)-charge. If it is positive, then the orientation (U(1)-flux) of the two individual strings must point away from the contact point. If the net charge is negative, then the orientation (U(1)-flux) of both strings must be such that they point towards the junction. Hence, under the condition that the weighted sum of their U(1)-charges does not cancel, two “basic” strings can compose a longer string only if the orientation of the two individual strings is opposite. This happens to be the case for the junctions of fig.6a, where two long strings with quantum numbers

\[
(3, 1)_x \times (1, 2)_y \rightarrow (3, 2)_{-x+y}
\]

are composed. On the other hand the orientations of the strings in fig.6b are not compatible with a non-zero total weighted U(1)-charge at the meeting point. Therefore, to avoid an inconsistency, when the states of fig.6b are composed, we have to demand that the total weighted U(1)-charge of such a junction has to vanish. From either of the two situations of fig.6b, we then get the requirement

\[
3(-x) + 2y = 0.
\]

If we choose a normalization of the U(1)-generator such that $x = 2$, we obtain $y = 3$. Thus, it is the number of D3-branes present in each stack (except for the U(1)-brane itself), which causes the value of the U(1)-charges. Together with the composition rule,
Figure 7: Massless and heavy gauge-bosons which arise from “basic” strings.

this will eventually determine all the U(1)-charges (which will be identified with the SM-hypercharge below) of open string states, which correspond to SM fields.

The NS-sector of the open DD-strings contains the bosonic four-dimensional gauge-fields $A^a_{\mu b_{\mu/2}}[k_4; i, j]$ and scalars $A^a_{m b_{m/2}}[k_4; i, j]; m = 5, \ldots, 10$ ($i$ and $j$ represent the Chan-Paton labels), whose momenta $k_4$ are restricted along the four longitudinal D3-brane directions due to the DD boundary-conditions. Those open strings, which start and end on the same brane-stack (see fig.7), lead to three massless gauge-boson states

$$(8, 1)_0, (1, 3)_0, (1, 1)_0,$$  \hspace{1cm} (70)

while the “basic” open strings, which directly connect one brane with another, give rise to two states with mass at the GUT scale

$$(3, 1)_{-2}, (3, 1)_2$$  \hspace{1cm} (71)

plus two further states with mass at the TeV scale

$$(1, 2)_3, (1, 2)_{-3}.$$  \hspace{1cm} (72)

Note the natural occurrence of the mass split between the GUT and the TeV scale between triplet and doublet states. The composition of the “basic” strings (see fig.8) adds another four “composed” states with mass at the GUT scale

$$(3, 1)_{-2} \otimes (1, 2)_3 \rightarrow (3, 2)_1,$$  \hspace{1cm} (3, 1)_{-2} \otimes (1, 2)_{-3} \rightarrow (3, 2)_{-5},$$  \hspace{1cm} (73)

$$(3, 1)_2 \otimes (1, 2)_3 \rightarrow (3, 2)_5,$$  \hspace{1cm} (3, 1)_2 \otimes (1, 2)_{-3} \rightarrow (3, 2)_{-1}.$$  \hspace{1cm} (74)
It remains to incorporate in a natural way a sector of nearly massless matter states, which can account for the MSSM fields with mass at the TeV scale. The most immediate way to introduce light matter would be to have some kind of tensionless string stretching between the brane stacks or some kind of “bridge” enabling open strings stretching along it to stay massless. However, for lack of those gadgets, we will simply assume $X^5$ to be compactified on a circle in such a way that the locations of the two brane-stacks are identified as (nearly) equivalent points $-l_{\sigma} \sim l_{\sigma}$ (see fig.9). The heavy GUT excitations found above now originate from open strings stretching once around the whole circle. In addition, the light MSSM matter-supermultiplets stem from open strings, which connect the brane-stacks over the small separating distance. Likewise as before, we will now build the massless fields out of two sorts of “basic” strings, as depicted in fig.10. The $Q, \bar{U}, \bar{E}$ chiral superfields, which are later on identified with the respective MSSM matter-fields, are composed as follows

$$Q = (3, 2)_1 = (3, 1)_{-2} \otimes (1, 2)_3 \quad \text{(75)}$$

$$\bar{U} = (\bar{3}, 1)_{-4} \subset (3, 1)_{-2} \otimes (3, 1)_{-2} \quad \text{(76)}$$

$$\bar{E} = (1, 1)_6 \subset (1, 2)_3 \otimes (1, 2)_3 \quad \text{(77)}$$

where we used $3 \otimes 3 = \bar{3} + 6$ and $2 \otimes 2 = 1 + 3$ in the last two cases and picked out the antisymmetric part while dismissing the symmetric one. Note, that the only composition of open strings, which could lead to an inconsistency in orientation with respect to the
Figure 9: Light MSSM and heavy GUT fields as open string excitations which arise from a configuration with compact $X^5$.

combined weighted U(1)-charge, is the one for $Q$. However, to avoid this, required the particular assignment of U(1)-charges for the “basic” strings as we discussed before.

Since we have now found the fields which will be identified with the MSSM matter, let us now discuss the number of generations by concentrating on the fermions in the chiral superfields. The fermions originate from the R-sector of the open DD-strings and in ten dimensions would be given by the Majorana-Weyl spinor $u^{ij}_\alpha(k_4;\alpha; k_4; i, j)$. Here, $\alpha = 1, \ldots, 8$ is a spinor-index running over the physical (on-shell) degrees of freedom and again $i, j$ are the Chan-Paton labels. By a dimensional reduction to four dimensions, $u$ turns into 4 two-component Weyl-spinors $\lambda^i_A; A = 1, \ldots, 4$. In an $\mathcal{N} = 1$ description in four dimensions $\lambda^i_A$ gets combined with the NS-sector gauge-field $A^{ij}_\alpha$ into a vector-superfield, while the remaining 3 spinors $\lambda^i_1, \lambda^i_2, \lambda^i_3$ are each paired with two NS-sector scalars $(A^{ij}_5, A^{ij}_8), (A^{ij}_6, A^{ij}_9), (A^{ij}_7, A^{ij}_{10})$ to build 3 chiral superfields. Hence it is generic to arrive at a multiplicity of 3 for the chiral matter fermions or in other words at a 3 generation model. The basic reason being that we happen to live in a world with 6 internal dimensions. The important task, however, is to lift the mass degeneracy between them or equivalently reducing the $D = 4, \mathcal{N} = 4$ extended supersymmetry of the above D3-brane configuration in type IIB string-theory to a minimal $D = 4, \mathcal{N} = 1$ supersymmetry.

For example, consider a complex description of the internal coordinates in terms of $Z_{58}, Z_{69}, Z_{710}$, where $Z_{ij} = X_i + iX_j$. They transform under an SU(3) subgroup of the SO(6) internal tangent space group. Since the Lie-algebra of SO(6) is isomorphic to that
of SU(4), the (say) positive-chirality spinor of SO(6) transforms as the fundamental 4 of SU(4). Its decomposition under the above SU(3) is $4 = 3 + 1$, which corresponds to the split

$$
\begin{pmatrix}
\lambda^1_{ij} \\
\lambda^2_{ij} \\
\lambda^3_{ij} \\
\lambda^4_{ij}
\end{pmatrix} =
\begin{pmatrix}
\lambda^1_{ij} \\
\lambda^2_{ij} \\
\lambda^3_{ij} \\
0
\end{pmatrix}
+ \begin{pmatrix}
0 \\
0 \\
0 \\
\lambda^4_{ij}
\end{pmatrix}
.$$  

(78)

In order to break the degeneracy between the 3 chiral matter fermions one could e.g. introduce 3 further D7-branes along the directions 12345689, 123457810, 123467910. Together, they preserve 1/8 of the initial 32 supercharges, which amounts to an $\mathcal{N} = 1$ supersymmetric theory in four dimensions. The conditions which are imposed by the presence of the D7-branes on the supersymmetry parameters $\epsilon_L, \epsilon_R$ (which are 16-component Majorana-Weyl spinors of the same chirality for IIB) are

$$
\epsilon_L = \Gamma_{D3}\Gamma_{R_1}\epsilon_R; \quad \Gamma_{R_1} = \Gamma^5\Gamma^6\Gamma^8\Gamma^9, \quad \Gamma_{D3} = \Gamma^4\Gamma^2\Gamma^3\Gamma^4
$$  

(79)

$$
\epsilon_L = \Gamma_{D3}\Gamma_{R_2}\epsilon_R; \quad \Gamma_{R_2} = \Gamma^5\Gamma^7\Gamma^8\Gamma^{10}
$$  

(80)

$$
\epsilon_L = \Gamma_{D3}\Gamma_{R_3}\epsilon_R; \quad \Gamma_{R_3} = \Gamma^6\Gamma^7\Gamma^9\Gamma^{10}
$$  

(81)

Taken together, they imply

$$
\epsilon_L = \Gamma_{D3}\epsilon_R.
$$  

(82)
Therefore, we can place D3-branes at the common four-dimensional intersection of the three D7-branes without breaking further supersymmetry. Moreover, the introduction of the D7-branes entails a further reduction of the SO(6) tangent space group to

$$SO(6) \supset SU(3) \supset SO(2) \times SO(2) \times SO(2) = U(1) \times U(1) \times U(1).$$  \hspace{1cm} (83)

The fermion triplet, which we had under SU(3), now gets split into

$$
\begin{pmatrix}
\lambda_{ij}^1 \\
0 \\
0
\end{pmatrix},
\begin{pmatrix}
0 \\
\lambda_{ij}^2 \\
0
\end{pmatrix},
\begin{pmatrix}
0 \\
0 \\
\lambda_{ij}^3
\end{pmatrix},
$$

since each one transforms with a phase-factor under a different U(1). Thus the degeneracy between them can be naturally lifted and different \(N = 1\) masses attributed to them. The light vector of the vector-superfield \((\lambda_{ij}^d, A_{\mu}^i)\) however, does not appear at low-energies in a realistic theory. Hence, ultimately it should be projected out on the basis of a further discrete symmetry of the model.

### 6.2 Field-Theory Perspective

Let us determine to what SUSY GUT the string states found above belong to. Since in the limit of vanishing D3-brane separations, we would recover an SU(6) gauge theory, it is natural to compare with the spectrum of an SU(6) SUSY GUT [32], whose gauge-group is spontaneously broken down to the SM SU(3) \(\times\) SU(2) \(\times\) U(1)_{Y}. Besides a gauge-field transforming in the 35 adjoint representation of SU(6), its matter content comprises the following chiral supermultiplets [32]

- Higgs-fields: \(\Sigma = 35, \ H = 6, \ \bar{H} = \bar{6}, \ Y = 1\)
- Fermionic Matter: \(\psi_f^I = 15, \ \bar{\psi}_f^I = \bar{6}, \ \bar{\psi}_f^2 = \bar{\eta}, \ \psi_2 = 15, \ \bar{\psi} = \bar{15}, \ \eta = 20\)

where \(f = 1, 2, 3\) is a family index. For a comparison to the afore obtained string spectrum it is necessary to decompose the fields first under SU(5) and afterwards under SU(3) \(\times\)
SU(2) \times U(1)_Y. Under SU(5) we have

\[ 35 = 1 + 5 + 5 + 24 \]
\[ 20 = 10 + 10 \]
\[ 15 = 10 + 5 \]
\[ 6 = 5 + 1 . \]

The fundamental 5, the antisymmetric tensor rep. 10 and adjoint 24 of SU(5) themselves decompose under SU(3) \times SU(2) \times U(1)_Y as follows

\[ 24 = (1,1)_0 + (1,3)_0 + (3,2)_{-5} + (\bar{3},2)_5 + (8,1)_0 \]
\[ 10 = (1,1)_6 + (\bar{3},1)_{-4} + (3,2)_1 \]
\[ 5 = (1,2)_3 + (3,1)_{-2} . \]

In the string description we found for the external \( \mu \) components of the NS-sector the massless \((8,1)_0, (1,3)_0, (1,1)_0\) states, which represent the familiar Standard Model gluons and the four electroweak gauge-bosons \( W_1, W_2, W_3, B \), which become upon spontaneous symmetry breaking the physical \( W^+, W^-, Z^0 \) and the photon \( A \). The heavy \((3,2)_{-5}\) and \((\bar{3},2)_5\) describe the twelve X- and Y-leptoquark gauge-bosons of broken SU(5) and were constructed as stringy “composites”. Together with the “basic” string states \((3,1)_{-2}, (\bar{3},1)_2, (1,2)_3, (1,2)_{-3}\) these states furnish the broken 35 of the SU(6) gauge-fields. Similarly, the internal \( m \) components of the NS-sector deliver the Higgs-adjoint \( \Sigma \). We already noted above that the Higgs-triplets naturally take masses at the GUT-scale due to the geometry of the brane set-up, whereas the two Higgs-doublets of the MSSM, which belong to the chiral superfields

\[
\text{Higgs-Bosons + Higgsinos: } \quad \tilde{H}_1 = (1,2)_{-3} \quad \tilde{H}_2 = (1,2)_{3}
\]

acquire a small mass. This mass, which should be at the TeV scale, is caused by the little split between the U(1)- and the SU(2)-branes in the second brane-stack. Therefore, the brane configuration, which resulted from the requirement of a small four-dimensional cosmological constant \( \Lambda_4 \), also allows for a natural understanding of the doublet-triplet splitting in spontaneously broken GUT-theories.

In the same vein the Higgs-fundamentals \( H, \tilde{H} \) and the fermionic SU(6) matter \( \tilde{\phi}^1_f, \tilde{\phi}^2_f \) can be built solely out of the two “basic” open strings. The matter fermions \( \psi^1_f, \psi^2, \tilde{\psi}, \eta \) contain the \( 5, \bar{5}, 10, \bar{10} \). Whereas the \( 5, \bar{5} \) are built out of the “basic” strings, the \( 10, \bar{10} \)
are exclusively “composite” string states. The chiral superfields representing the matter-content of the MSSM

\[
\begin{align*}
\text{Squarks + Quarks:} & \quad Q = (3, 2)_1 \quad \bar{U} = (\bar{3}, 1)_{-4} \quad \bar{D} = (\bar{3}, 1)_2 \\
\text{Sleptons + Leptons:} & \quad L = (1, 2)_{-3} \quad \bar{E} = (1, 1)_6
\end{align*}
\]

fill up the the $\bar{5}$ and $10$ of SU(5). Their SU(6) origin is from $\bar{3}^1, \bar{3}^2, \bar{3}^3, \psi_1, \psi_2, \eta$ [32]. We now discern, that the abelian charge which arose from coupling of open strings to the single U(1)-brane, indeed has to be identified with the SM hypercharge $Y$.

In general, to build bosonic or fermionic matter with GUT-mass, we have to use in the string description the heavy “basic” $(3, 1)_{-2}, (\bar{3}, 1)_2$ states which wind from the SU(3)-brane stack to the U(1)-brane around the circle. For light states instead, we use the “basic” $(3, 1)_{-2}, (\bar{3}, 1)_2$ states which connect the same brane-stacks but this time via the small gap in fig.9. In order to secure a small cosmological constant $\Lambda$, the stringy description generically predicts light mass for all doublets $(1, 2)_3$ or composites thereof like $(1, 1)_6$. However, in some cases (e.g. $\psi_2, \bar{\psi}$) these states also have to become heavy to decouple from the light spectrum. This can be achieved by coupling these states in the superpotential to other generically heavy states.

We want to conclude with two remarks. First, it is of interest to explore the low-energy value for $\sin^2 \Theta_W$. On a stack of $N$ D3-branes the effective gauge coupling is given by $g_{\text{eff}}^2 = g_s N$ [33]. Therefore, if we plug $g_2^2 = 2g_s, g_Y^2 = g_s$ into

\[
\sin^2 \Theta_W(M_s) = \frac{g_Y^2}{g_s^2 + g_Y^2} = \frac{1}{3} ,
\]

we arrive at a value, which is close to the traditional GUT value $\frac{3}{8}$. Using the representation $\frac{1}{\alpha} = \frac{1}{\alpha_Y} + \frac{3}{\alpha_2}$ for the electromagnetic fine-structure “constant” plus $\sin^2 \Theta_W = \frac{\alpha}{\alpha_2}$, we can evaluate the difference $\frac{1}{\alpha(Y)} - \frac{3}{\alpha_2(M_Z)}$ as $(1 - 3\sin^2 \Theta_W(M_Z))\frac{1}{\alpha(M_Z)}$. Alternatively, using the 1-loop running of the gauge-couplings

\[
\frac{1}{\alpha_i(E)} = \frac{1}{\alpha_i(M_Z)} - \frac{b_i}{2\pi} \ln \left( \frac{E}{M_Z} \right) ,
\]

plus the condition that $\frac{1}{\alpha_Y} = \frac{2}{\alpha_2}$ at the string-scale $M_s$, the difference can be evaluated as $\frac{1}{2\pi}(b_Y - 2b_2)\ln(M_s/M_Z)$. Combining both expressions, we obtain the 1-loop running of the Weinberg-angle

\[
\sin^2 \Theta_W(M_Z) = \frac{1}{3} + \frac{\alpha(M_Z)}{6\pi} (2b_2 - b_Y) \ln \left( \frac{M_s}{M_Z} \right) .
\]
Moreover, let us assume that the MSSM is valid from the weak scale $M_Z$ up to the string-scale $M_s \simeq 40M_{\text{GUT}}$. This assumption selects the $\beta$-function one-loop coefficients of the MSSM

$$b_Y = 11, \quad b_2 = 1$$

for the whole energy-region. Together with the experimental value $\frac{1}{\alpha(M_Z)} = 127.9$ this leads to a value of $\sin^2 \Theta_W(M_Z) = 0.196$, which is too low compared with the measured value $\sin^2 \Theta_W(M_Z) = 0.231$. Hopefully, this result could be corrected towards the right direction, if we abandon the idea of a universal validity of the MSSM up to the string-scale. Instead, it would be natural to allow for some intermediate scale $M_I \simeq \sqrt{1\,\text{TeV}} \times M_s \simeq 3 \times 10^{10}\text{GeV}$, since we saw above that some light doublets should couple to $M_s$-massive states and thereby acquire mass of order $M_I$.

Second, in order to embrace an $\mathcal{N} = 1$ supersymmetric four-dimensional theory at low-energies, we did not aim at breaking all supersymmetry already at the string-scale. This will probably be only reasonable if one lowers the string-scale to the TeV-scale, which we did not intend. Therefore, we left open the precise mechanism for low-energy supersymmetry breaking. Usually the question of obtaining a realistic mechanism for supersymmetry breaking struggles with the fact that all known mechanisms produce a large cosmological constant. In this respect, we hope that the above described scenario will likewise suppress these contributions, such that it will be eventually possible to apply one of the various mechanisms to actually determine the soft-breaking terms of the complete MSSM.

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Figure 11: The function $A(x^5)$, which determines the warp-factor.

A The Warped Geometry and the Effective D=4 Action for Unequal Wall Tensions

In this appendix, we will deal with the case of unequal wall tensions $T_1 \neq T_2$. The Ansatz (4) then yields the solution

$$A(x^5) = \frac{k_1}{2} |x^5 + l| + \frac{k_2}{2} |x^5 - l| = \begin{cases} 
  x^5 \geq l : & \frac{1}{2} K_{12} x^5 + \frac{1}{2} k_{12} l \\
  -l \leq x^5 \leq l : & \frac{1}{2} k_{12} x^5 + \frac{1}{2} K_{12} l \\
  x^5 \leq -l : & -\frac{1}{2} K_{12} x^5 - \frac{1}{2} k_{12} l 
\end{cases}, \quad (89)$$

where $K_{12} = k_1 + k_2$ and $k_{12} = k_1 - k_2$. Without loss of generality, we will assume that $k_1 \geq k_2$ subsequently. The function $A(x^5)$, which determines the warp-factor is displayed in fig.11. The corresponding warp-factor $e^{-A(x^5)}$ is upper-bounded by $e^{-k_2 l}$ throughout the whole fifth dimension. From the Einstein equation (5) we receive the expressions for $\Lambda$ and the wall tensions

$$\Lambda(x^5) = \begin{cases} 
  \Lambda_e, & |x^5| \geq l \\
  \Lambda_i, & |x^5| < l 
\end{cases} = -\frac{3 M^3}{4} \begin{cases} 
  K_{12}^2, & |x^5| \geq l \\
  k_{12}^2, & |x^5| < l 
\end{cases}, \quad (90)$$

$$T_1 = 3 M^3 k_1, \quad T_2 = 3 M^3 k_2. \quad (91)$$
The next task is again the determination of the effective four-dimensional action. Along the same lines as above by employing (13), we get for the Einstein-Hilbert term

$$S_{EH} = - \int d^4x \sqrt{g} M^3 \left\{ R(g) \int_0^\infty dx^5 e^{-A} + \int_0^\infty dx^5 e^{-2A} \left[ 5(A')^2 - 4A'' \right] \right\}$$

$$= -e^{-K_{12l}/2} \int d^4x \sqrt{g} M^3 \left\{ 4R(g) \left[ \frac{1}{K_{12l}} \cosh \left( \frac{k_{12l}}{2} \right) + \frac{1}{k_{12l}} \sinh \left( \frac{k_{12l}}{2} \right) \right] 
+ \frac{5}{4} e^{-K_{12l}/2} \left[ 2K_{12l} \cosh (k_{12l}) + 2k_{12l} \sinh (k_{12l}) \right] 
- 4e^{-K_{12l}/2} \left[ k_1e^{k_{12l}} + k_2e^{-k_{12l}} \right] \right\} . \tag{92}$$

For the remaining wall- and bulk cosmological constant terms we obtain

$$S_{SM_1} + S_{SM_2} + S_\Lambda = -e^{-K_{12l}} \int d^4x \sqrt{g} \left\{ e^{k_{12l}T_1} + e^{-k_{12l}T_2} 
+ 2 \frac{\Lambda_e}{K_{12l}} \cosh(k_{12l}) + 2 \frac{\Lambda_i}{k_{12l}} \sinh(k_{12l}) \right\} . \tag{93}$$

Pulling out an overall factor of $e^{-K_{12l}/2}$ in front, the final effective action reads

$$S_{EH} + S_{SM_1} + S_{SM_2} + S_\Lambda$$

$$= -e^{-K_{12l}/2} \int d^4x \sqrt{g} \left\{ 4M^3 R(g) \left[ \frac{1}{K_{12l}} \cosh \left( \frac{k_{12l}}{2} \right) + \frac{1}{k_{12l}} \sinh \left( \frac{k_{12l}}{2} \right) \right] + \frac{5}{2} M^3 e^{-K_{12l}/2} \left[ K_{12l} \cosh (k_{12l}) + k_{12l} \sinh (k_{12l}) \right] + e^{-K_{12l}/2} \left[ e^{k_{12l}T_1} - 4k_1 M^3 \right] 
+ e^{-k_{12l}T_2} - 4k_2 M^3 + 2 \frac{\Lambda_e}{K_{12l}} \cosh(k_{12l}) + 2 \frac{\Lambda_i}{k_{12l}} \sinh(k_{12l}) \right\} . \tag{94}$$

At the classical level the normalization of the action is irrelevant. Let us therefore by the same reasoning as in the main text drop the overall scale-factor and arrive at the effective action

$$S_{D=4} = - \int d^4x \sqrt{g} \left\{ M_{\text{eff}}^2 R(g) + \Lambda_4 \right\} , \tag{95}$$

with the effective four-dimensional Planck-scale $M_{\text{eff}}$ and the four-dimensional cosmological constant $\Lambda_4$ now given by

$$M_{\text{eff}}^2 = 4M^3 \left[ \frac{1}{K_{12l}} \cosh \left( \frac{k_{12l}}{2} \right) + \frac{1}{k_{12l}} \sinh \left( \frac{k_{12l}}{2} \right) \right] \tag{96}$$

$$\Lambda_4 = e^{-K_{12l}/2} \left( \frac{5}{2} M^3 \left[ K_{12l} \cosh (k_{12l}) + k_{12l} \sinh (k_{12l}) \right] + e^{k_{12l}T_1} - 4k_1 M^3 \right) 
+ e^{-k_{12l}T_2} - 4k_2 M^3 + 2 \frac{\Lambda_e}{K_{12l}} \cosh(k_{12l}) + 2 \frac{\Lambda_i}{k_{12l}} \sinh(k_{12l}) \right\} . \tag{97}$$

32
Again, there exists a huge suppression-factor $e^{-K_{12}/2}$ multiplying the whole cosmological constant, which serves to bring $\Lambda_4$ down to its observed upper bound if generically $k_1, k_2 \simeq M_{Pl}$. When the above obtained values (90),(91) for $T_1, T_2, \Lambda_e, \Lambda_i$ are substituted in the obtained action, we arrive at a vanishing $\Lambda_4$, which checks the derivation of the action, since in that special case the fine-tuning of the parameters requires a flat four-dimensional metric $g_{\mu\nu} = \eta_{\mu\nu}$. For the particular case of coinciding wall-tensions, $T_1 = T_2 = T$ (which entails $k_1 = k_2 = k$), we arrive at the effective action given by (29),(31),(34), which was discussed in the main text.

Again, let us now lift the fine-tuning of the parameters imposed by $k_{12} = 2M_{GUT}$. For the particular case of coinciding wall-tensions, $T_1 = T_2 = T$ (which entails $k_1 = k_2 = k$), we arrive at the effective action given by (29),(31),(34), which was discussed in the main text.

$$\Lambda(x^5) = \begin{cases} \Lambda_e, & |x^5| \geq l \\ \Lambda_i, & |x^5| < l \end{cases} = -\frac{1}{12M^2} \left( \frac{(T_1 + T_2)^2}{(T_1 - T_2)^2}, \frac{|x^5| \geq l}{|x^5| < l} \right),$$

(98)

which corresponds to a non-trivial four-dimensional metric $g_{\mu\nu} \neq \eta_{\mu\nu}$ in the Ansatz

$$ds^2 = e^{-\Lambda(x^5)}g_{\mu\nu}dx^\mu dx^\nu + (dx^5)^2.$$  

(99)

From (97) it is evident, that in order to arrive at a small $\Lambda_4$, we require

$$k_1 - k_2 \equiv k_{12} \lesssim \frac{1}{l} = 2M_{GUT}.$$  

(100)

Generically, we choose $k_1, k_2 \simeq M_{Pl}$, $T_1, T_2 \simeq M_{Pl}^4$, $\Lambda_e \simeq M_{Pl}^5$ and the fundamental five dimensional Planck-scale $M \simeq M_{Pl}$. Again, as explained in the main text, $\Lambda_i$ has to be chosen with an upper bound of $(3 \times 10^{18}\text{GeV})^5$, which roughly corresponds to the traditional string-scale. Then, we recognize from (32), that the suppression through the exponential factor is just sufficient to bring the contributions to the four-dimensional cosmological constant down to its observed value. The effective four-dimensional Planck-scale $M_{eff} \simeq 24M_{Pl}$ again comes out slightly too high. It can however be brought down, e.g. to $M \simeq M_{Pl}$, if we choose $M \simeq 1.5 \cdot 10^{18}\text{GeV}$, close to the traditional string-scale.

### B The Effective Potential for Bulk Scalars in the Case of Unequal Wall-Tensions

Here, we want to extend the analysis of a bulk scalar contribution to the effective potential ($= \Lambda_4$) to the case of unequal wall tensions. For the action of the scalar $\Phi$ with mass $m,$
\[ S_\Phi = - \int d^4x \int_{-\infty}^{\infty} dx^5 \sqrt{G} \left\{ \frac{1}{2} G^{MN} \partial_M \Phi \partial_N \Phi + \frac{1}{2} m^2 \Phi^2 \right\} \\
- \int d^4x \int_{-\infty}^{\infty} dx^5 \left\{ \sqrt{g_1^{(4)}} \lambda_1 (\Phi^2 - v_1^2) \delta(x^5 + l) + \sqrt{g_2^{(4)}} \lambda_2 (\Phi^2 - v_2^2) \delta(x^5 - l) \right\}. \]

Assuming only an \( x^5 \) dependence of \( \Phi \), we arrive at the field equation

\[ (e^{-2A} \Phi')' - e^{-2A} m^2 \Phi = 4 \left[ e^{-2A(-l)} \lambda_1 (\Phi^2 - v_1^2) \Phi \delta(x^5 + l) \right. \\
+ \left. e^{-2A(l)} \lambda_2 (\Phi^2 - v_2^2) \Phi \delta(x^5 - l) \right], \]

which, away from the walls, has the solution

\[ \Phi(x^5) = \begin{cases} 
  a e^{(1+\Gamma)A} + b e^{(1-\Gamma)A}, & x^5 < -l \\
  c e^{(1+\gamma)A} + d e^{(1-\gamma)A}, & |x^5| \leq l \\
  e e^{(1+\Gamma)A} + f e^{(1-\Gamma)A}, & x^5 > l 
\end{cases} \]

with

\[ a = \Gamma = \sqrt{1 + 4m^2/K_{12}^2}, \quad \gamma = \sqrt{1 + 4m^2/k_{12}^2}. \]

In order to obtain a normalizable solution for \( \Phi \), we set the coefficients \( a = e = 0 \). Moreover, imposing continuity of \( \Phi \) at the walls determines \( b \) and \( f \) in terms of \( c, d \)

\[ b = e^{ik_2l} \tilde{b}, \quad \tilde{b} = ce^{\gamma k_2l} + de^{-\gamma k_2l} \]

\[ f = e^{ik_1l} \tilde{f}, \quad \tilde{f} = ce^{\gamma k_1l} + de^{-\gamma k_1l}. \]

To fix the remaining coefficients \( c \) and \( d \) one would have to plug the above bulk solution in the field equation and integrate out the fifth dimension to incorporate the wall boundary conditions. Since this leads to a complicated cubic equation in the unknowns \( c, d \), it is easier to determine them by inserting the bulk solution into the scalar action and integrating over \( x^5 \) to arrive at an effective potential for the wall-distance \( l \). For positive couplings \( \lambda_1, \lambda_2 \) this effective potential will be positive definite. Hence, to minimize the potential, we must have \( \Phi(-l) = v_1 \) and \( \Phi(l) = v_2 \). This allows for a determination of \( c, d \) in terms of the parameters \( v_1, v_2 \)

\[ c = v_2 e^{-(1-\gamma)k_1l} - v_1 e^{-(1-\gamma)k_2l} \]

\[ d = v_2 e^{-(1+\gamma)k_1l} - v_1 e^{-(1+\gamma)k_2l} \]
The effective potential\textsuperscript{13} eventually becomes

\begin{equation}
V_b(l) = \frac{k_{12}}{2} \sinh(\gamma k_{12} l) \left[ c^2(\gamma + 1)e^{\gamma K_{12} l} + d^2(\gamma - 1)e^{-\gamma K_{12} l} \right] \\
+ \frac{(\Gamma - 1)K_{12}}{4} [\vec{b}^2 + \vec{f}^2].
\end{equation}

A numerical analysis of this potential shows, that also in the case with differing tensions a bulk scalar, with couplings to the walls, leads generically to an effective potential, which is likewise sufficiently suppressed. Therefore, it does not generate a huge four-dimensional cosmological constant, which could have been spoiled the embedding of the mechanism into string-theory.

References


\textsuperscript{13}Here we use the relations \((1 \pm \gamma)^2 M_{\text{Pl}}^2 + m^2 = \gamma(\gamma \pm 1) M_{\text{Pl}}^2\) and \((1 - \gamma^2) M_{\text{Pl}}^2 + m^2 = 0\).


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